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# Graphic Statics and Bridge Trusses

174 Illustrations

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GRAPHIC STATICS
STRESSES IN BRIDGE TRUSSES

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# PREFACE

The volumes of the International Library of Technology are made up of Instruction Papers, or Sections, comprising the various courses of instruction for students of the International Correspondence Schools. The original manuscripts are prepared by persons thoroughly qualified both technically and by experience to write with authority, and in many cases they are regularly employed elsewhere in practical work as experts. The manuscripts are then carefully edited to make them suitable for correspondence instruction. The Instruction Papers are written clearly and in the simplest language possible, so as to make them readily understood by all students. Necessary technical expressions are clearly explained when introduced.

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In the table of contents that immediately follows are given the titles of the Sections included in this volume, and under each title are listed the main topics discussed.

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§ 67

## STRESSES IN BRIDGE TRUSSES
(*Continued*)

## STRESSES IN BRIDGE TRUSSES
(*Continued*)

# GRAPHIC STATICS

## COPLANAR CONCURRENT FORCES

**1. The Graphic and the Analytic Method Compared.**—The graphic method of dealing with problems in statics, although not so accurate as the analytic, is in a great many cases much simpler than the latter and gives results that are sufficiently close for all engineering purposes. It is not, however, to be supposed that the graphic method is preferable in all cases; for the constructions it requires are sometimes of exceeding complexity compared with the simple formulas employed by the analytic method. Moreover, the latter method is broader and better adapted to discussions and investigations of a general character.

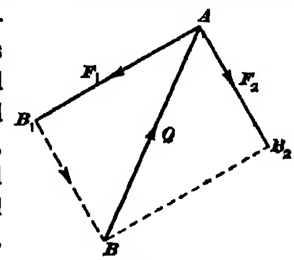


FIG 1

As an illustration, let it be required to find the equilibrant of two forces $F_1$ and $F_2$, Fig 1, acting at a point $A$ in given directions. From $A$ (or from any other point) two lines are drawn parallel, respectively, to the given directions of the forces. On these lines, and in the proper directions, are laid off the vectors $A B_1$ and $A B_2$, to represent $F_1$ and $F_2$, using any convenient scale. On the lines $A B_1$ and $A B_2$ the parallelogram $A B_1 B B_2$ is constructed, whose diagonal $B A$ is the required equilibrant. Its magnitude is determined by measuring the line $B A$ and multiplying its length by the scale used Thus, if $F_1$ and $F_2$ are laid off to a scale of 100 pounds to the inch, and $B A$ measures $3\frac{7}{16}$ inches, the magnitude of the equilibrant $Q$ is

$$3\tfrac{7}{16} \times 100 = 344 \text{ pounds, nearly.}$$

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Instead of constructing the whole parallelogram, the triangle $A B_1 B$ may be constructed, by drawing $B_1 B$ equal and parallel to $F_2$, and then drawing $B A$.

Here the graphic solution is evidently simpler and can be accomplished much more rapidly than the analytic solution.

**2.** Suppose, now, that a weight of 1,000 pounds, Fig. 2, is placed on a beam $A_1 A_2$, at distances of 9 and 3 feet from the supports $A_1$ and $A_2$, and that it is required to find the reactions at those points. Graphically, the problem may be solved as follows: From $A_1$, draw $A_1 W'$ to represent the weight of 1,000 pounds to any convenient scale; draw $W' A_2$, the vertical $A B_1$, and the horizontal $B_1 B_2$. Then will $A B_1$

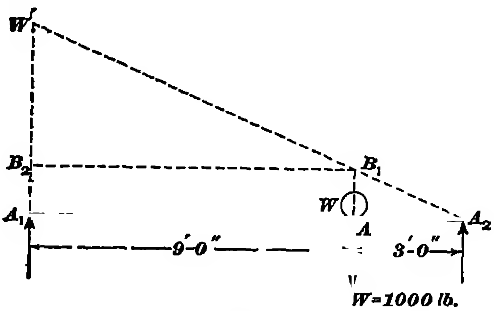


FIG. 2

(or $A_1 B_2$) represent the reaction $R_1$ at $A_1$, and $B_2 W'$ the reaction $R_2$ at $A_2$. For, if moments are taken about $A_2$, we must have,

$$R_1 \times A_1 A_2 = W \times A A_2, \text{ or } \frac{A_1 A_2}{A A_2} = \frac{W}{R_1}$$

Now, the similar triangles $A_1 A_2 W'$ and $A A_2 B_1$ give

$$\frac{A_1 A_2}{A A_2} = \frac{A_1 W'}{A B_1}$$

Comparing this with the preceding proportion and noticing that $A_1 W' = W$, it follows that $A B_1 = R_1$ We must have, also, $R_2 = W - R_1$. The figure gives,

$$B_2 W' = A_1 W' - A_1 B_2 = A_1 W' - A B_1 = W - R_1 = R_2$$

In this case, the geometric construction, although simple, can be advantageously dispensed with, as the reactions can be much more readily calculated from the general equation

of equilibrium referred to above, viz., $12 R_1 = 3\ W$; whence, $R_1 = \frac{1}{4} W = 250$ pounds, and $R_2 = W - R_1 = 1{,}000 - 250 = 750$ pounds.

This example shows that the graphic method is not always the simpler, and that some judgment should be exercised in the choice of the method to be employed in any particular case. There are geometric constructions by which centers of gravity, statical moments, bending moments, and shearing stresses (terms used in the mechanics of materials) can be determined. But, unless these constructions lead to the required results more easily and quickly than the formulas employed by the analytic method, the formulas are to be preferred.

**3. Graphic Determination of the Resultant and Equilibrant of Any Number of Concurrent Forces.**

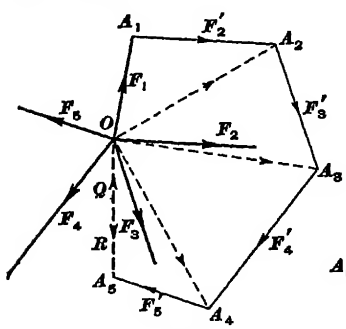


FIG 3

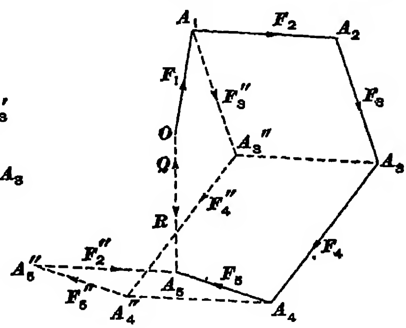


FIG 4

The construction to be presently described has already been explained, but is repeated here in a slightly different form for the sake of convenience. Let $F_1, F_2, F_3, F_4, F_5$, Fig. 3, be coplanar concurrent forces acting at $O$ If, from the end $A_1$ of the vector $O A_1$ representing the force $F_1$, the vector $A_1 A_2$, equal and parallel to $F_2$, is drawn, the vector $O A_2$ will represent the resultant of $F_1$ and $F_2$. These two forces can, therefore, be replaced by the single force $O A_2$. Likewise, by drawing $A_2 A_3$ equal and parallel to $F_3$, and joining $O$ to $A_3$, the resultant $O A_3$ of $O A_2$ and $F_3$, or, what is the same thing, of $F_1, F_2$, and $F_3$, is obtained Similarly, $O A_4$ is the resultant

of $OA_3$ and $F_4$, or of $F_1, F_2, F_3$, and $F_4$; and, finally, $OA_5$, obtained in a similar manner, is the resultant of $OA_4$ and $F_5$ —that is, the resultant $R$ of the five given forces. The same vector, taken in the opposite direction, as shown by the arrow $Q$, evidently represents the equilibrant of the same given forces.

Noting that the sides $F_1$, $F_2'$, $F_3'$, $F_4'$, $F_5'$ of the polygon $OA_1A_2A_3A_4A_5O$ represent the given forces in magnitude and direction, and that the closing side $R$ or $Q$ represents either the resultant or the equilibrant of the given forces; and noting also that for the construction of this polygon it is not necessary to draw the auxiliary vectors $OA_2$, $OA_3$, etc., the following general rule may be stated:

**Rule.**—*To find the resultant or the equilibrant of any number of concurrent forces, draw, in cyclic order, vectors representing the given forces, and join the end of the last vector with the origin of the first. The vector thus obtained, if taken in cyclic order with the two vectors between which it lies, will represent the equilibrant of the given forces; if taken in non-cyclic order, it will represent the resultant.*

**4. The Force Polygon.**—The figure $OA_1A_2A_3A_4A_5$, Fig. 3, though not generally a closed figure, is called the **force polygon** of the given forces, and the line $OA_5$ is called the **closing line** of the polygon.

Usually, the lines of action of the forces are given by the center lines of the members of a structure or parts of a machine. The assemblage of these lines of action is called the **space diagram** of the given forces. It is generally more convenient to construct the force polygon separately; that is, not taking the actual meeting point of the forces as a starting point. Thus, Fig. 4 represents the force polygon (or polygons) of the forces given in Fig. 3. The point $O$ may be taken at any convenient place, and the vectors $OA_1$ $A_1A_2$, $A_2A_3$, etc., equal and parallel, respectively, to the given forces, drawn in cyclic order. The closing line $A_5O$ represents either the resultant or the equilibrant, according to the direction in which it is taken. That this gives the

same result as the one obtained before is evident from the equality of the two polygons.

The order of succession in which the vectors are drawn is immaterial. Thus, after drawing $OA_1$, $A_1A_3''$ may be drawn to represent $F_3$, then $A_3''A_4''$ to represent $F_4$, then $A_4''A_2''$ to represent $F_2$, and, finally, $A_2''A_5$ to represent $F_5$. The point $A_5$ thus determined will coincide with the one determined before, and the line $A_5O$ will, therefore, be the same in both cases. The reason for this is obvious. If $A_2A_3''$ and $A_4A_4''$ are drawn, the quadrilaterals $A_2A_3''$ and $A_3A_4''$ will be parallelograms, since, by construction, $A_1A_3''$ is equal and parallel to $A_2A_3$, and $A_3''A_4''$ is equal and parallel to $A_3A_4$. The parallelogram $A_2A_3''$ shows that $A_2A_3''$ is equal and parallel to $A_1A_2$ $(= F_2)$; and, as in the parallelogram $A_3A_4''$ the side $A_4A_4''$ is equal and parallel to $A_3A_3''$, it follows that $A_4A_4''$ is equal and parallel to $A_1A_2$ $(= F_2)$. As $A_4''A_2''$ is equal and parallel to $A_4A_2$ $(= F_2)$, the line drawn from $A_2''$ to $A_5$ must be equal and parallel to $A_4A_4''$, and, therefore, to $F_5$. So that, if the point $A_5$ had not been already determined, it could be located by drawing from $A_2''$ the line $A_2''A_5$ equal and parallel to $F_5$, as was stated above. It can be shown in the same manner that, whatever the order followed in drawing the sides of the force polygon, the result will always be the same.

**5.** The rule for the composition of concurrent forces can now be stated in the following concise manner:

**Rule.**—*To find the resultant or the equilibrant of several concurrent forces, draw the force polygon of the given forces, and close it; the closing line will represent either of the required forces, according to the direction in which it is taken.*

**6. Conditions of Equilibrium.**—Should the end of the last vector of the force polygon coincide with the origin of the first, the closing line will vanish; that is, there will be no resultant. This will indicate that the given forces are in equilibrium. Thus, if the given forces were $F_1$, $F_2$, $F_3$, $F_4$, $F_5$, and $Q$, Fig. 3, the last vector of the force polygon would be $A_5O$; the force polygon would be closed, and the forces

would have no resultant. In this case, any of the forces may be considered as the equilibrant of the others. The following principles may, therefore, be stated.

*If the force polygon of a system of concurrent forces closes, the forces are in equilibrium.*

Conversely, *in order that several concurrent forces may be in equilibrium, the force polygon must close* For, otherwise, the forces would necessarily have a resultant represented by the closing line of the polygon.

## COPLANAR NON-CONCURRENT FORCES

### THE FORCE DIAGRAM

**7. Determination of the Magnitude and Direction of the Resultant or Equilibrant of Any Number of Coplanar Non-Concurrent Forces.**—Let $F_1$, $F_2$, $F_3$, $F_4$,

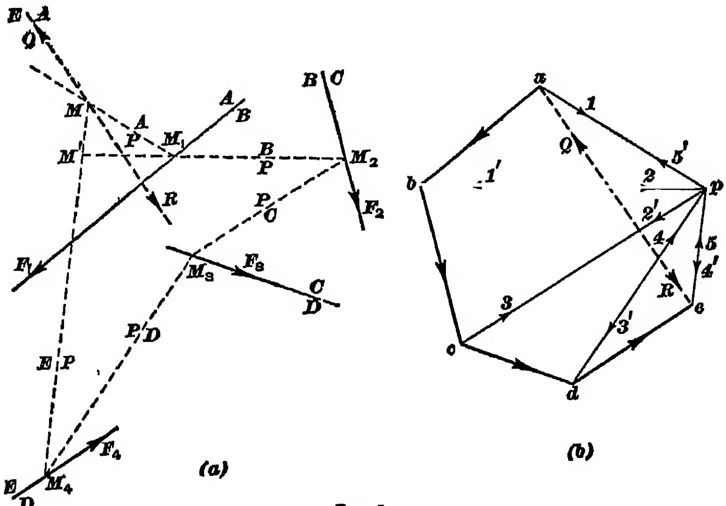


FIG. 5

Fig. 5 (*a*), be non-concurrent forces acting on a rigid body. It was shown in *Analytic Statics*, Part 2, that, so far as the magnitude and direction of the resultant are concerned, the

forces may be treated as concurrent; or, rather, the magnitude and direction of their resultant are the same as the magnitude and direction of the resultant of a system of concurrent forces equal and parallel to the given forces. Therefore, if a force polygon *a b c d e*, Fig. 5 (*b*), is constructed by drawing *a b* equal and parallel to $F_1$, *b c* equal and parallel to $F_2$, etc., the closing vector *a e* or *e a* will represent the magnitude and direction of either the resultant or the equilibrant of the given forces, according as it is taken in non-cyclic or in cyclic order with the two vectors *a b* and *d e* between which it lies.

**8. Notation.**—The following notation, which is illustrated in Fig. 5 (*a*), is very convenient and very often used: The line of action of a force, instead of being indicated by letters at its extremities, is indicated by two capital letters opposite each other, one on each side of the line. For example, *A B* is the line of action of $F_1$, and *C D* is the line of action of $F_3$. Each letter is common to the lines of action of at least two forces, including the resultant (or the equilibrant). Thus, the lines of action of the four given forces and their resultant (the latter to be determined presently) are: *A B*, *B C*, *C D*, *D E*, and *E A*.

In the force polygon, shown in Fig. 5 (*b*), each vector is denoted by two small letters corresponding to the capital letters indicating the line of action of the force represented by the vector. For instance, *a b* represents $F_1$, whose line of action is *A B*. Having drawn *a b*, the next vector *b c* must be drawn parallel to *B C*, the letter *B* being common to *A B* and *B C*. In the same way, after drawing *b c*, the next vector must be drawn parallel to the other line of action having *C* for one of its letters, that is, to *C D*. It is convenient, although not necessary, to draw the vectors of the force polygon so that the letters will be in alphabetical order. This mode of notation is known as **Bow's notation.**

If the magnitude of each force is indicated by *F*, with a subscript consisting of the capital letters marking the line of action of the force, we may write the following convenient

equations, showing the correspondence between the forces in the space diagram, Fig. 5 (*a*), and their magnitudes and directions as given by the force polygon, Fig. 5 (*b*):

$$F_{AB} = ab,\ F_{BC} = bc,\ F_{CD} = cd, \text{ etc.}$$

It should be noticed that the pairs of letters $AB$, $BC$, etc. indicate lines, without any reference to directions, so that $AB$ has the same meaning as $BA$. The lines $ab$, $bc$, etc. of the force polygon, on the contrary, are vectors, whose directions are indicated by the order of the letters. Thus, $ab$ is a vector whose arrowhead points from $a$ toward $b$, and $ba$ would be a vector with its arrowhead pointing from $b$ toward $a$. We thus have $ab = F_{AB}$; but never $ab = ba$, the true relation between $ab$ and $ba$ being $ab = -ba$. The resultant of the given forces is represented by $ae$; their equilibrant, by $ea$.

**9. Line of Action of the Resultant (or of the Equilibrant)—The Force Diagram.**—Having determined the magnitude and direction of the resultant $R = ae$ (or of the equilibrant $Q = ea$), Fig. 5 (*b*), the line of action, or the position of this force in the space diagram, is determined in the following manner: From any convenient point $p$, Fig. 5 (*b*), draw lines $pa$, $pb$, $pc$, etc. to the vertices of the force polygon. The lines $ap$ and $pb$, considered as vectors in noncyclic order with $ab$, as indicated by the arrows *1* and *1'*, evidently represent the components of $F_{AB}$, or $F_1$, in the directions $ap$ and $pb$; this follows from the principle of the triangle of forces. Likewise, $bp$ and $pc$ represent the components of $F_{BC}$, or $F_2$, in the directions $bp$ and $pc$; this is indicated by the arrows 2 and 2'. The same is true of the other lines radiating from $p$. The figure formed by these lines and those of the force polygon is called a **force diagram** of the given forces. The point $p$ is called the **pole** of the force diagram, and the radiating lines $pa$, $pc$, etc. are called **rays**. The position of the pole $p$ being arbitrary, an infinite number of force diagrams may be drawn for the same system of forces, but they all give the same line of action for the resultant (or the equilibrant).

**10.** To determine this line of action, draw from any point $M_1$, Fig. 5 (*a*), on the line of action $AB$ of $F_1$, two lines $PA$ and $PB$, parallel, respectively, to the components *1* and *1'* of $F_{AB}$, represented in the force diagram by the vectors $ap$ and $pb$, respectively. Notice that the lines $PA$ and $PB$ are denoted by capital letters corresponding to the small letters denoting the components *1* and *1'*. Produce $PB$ to its intersection with $BC$ at $M_2$ (the line $BC$ being that line in the space diagram having one of its letters, $B$, the same as one of the letters of the line $PB$ just drawn). From $M_2$ draw $PC$ parallel to $pc$, or *2'*, and produce it to its intersection $M_3$ with $CD$, the latter being that line in the space diagram one of whose letters, $C$, is the same as one of the letters of the line $PC$ just drawn. From $M_3$ draw $PD$ parallel to $pd$, meeting $DE$ at $M_4$. Finally, from $M_4$ draw $PE$ parallel to $pe$, meeting at $M$ the line $PA$ drawn from $M_1$. The line $EA$, drawn through $M$ parallel to $ae$, is the line of action of either the resultant $R$ or the equilibrant $Q$.

The correctness of this construction is very easily demonstrated. Each of the forces $F_{AB}$, $F_{BC}$, etc. can be replaced by its two components, as given in the force diagram. The force $F_{AB}$ may be replaced by two components, equal, respectively, to $ap$ and $pb$, acting along the lines $AP$ and $PB$, and applied anywhere on those lines. These components will be designated by $F_{AP}$ and $F_{PB}$. The force $F_{BC}$ may be replaced by its two components $F_{BP}$ and $F_{PC}$, equal and parallel to $bp$ and $pc$, respectively, and acting along the lines $BP$ and $PC$. Similarly, $F_{CD}$ may be replaced by the components $F_{CP}$ and $F_{PD}$, and $F_{DE}$ by the components $F_{DP}$ and $F_{PE}$. It should be noted, now, that the component $F_{PB}$ of $F_{AB}$ is equal and opposite to the component $F_{BP}$ of $F_{BC}$, as is plainly shown by the opposite vectors $pb$ and $bp$. Likewise, the component $F_{PC}$ of $F_{BC}$ is equal and opposite the component $F_{CP}$ of $F_{CD}$, and the component $F_{PD}$ of $F_{CD}$ is equal and opposite to the component $F_{DP}$ of $F_{DE}$. These components, therefore, balance in pairs, and may be removed. This leaves the system reduced to the two forces $F_{AP}$ and $F_{EP}$, whose resultant (or equilibrant) must be the same as the resultant (or

equilibrant) of the given forces. The line of action of that resultant (or equilibrant) must, therefore, pass through $M$, the point of intersection of its two components. In the force diagram, the two components of the equilibrant $Q$ are represented by the vectors $p\,a$ and $p\,e$, whose directions are indicated by the arrows *5′* and *5*.

## THE FUNICULAR OR EQUILIBRIUM POLYGON

**11. Definition and General Properties of the Funicular.**—The polygon $M\,M_1\,M_2\,M_3\,M_4$, Fig. 5 ($a$), whose vertices are on the lines of action of the forces (the resultant or the equilibrant included), and whose sides are parallel to the rays of the force diagram, is called an **equilibrium polygon,** a **funicular polygon,** or simply a **funicular** of the given forces. Since both the pole $p$ of the force diagram, Fig. 5 ($b$), and the starting point $M_1$, Fig. 5 ($a$), of the funicular are arbitrary, it is evident that to any system of forces there corresponds an infinite number of funiculars. The vertex $M$ of the funicular, however, is always on the line of action of the resultant $R$ or the equilibrant $Q$ of the given forces.

**12.** The sides of the funicular are called **strings.** As already explained, they are the lines of action of the components represented in magnitude and direction by the rays of the force diagram.

In the space diagram, *the two strings parallel to the rays representing the components of one of the forces intersect on the line of action of that force. Each string is common to two forces whose lines of action have a common letter. Also, each string is drawn between the lines of action of the two forces whose vector representatives in the force polygon intersect on the ray parallel to the string in question.* Thus, the string $P\,B$ is parallel to the ray $p\,b$, and is drawn between $A\,B$ and $B\,C$, whose corresponding vectors $a\,b$ and $b\,c$ meet on the ray $p\,b$.

These are convenient relations to remember, as they facilitate the work of construction, making it to some extent mechanical.

**13.** There is another important and useful property of the funicular, namely: *If any two strings of the funicular are produced, their point of intersection will be a point on the line of action of the resultant (or the equilibrant) of the forces acting through the other vertices of the new funicular thus formed.*

For example, if $PB$, Fig. 5 $(a)$, is produced to its intersection $M'$ with $PE$, a new funicular $M_2 M_3 M_4 M'$ will be formed, and $M'$ will be a point in the line of action of either the resultant or the equilibrant of the forces $F_2$, $F_3$, $F_4$, acting through the vertices $M_2$, $M_3$, $M_4$. This follows from the general principles explained in connection with the determination of the point $M$. When the force $F_1$ is left out of consideration and the funicular is constructed, beginning at $M_2$, $PB$ is its first string, and the point $M'$ where this string intersects the last one $PE$ determines a point in the line of action of either the resultant or the equilibrant of the forces considered. The magnitude and direction of the resultant are given by the vector $b\,e$ (not drawn) in the force polygon, Fig. 5 $(b)$.

**14. The Funicular as a Jointed Frame.**—Suppose the strings $PA$, $PB$, $PC$, $PD$, $PE$, Fig. 5 $(a)$, to be replaced by bars jointed at the points $M_1$, $M_2$, etc., and suppose also the forces $F_1$, $F_2$, $F_3$, $F_4$ and the equilibrant $Q$ to act at these joints, respectively. It is evident, then, that the frame formed by the bars will be in equilibrium, for each force can be replaced by two components—one along each of the bars at the intersection of which the force acts. But it has been explained that, for either component of any of the forces, there is an equal and opposite component of another force, balancing the former component. For example, the component of $F_2$ along $M_2 M_3$ is equal and opposite to the component of $F_3$ along the same line. Each bar being thus in equilibrium, the whole frame is in equilibrium.

It is also obvious that the equal and opposite components acting along any of the bars measure the stress in that bar. The following proposition may, therefore, be stated:

*When a frame with no diagonal members is in equilibrium under the action of forces applied at its joints, the frame is a funicular of the applied forces, and the rays of the force diagram represent the magnitudes of the stresses in the members of the frame to which they are parallel.*

**15. Special Conditions.**—In the case represented in Fig. 5 ($b$), the origin $a$ of the first vector and the end $e$ of the last vector of the force polygon do not coincide. In this case, the forces have a single resultant $R$ represented in magnitude and direction by the vector $a\,e$, and whose line of action $AE$, Fig. 5 ($a$), is determined by constructing

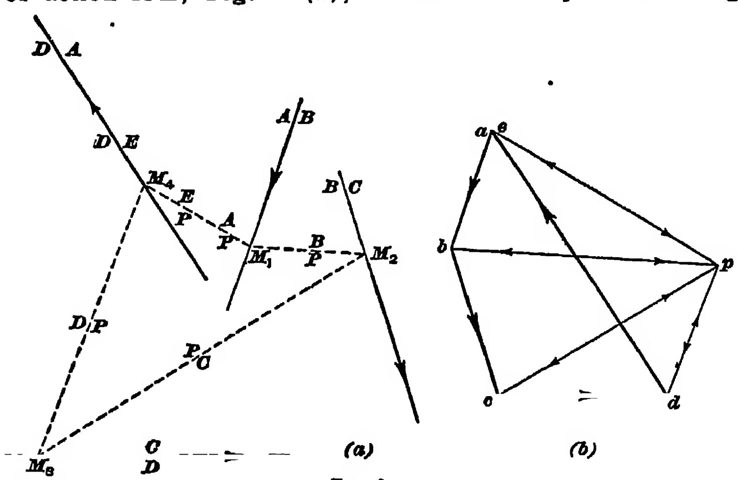


Fig 6

the funicular as explained above. Since $PA$ and $PE$ are parallel, respectively, to the two intersecting lines $p\,a$ and $p\,e$, they must intersect and thus determine a point $M$ in the line of action of the resultant (or equilibrant).

If the points $a$ and $e$ of the force polygon coincide; that is, if the force polyon closes, the forces either form a balanced system or are equivalent to a couple. Which of these two conditions obtains is determined by means of the funicular as follows:

1. In Fig. 6 is represented a system of four forces, $F_{AB}$, $F_{BC}$, $F_{CD}$, and $F_{DE}$, whose force polygon $a\,b\,c\,d\,e$ closes, that

is, when the vectors $ab$, $bc$, $cd$, and $de$ are drawn to represent the given forces, the end $e$ of the last vector $de$ falls on the origin $a$ of the first. Taking any pole $p$, and drawing the rays $pa$, $pb$, $pc$, and $pd$, the ray $pe$ coincides with $pa$. Starting from any point $M_1$ on $AB$, $PA$ and $PB$ are drawn parallel, respectively, to $pa$ and $pb$. From $M_2$, where $PB$ intersects $BC$, $PC$ is drawn parallel to $pc$, meeting $CD$ at $M_3$. From $M_3$, $PD$ is drawn parallel to $pd$, meeting $PA$ at $M_4$. Finally, from $M_4$, $PE$ is drawn parallel to $pe$. As $pa$ coincides with $pe$, it is evident that $PA$ and $PE$ must either

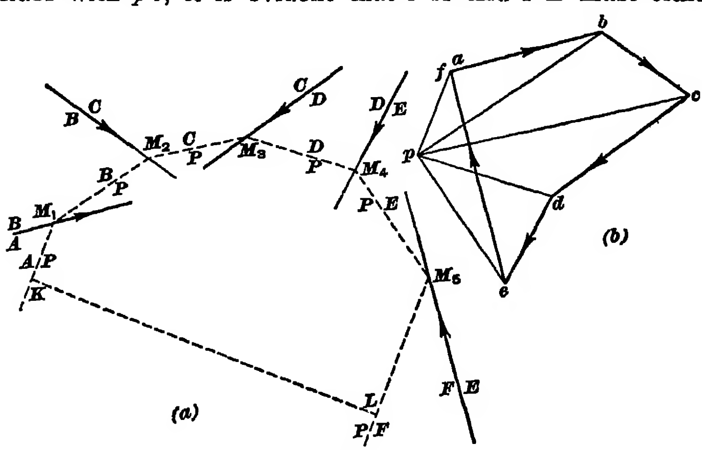


FIG 7

coincide or be parallel. In the case represented in the figure, they coincide; this shows that the given forces form a balanced system. For, as already stated, the sides of the funicular are lines of action of components represented in magnitude and direction by the rays of the force diagram. In the case considered in Fig. 5, all these components balanced in pairs, except the extreme components $ap$ and $pe$; and this left the system reduced to two forces acting along $PA$ and $PE$, whose resultant was the resultant of the given forces. In the present case, the components $bp$ and $pb$, $cp$ and $pc$, $dp$ and $pd$ form balanced pairs, as before, and as the remaining components $ap$ and $pe$ are equal in magnitude and opposite in direction, and have the same line of action $PA$ or $PE$.

they balance each other. The system is, therefore, in equilibrium.

2. In Fig. 7 are represented five forces $F_{AB}$, $F_{BC}$, $F_{CD}$, $F_{DE}$, and $F_{EF}$, whose force polygon *a b c d e f* closes, the end of the vector *e f* coinciding with the origin of the vector *a b*. As before, the force diagram is completed by drawing the rays from any pole *p*, and then the funicular $M_1 M_2 M_3$, etc. is constructed. The given forces can be replaced by two forces represented by the vectors *p a* and *f p* (= *a p*). Here, however, these components, although equal and opposite, have not the same line of action, but act along the parallel lines *P A* and *P F*. *The system is, therefore, equivalent to a couple whose moment is obtained by multiplying the magnitude of the force represented by the vector p a by the perpendicular K L between the sides of the funicular parallel to that vector.*

**16. Graphic Conditions of Equilibrium.**—From the foregoing discussion, the necessary and sufficient graphic conditions for the equilibrium of any system of coplanar forces follow at once. They are:

1. *The force polygon must close;* for, otherwise, the vector drawn from the origin of the first vector to the end of the last (that is, the closing line of the polygon) would give the magnitude and direction of the resultant, which obviously would not be zero.

2. *The last string P E*, Fig. 6, *of the funicular must coincide with the first P A*; for, otherwise, the forces would be equivalent to a couple, as explained in the last article.

Conversely, *if the force polygon of a system of forces closes, and the last string of the funicular coincides with the first, the forces form a balanced system.*

Note —When *the* funicular is spoken of, any funicular that is drawn is meant

**17. Parallel Forces.**—When the given forces are all parallel, the construction is much simplified, as in such a case the force polygon reduces to a straight line In Fig. 8, $F_{AB}$, $F_{BC}$, $F_{CD}$, $F_{DE}$, and $F_{EF}$ are given parallel forces acting along the lines *A B*, *B C*, etc. in the directions shown by the

arrows. It should be noted that letters which are common to the lines of action of two forces are written only once and placed between the two lines to which they are common. This is often done when there is no danger of confusion; otherwise it is preferable to repeat the letters, as in the funicular $M_1 M_2 M_3$, etc., shown in the same figure. Were there no lines crossing the polygon, it would be sufficient to write the letter $P$ once anywhere inside of it. In the present case, however, this might be confusing.

To construct the force diagram, draw an indefinite line $XY$, Fig. 8 ($b$), parallel to the common direction of the forces. From any convenient point, as $a$, lay off the vector $ab$ to represent $F_{AB}$ (no arrowhead is necessary, as the direction of the vector is indicated by the order of the letters $a$ and $b$).

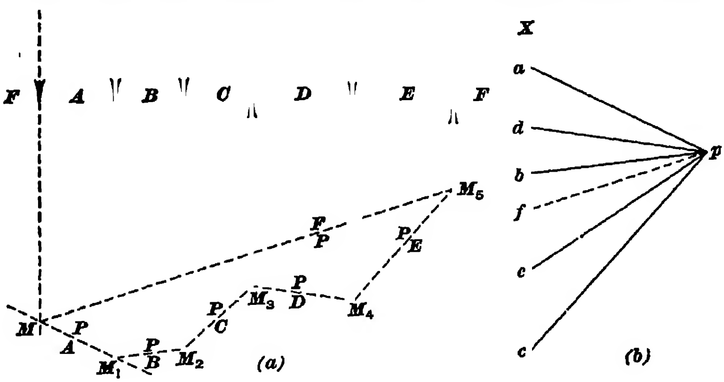


FIG 8

From $b$ lay off the vector $bc$ to represent $F_{BC}$; from $c$ lay off the vector $cd$ to represent $F_{CD}$, and so on for the other forces. The last vector is $ef$, representing $F_{EF}$. The closing line of the polygon is the line $af$ joining the origin of the first vector with the end of the last. The resultant of the given forces is, therefore, represented in magnitude and direction by $af$, the equilibrant, by $fa$.

The line of action $AF$ of the resultant (or the equilibrant) is found in the usual manner, by selecting a pole $p$, drawing the rays $pa$, $pb$, etc., and then constructing the funicular $M_1 M_2 M_3 M_4 M_5 M$.

# GENERAL SOLUTION OF SOME IMPORTANT PROBLEMS

## PROBLEMS ON CONCURRENT FORCES

**18. Problem I.**—*To resolve a force into two components along lines meeting on the line of action of the force.*

This problem has been of constant occurrence in *Analytic Statics*, and its graphic solution has been indicated as a necessary step in the analytic solution. It is repeated here as a review of the principle of the parallelogram of forces and the derived principle of the triangle of forces, which principles form the foundation of all graphic methods.

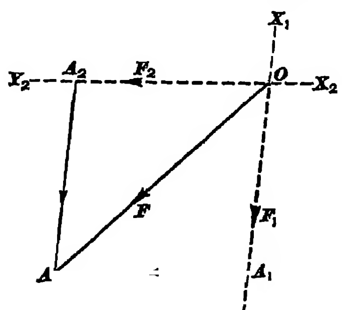


FIG 9

Let $F$, Fig. 9, be a force acting at $O$, and let $X_1\,Y_1$ and $X_2\,Y_2$ be lines along which it is desired to resolve $F$. The resolution may be accomplished by drawing from $A$ the line $A\,A_1$ parallel to $Y_2\,X_2$, meeting $X_1\,Y_1$ at $A_1$; or $A\,A_2$ parallel to $Y_1\,X_1$, meeting $X_2\,Y_2$ at $A_2$. In the former case, the component $O\,A_1 = F_1$ along $X_1\,Y_1$ is determined in magnitude, position, and direction, and the component $A_1\,A = F_2$ in magnitude and direction, its position being understood to be $O\,A_2$, along $X_2\,Y_2$; in the latter case, $O\,A_2 = F_2$ is determined in magnitude, position, and direction, and $A_2\,A = F_1$ in magnitude and direction, its position being understood to be $O\,A_1$ along $X_1\,Y_1$.

Usually, it is not necessary to show the components in position, and then either of the triangles $O\,A\,A_1$, $O\,A\,A_2$ is sufficient for the solution of the problem. Should it be desired to show the component in position, it is better to

construct the two triangles, or the parallelogram $OA_1AA_2$. The components can also be determined by means of one triangle, as $OAA_1$, and then $OA_2 = A_1A$ may be laid off along $OY_2$ without drawing $AA_2$.

The last remark, be it understood, refers to the case in which the components are to be shown in position in the space diagram. Usually, however, as has been repeatedly observed, it is neither necessary nor convenient to show the magnitudes of all the forces in the space diagram, and the geometric constructions are better effected separately. This is almost invariably done in the graphic solution of static problems, and the principle of the triangle of forces, from which the polygon of forces is directly derived, is used.

**19.** The general method used in graphic statics for the solution of the problem stated and solved above is as follows: Let the given force be $F_{AB}$, Fig. 10, and let the lines of action of the required components be $BC$ and $CA$. At any convenient place draw a vector $ab$ to represent $F_{AB}$. From $a$ and $b$ draw the indefinite lines $ac'$ and $bc''$, parallel, respectively, to $AC$ and $BC$, and meeting at $c$. The vectors $ac$ and $cb$, both in non-cyclic order with $ab$, are the required components of $F_{AB}$.

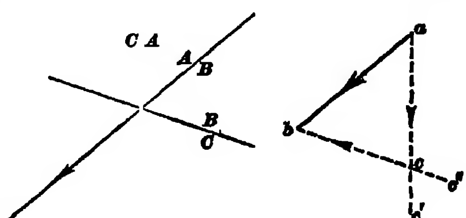


FIG 10

**20. Problem II.**—*In a balanced system of concurrent forces, the lines of action, magnitudes, and directions of all the forces but two are known, and also the lines of action of the other two. It is required to find the magnitudes and directions of these two.*

Let the forces be six, of which $F_{AB}$, $F_{BC}$, $F_{CD}$, $F_{DE}$, Fig. 11 ($a$), are completely known, while of the other two, only the lines of action $EF$ and $AF$ are known. Starting at any convenient point $a$, Fig. 11 ($b$), draw the force polygon $abcde$ for the known forces. Through $a$ draw the indefinite

line $a'f'$, parallel to $AF$ of Fig. 11 ($a$); and through $e$ draw the indefinite line $e'f''$, parallel to $EF$ of Fig. 11 ($a$), and intersecting $a'f'$ at $f$. Then will $ef$ and $fa$ be the required

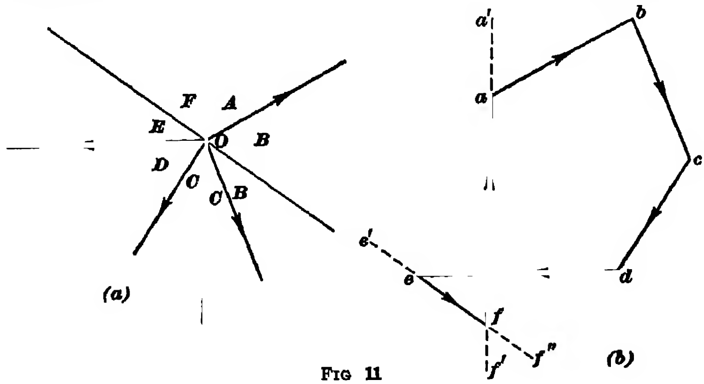


FIG 11

forces, the arrows being marked so that each vector will be in cyclic order with the two between which it lies.

## PROBLEMS ON NON-CONCURRENT AND ON PARALLEL FORCES

**21. Problem III.**—*A rigid body is in equilibrium under the action of several forces of which all but two are wholly known Of the other two, the line of action of one and a point in the line of action of the other are known. It is required to determine completely the two latter forces.*

Let the forces be six, of which $F_{AB}$, $F_{BC}$, $F_{CD}$, and $F_{DE}$ are entirely known, as shown in Fig. 12 ($a$). Of the fifth force $F_{EF}$, only the line of action $EF$ is known, and of the sixth force $F_{FA}$, only a point $M_4$ on its line of action is known.

Draw the force polygon $abcde$, Fig 12 ($b$), for the known forces, and, taking any pole $p$, draw the rays $pa$, $pb$, $pc$, $pd$, and $pe$ Through $e$ draw the indefinite line $e'f'$ parallel to the given line of action $EF$. Construct the funicular for the given forces, drawing the first string $PA$ so that it will pass through $M_4$, which is done by drawing $PA$ through $M_4$ and parallel to $pa$, to meet $AB$ at $M$. and then drawing $PB$,

$PC$, etc. in the usual manner. The string $PE$, from the last of the known forces $DE$, meets the given line of action of $EF$ at $M_5$. Draw $M_5 M_6$, and from the pole $p$ draw $pf$ parallel to $M_5 M_6$, meeting $e'f'$ at $f$. Finally, draw $fa$; then will the vector $ef$ represent $F_{EF}$ and the vector $fa$ will represent

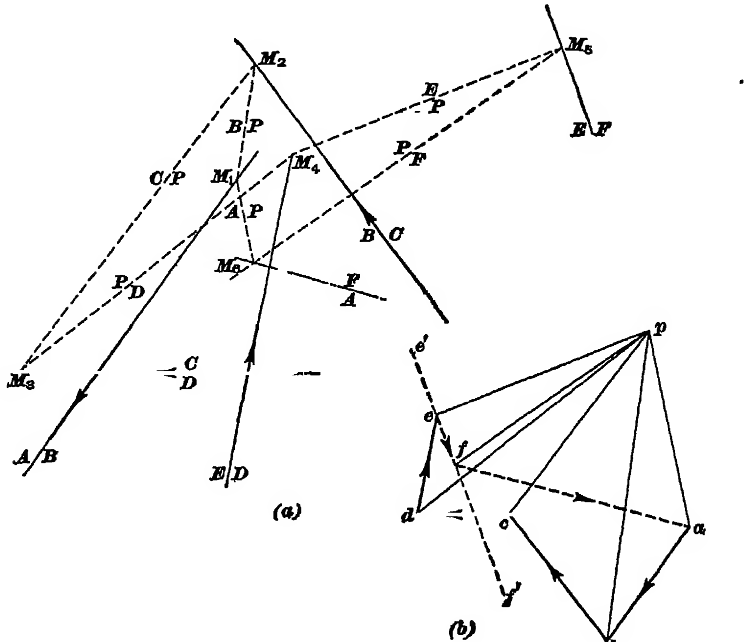


FIG 12

$F_{FA}$, the required force through $M_6$. The line of action of the latter force is $FA$, drawn through $M_6$ parallel to $fa$.

**22. Special Case of the Preceding Problem: Four Forces.**—An important special case of the preceding problem is that in which the number of forces is four, and the lines of action of the two completely known forces intersect. The solution may be effected either by the general method explained above, or by the following special method.

Let $F_{AB}$ and $F_{BC}$, Fig 13, be the wholly known forces; $CD$, the line of action of one of the partly unknown forces; and $K$, a point in the line of action of the other.

First determine the resultant $R$ of $F_{AB}$ and $F_{BC}$, by the force triangle $a\,b\,c$. The line $JH$, drawn parallel to $a\,c$ through the intersection $H$ of $A\,B$ and $B\,C$, is obviously the line of action of $R$. It is also obvious that $R$ is the equilibrant of the yet unknown forces $F_{CD}$ and $F_{AD}$. These three forces must, therefore, be concurrent, so that, if $I$ is the intersection of $H\,J$ and $C\,D$, the line of action of $F_{DA}$ is determined by joining $K$ to $I$. Having the lines $C\,D$ and $D\,A$, the force polygon is completed by drawing $c\,d''$ parallel to $C\,D$, and $a\,d'$ parallel to $A\,D$. The intersection of these two parallels is the fourth vertex of the force polygon

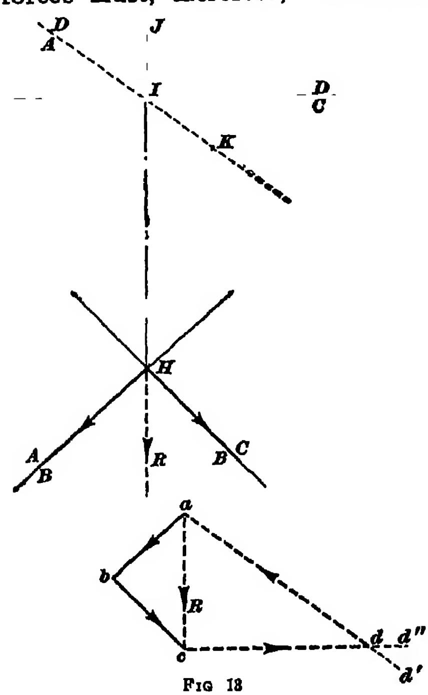


FIG 13

**23. Problem IV: Polygonal Frame.**—*A polygonal frame (that is, a frame without diagonal members) is in equilibrium under the action of forces acting at the joints. Of these forces, one is wholly known, and the lines of action of all but one of the others are known. It is required to determine the unknown elements of the forces not wholly known, and also the magnitudes and characters of the stresses in the members of the frame.*

Let $J_1 J_2 J_3 J_4 J_5 J_6$, Fig. 14, be the given frame, $F_{AB}$, the force that is wholly known, and $B\,C$, $C\,D$, $D\,E$, $E\,F$ the lines of action of the forces acting at $J_2$, $J_3$, etc The force $F_{FA}$ acting at $J_4$ is entirely unknown. The letters $B$, $C$, $D$, it will be noted, are written only once, each in the space bounded by the lines to which the letter is common; the

letters $A$ and $E$ are repeated for convenience. The letter $P$, being common to all the sides of the frame (since these sides, as explained in Art. 14, form the strings of the funicular of the forces $F_{AB}$, $F_{BC}$, etc.), is written only once, in the center of the polygon.

The forces $F_{AB}$, $F_{BC}$, etc. are external forces, which induce

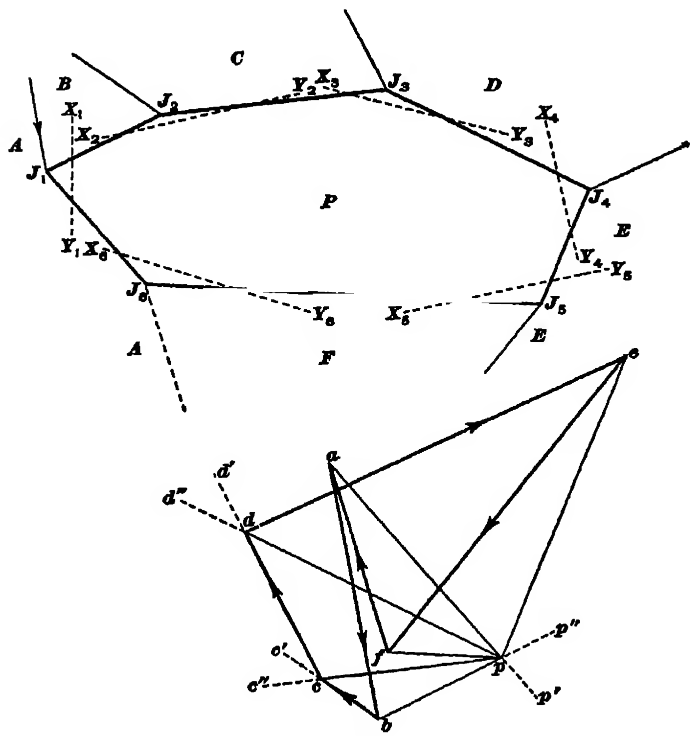

FIG 14

in the bars internal forces. The latter occur in pairs, each pair consisting of the equal and opposite forces acting along a bar. Either of these forces measures the stress in the bar

In the general problems solved and discussed in previous articles, the funicular has not been given, and the pole of the force diagram has been assumed arbitrarily. Here, however, the pole is determined by the data. Having drawn the

vector $ab = F_{AB}$, the rays from $a$ and $b$ must be parallel to the strings $PA$ and $PB$, respectively—that is, to $J_1 J_6$ and $J_1 J_2$. Therefore, if from $a$ and $b$ are drawn $ap'$ and $bp''$ parallel, respectively, to $AP$ and $BP$, their intersection $p$ will be the pole. The next line in the force polygon must be parallel to $BC$, and the next ray must be parallel to $PC$ Therefore, if $bc'$ and $pc''$, parallel, respectively, to $BC$ and $PC$, are drawn, their intersection $c$ will be the end of the vector $bc$, representing $F_{BC}$. Likewise, the intersection $d$ of the lines $cd'$ and $pd''$, parallel, respectively, to $CD$ and $PD$, is the end of the vector $cd$ representing $F_{CD}$. The forces $F_{DE}$ and $F_{EF}$ are determined in the same manner. Finally, the closing vector $fa$ of the force polygon determines $F_{FA}$ in magnitude and direction.

**24.** The stresses in the members are determined by the rays of the force diagram. If, for instance, the two members $PB$ and $PA$ meeting at the joint $J_1$ are cut by a plane $X_1Y_1$, and the part at the right of $X_1Y_1$ is removed, the part on the left will be in equilibrium under the action of the force $F_{AB}$, and two forces (now considered as external) acting along $PA$ and $PB$; the latter two forces measure the stresses in those two members (See *Analytic Statics*, Part 1 )

Now, in the force diagram, $ab$ represents $F_{AB}$, and, since $bp$ and $pa$ are parallel, respectively, to $PB$ and $PA$, or $J_1 J_2$ and $J_1 J_6$, the vectors $bp$ and $pa$, taken, as indicated by the order of the letters, in cyclic order with $ab$, will represent the equilibrants of $F_{AB}$ along the lines $J_1 J_2$ and $J_1 J_6$, and will, therefore, measure the stresses in $PB$ and $PA$, respectively. These stresses will be designated by the notation $S_{PB}$, $S_{PA}$ Their characters are easily determined by the directions of the vectors Thus, when the portion of the frame at the right of $X_1Y_1$ is removed and the forces $bp$ and $pa$ are applied, respectively, at the intersections of $X_1 Y_1$ with $J_1 J_2$ and $J_1 J_6$, it is seen that $bp$, whose direction is from $J_1$ to $J_2$, acts away from the joint, showing that the stress in $PB$ is a pull The force $pa$, on the contrary, will act toward the joint $J_1$, which shows that the stress in $PA$ is a thrust.

The stresses in the other members are determined in a similar manner.

**25. Stress Diagram.**—It will be noticed that, for the determination of the stresses, the triangle formed by each vector of the force polygon and the rays drawn to its extremities is considered as a force triangle. If there were more than one external force acting at any joint, the polygon formed by the vectors representing these forces and by the rays parallel to the members meeting at the joint would be used as a secondary force polygon, the same as the force triangle has been used above.

When the force diagram is employed as a combination of force polygons for the determination of stresses, it is called a **stress diagram.** If each joint of the frame is denoted by the letters indicating the members meeting at the joint, the triangle having corresponding letters in the stress diagram will be the force triangle for that joint. Thus, the joint $J_4$ may be called joint $D E P$, and the triangle $d e p$ in the stress diagram is the force triangle for the equilibrium of the forces acting at $J_4$, from which triangle the stresses $S_{PD}$ and $S_{PE}$ are determined

Arrowheads might be drawn on the rays of the stress diagram to indicate the direction in which forces represented by those rays are to be applied to the members cut; but, as in this case each ray would have two arrows pointing in opposite directions, these arrows might prove a source of confusion. Besides, the arrowheads of the force polygon are sufficient for the determination of the directions of the forces represented by the rays. Consider, for example, the joint $J_1$, or $A B P$, whose corresponding force triangle is $a b p$. Since, in this triangle, the vectors must be in cyclic order, and since the origin and the end of the vector $a b$ are known, the direction of each of the other two vectors follows at once. The end $b$ of $a b$ must be the origin of the next vector, which is, therefore, $b p$ and not $p b$. Likewise, the other vector is $p a$ and not $a p$.

## ILLUSTRATIVE EXAMPLES

**26. Triangular Truss.**—A triangular frame or truss $J_1 J_2 J_3$, Fig. 15, supported at the joints $J_1$ and $J_3$, which are on the same horizontal line, carries a weight of 800 pounds at the upper joint $J_2$. The dimensions being as shown, it is required to determine the reactions of the supports, and also the stresses in the three members of the truss.

Owing to the symmetry of the structure, the reactions, supposed to be vertical, are each equal to one-half the load, or 400 pounds. Their lines of action are indicated by $A B$ and $B C$. The line of action of the applied weight is the

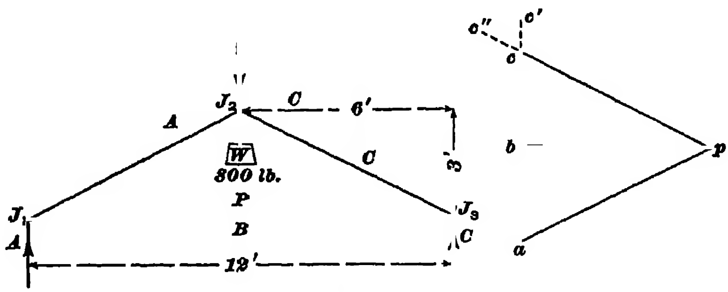


FIG. 15

vertical $A C$, through $J_2$. For convenience, the magnitude and direction of this weight are represented by a vector with its arrowhead pointing toward $J_2$ from above. The notation does not require any explanation, as it is the same notation used in previous articles.

As explained in Art. **14**, the truss $J_1 J_2 J_3$ is a funicular of the forces $F_{AB}$, $F_{BC}$, $F_{AC}$; and, as explained in Art **25**, the stress diagram is a combination of force triangles, each of which consists of vectors representing the forces acting at a joint. Starting with the joint $J_1$, the vector $a b = F_{AB}$, and the rays $a p$ and $p b$, parallel, respectively, to the members $A P$ and $P B$, are drawn. These rays meet at the pole $p$. Then $b p$ and $p a$ will represent $S_{BP}$ and $S_{PA}$, respectively. Since $b p$, when applied along $B P$, points away from the joint $J_1$, the stress $S_{BP}$ is a pull. The force $p a$, on the contrary, when

applied along $PA$, is directed toward the joint, which shows that $S_{PA}$ is a thrust.

The line $pc''$ may be drawn parallel to $PC$ to meet $bc'$ (parallel to $BC$, and, in this case, in line with $ab$) at $c$, and the triangle $bcp$ will give both $F_{BC}$ and the stress in $PC$; but this is not necessary, as, owing to the symmetry of the truss, $F_{AB} = F_{BC}$, and $S_{PA} = S_{PC}$.

**27. Scale Used.**—In the solution of this problem, a scale of 1 inch to the foot was used for distances, making the drawing of the truss 12 inches long and 3 inches high. For the forces, a scale of 100 pounds to the inch was used, making $ab$ 4 inches long If all the stress diagram $acp$ had been drawn, this scale might have been inconveniently large; but only the diagram $abp$ for the joint $ABP$ or $J_1$ was drawn, this being sufficient, as already explained. For this kind of work, a decimally graduated triangular scale (one having scales divided into tenths, twentieths, thirtieths, etc. of an inch) is preferable to the scales used in ordinary mechanical drawing The scale of fortieths is a convenient one to use. In the present case, $\frac{1}{40}$ inch represented 2.5 pounds, and assuming that the drawing was correct within $\frac{1}{40}$ inch, the forces determined by it may be considered correct within 2 or 3 pounds. The line $bp$ was found by actual measurement to be 8 inches, and the line $pa$, $8\frac{37}{40}$ inches. Denoting, therefore, compression and tension by the plus and the minus sign, respectively,

$$S_{BP} = -8 \times 100 = -800 \text{ pounds},$$

$$S_{AP} = S_{CP} = +8\tfrac{37}{40} \times 100$$

$$= +(800 + 37 \times 2.5) = +893 \text{ pounds}.$$

No fractions of a pound are written, as the results cannot be expected to be exact within less than 2 or 3 pounds.

In the solution of static problems by graphic methods, as large a scale as possible should be employed. The draftsman should use his judgment as to the degree of approximation attained If, for instance, he thinks that, on an average, the lengths of the lines are correct within $\frac{1}{50}$ inch, he may measure them in inches and fiftieths, and consider the forces, obtained

in this manner, as correct within about as many pounds or tons as are represented by $\frac{1}{50}$ inch, according to the scale used. Thus, if the scale used is 1 ton to the inch, a length of $\frac{1}{50}$ inch will represent $\frac{1}{50}$ ton $= \frac{2000}{50}$ pounds $= 40$ pounds, and a line found to measure $4\frac{7}{50}$ inches in length will represent $4 \times 2{,}000 + 7 \times 40 = 8{,}280$ pounds, within *about* 40 pounds.

**28. Roof Truss.**—A roof truss $J_1 J_3 J_8$, Fig. 16, with dimensions as shown, is loaded at the joints in the manner indicated in the figure The supports $J_1$ and $J_8$ are on the same level, the member $GP_5$, or $J_7 J_6$, is horizontal, and the members $P_1 P_2$ and $P_4 P_5$ are perpendicular, respectively, to $J_1 J_3$ and $J_3 J_8$ at their middle points. The forces acting at the joints are supposed to be the resultants of weights, and act, therefore, vertically. The reactions $F_{AG}$ and $F_{GF}$ are also supposed to be vertical. It is required to determine the magnitudes of these reactions and the magnitudes and characters of the stresses in the members of the truss.

From the symmetric distribution of the forces, each reaction is equal to one-half their sum, that is, $F_{AG} = F_{GF} = 2{,}000$ pounds. It will be observed that each triangle in the truss is designated by a single letter, common to the lines of the triangle Also, the lines $J_1 J_7$, $J_7 J_6$, $J_6 J_8$ have the common letter $G$. The reason for this is that in the force diagram the rays parallel to the three sides of each triangle radiate from a common point, and that the rays parallel to $J_1 J_7$, $J_7 J_6$, and $J_6 J_8$ also radiate from a common point, as will be seen presently.

For the determination of the stresses, any joint may be taken at random, and the force polygon for the forces acting at that joint constructed, provided that all the forces but two are known. Usually, one of the joints at the supports is the most convenient to begin with, not only because the stress diagram is thus more methodically constructed, but also because, in the majority of cases, the joints at the supports are the only ones for which the force polygon can be constructed independently of the force polygons corresponding to the other joints.

Starting with $J_1$, there are four forces acting at that joint; namely, $F_{GA}$, $F_{AB}$, $S_{BP_1}$, and $S_{P_1G}$. Of these, the first two are known; therefore, the other two may be determined by constructing the force polygon $g\,a\,b\,p_1\,g$, as in the problem solved in Art. 22; that is, by drawing, to any convenient scale, $g\,a = F_{GA} = 2{,}000$ pounds; $a\,b = F_{AB} = 500$ pounds, and then from $b$ a parallel $b\,p_1'$ to $B\,P_1$, and from $g$ a parallel

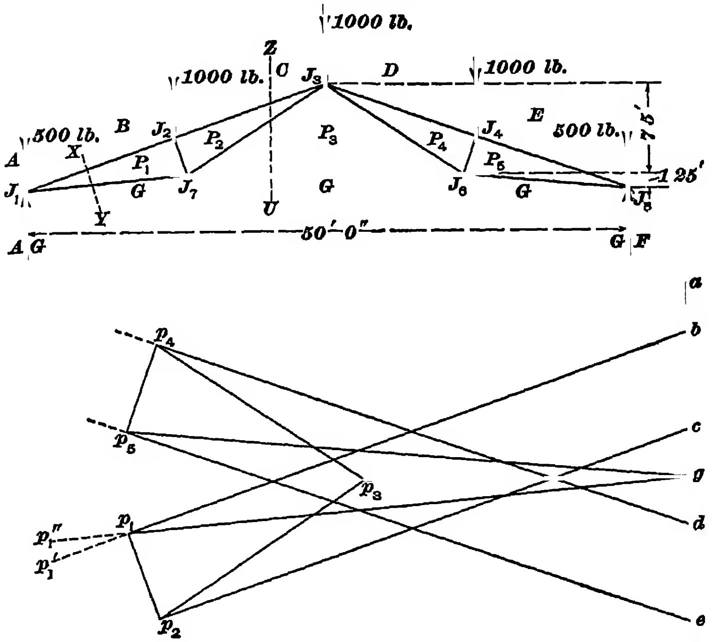


FIG 16

$g\,p_1''$ to $G\,P_1$, meeting $b\,p_1'$ at $p_1$. As $G\,A$ and $A\,B$ are parallel (or, in this case, coincident), the vectors $g\,a$ and $a\,b$ run along the same line, but in opposite directions The lengths of the lines $b\,p_1$ and $p_1\,g$, multiplied by the scale of reduction, give $S_{BP_1}$ and $S_{P_1G}$ If $b\,p_1$ were applied along $B\,P_1$, its direction would be toward the joint $J_1$; and, if $p_1\,g$ were applied along $P_1\,G$, its direction would be away from joint $J_1$. Hence, $S_{BP_1}$ is a thrust and $S_{P_1G}$ a pull, that is, $B\,P_1$ is in compression, and $P_1\,G$ is in tension. It is to be repeated

here that, in determining $S_{BP_1}$ and $S_{P_1G}$, what has been really determined in either case is one of the two equal and opposite forces constituting the stress in the member in question; the force, namely, consisting in the action of the member on the joint considered. Thus, the vector $b p_1$ represents the action of the member $B P_1$ on the joint $J_1$. As stated in various other places, the members $B P_1$ and $P_1 G$ may be imagined cut by a plane $X Y$, and the force polygon, as constructed, gives the external forces $b p_1$ and $p_1 g$ that must be applied at the intersections of $X Y$ with $J_1 J_2$ and $J_1 J_7$ in order to balance $g a$ and $a b$. These forces represent the action exerted by the part of the truss at the right of $X Y$ on the part at the left, which action is transmitted along $J_2 J_1$ and $J_7 J_1$. The action of the part of the truss at the left of $X Y$ on the part at the right is equal and opposite to the action before referred to; so that, if the equilibrium of the part on the right is considered, the external forces that must be applied to the members cut must be equal and opposite to those introduced when the equilibrium of the other part was considered; and, therefore, in the construction of the force polygon, the vectors $b p_1$ and $p_1 g$ may be used, but with their directions reversed, in which case they become $p_1 b$ and $g p_1$, respectively. The same thing may be expressed by saying that the actions of a member on the joints at its extremities are equal in magnitude, but opposite in direction.

The joint $J_2$ is to be considered next. The forces acting on it are $F_{P_1B}$, $F_{BC}$, $F_{CP_2}$, and $F_{P_2P_1}$, of which only the last two are unknown. In the stress diagram already drawn, $p_1 b = S_{P_1B}$. Laying off $b c = F_{BC} = 1{,}000$ pounds, and drawing from $p_1$ and $c$ parallels to $P_1 P_2$ and $C P_2$, respectively, $p_1 b c p_2 p_1$ is determined as the force polygon for the joint $J_2$; this determines $S_{CP_2}$ and $S_{P_2P_1}$.

Now pass to $J_7$, for which $S_{GP_1}$ and $S_{P_1P_2}$ are known and represented in the stress diagram by $g p_1$ and $p_1 p_2$ (notice the order of the letters). Drawing $p_2 p_3$ and $g p_3$, parallel, respectively, to $P_2 P_3$ and $G P_3$, $g p_1$ $p_2 p_3$ $g$ is determined as the force polygon for the joint $J_7$, from which $S_{P_2P_3}$ and $S_{GP_3}$ can be determined.

Since the truss is symmetrical, it is not strictly necessary to draw the rest of the stress diagram. When space permits, however, it is advisable to complete the diagram, as this affords a good check on the work. For joint $J_3$, one force. $F_{CD}$, is given, and two, $S_{P_3P_2}$ and $S_{P_2C}$, have already been determined. These forces are represented by $cd$, $p_3p_2$, and $p_2c$. The lines $dp_4$ and $p_3p_4$, parallel, respectively, to $DP_4$ and $P_3P_4$, complete the polygon $p_3p_2\,cd\,p_4p_3$ for the joint $J_3$, or $P_3P_2\,CDP_4P_3$. If the work has been accurately done, there must obtain $dp_4 = cp_2$, $p_4p_3 = p_2p_3$. The construction of the polygon for the joints $J_4$, $J_5$, and $J_6$ is left as an exercise for the student

It will be again noted that the force polygon for any joint has the same letters as the lines meeting at the joint, and that the directions of its vectors are obtained by starting with one whose direction is known and taking the others in cyclic order. Thus, for the joint $J_4$, or $DEP_5P_4D$, the polygon is $dep_5p_4d$. The direction of $ep_5$ shows that $EP_5$ is in compression, etc. In the case of $J_7$, where there is no external force applied, it will be observed that the direction of the vector parallel to $P_1G$ must be opposite what it is for the joint $J_1$. Now, for this joint, the force polygon is $gabp_1g$, in which the vector parallel to $P_1G$ is $p_1g$. Therefore, the direction of the vector represented by the same line in the polygon for $J_7$ must be $gp_1$, and the direction of the other vectors is determined as above.

By actual measurement, the following results, which the student should verify, have been found:

$$S_{BP_1} = S_{BP_5} = +\ 6{,}150 \text{ pounds}$$
$$S_{CP_2} = S_{DP_4} = +\ 5{,}850 \text{ pounds}$$
$$S_{GP_1} = S_{GP_5} = -\ 5{,}850 \text{ pounds}$$
$$S_{P_1P_2} = S_{P_5P_4} = +\ 950 \text{ pounds}$$
$$S_{P_2P_3} = S_{P_3P_4} = -\ 2{,}600 \text{ pounds}$$
$$S_{GP_3} = -\ 3{,}350 \text{ pounds}$$

These results are correct within about 30 pounds. Besides the construction of the diagrams, the work should be checked by the method of sections (see *Analytic Statics*, Part 2). If the truss is cut by the plane $ZU$, intersecting the three

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members $CP_8$, $P_8P_8$, and $P_8G$, and moments are taken about $J_8$, we get, remembering that $F_{GA} = 2{,}000$ pounds,

$$2{,}000 \times 25 - 500 \times 25 - 1{,}000 \times 12\ 5 - S_{GP_3} \times 7.5 = 0$$

whence, $$S_{GP_3} = \frac{25{,}000}{7.5} = 3{,}333 \text{ pounds}$$

which differs but little from the result found graphically and given above.

**29.** The vectors representing the external forces have been drawn as they have been required Usually, however, it is better to begin by constructing the complete force polygon (in this case a straight line) for the external forces. This procedure is strictly necessary when the reactions are to be determined graphically.

**30. Crane.**—A weight of 12 tons is supported by a crane consisting of a **mast** or **post** $EJ_4$, Fig. 17, that fits and may turn in a socket in a block $BL$; an inclined post or **jib** $J_2J_1$, from which the weight is suspended; a **stay** $J_1J_4$, which may be either a rod or a rope, and the **backstay** $J_4J_3$, which also may be either a rod or a rope, and is fastened to a horizontal plank $J_2K$. The inclinations of the various members to the vertical are shown in the figure. In order to avoid bending and upsetting, it is desirable that the resultant of the external forces applied at the crane should be vertical and pass through the support $E$ of the post. To accomplish this end, weights $W_1$, $W_2$ are placed on $J_2K$ to counteract the upsetting effect of the eccentric load acting through $J_1$ These weights are supposed to be equal and placed at equal distances from $J_3$, so that their resultant $W$ may be treated as a single weight acting

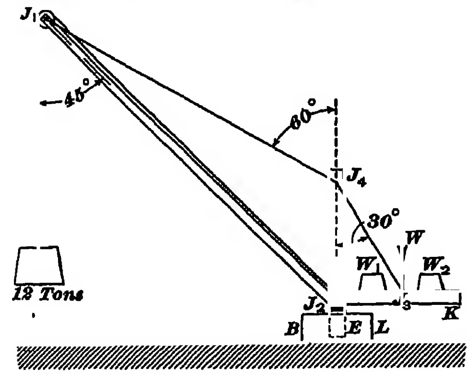

FIG 17

through $J_3$ It is required to find the weight $W$, the reaction at $E$, and the stresses in the various members.

As usual, external forces are assumed to act at the intersections of the center lines of the members directly affected by them, and internal forces along those center lines. Fig. 18 shows a skeleton diagram of the crane, and a force diagram whereby the weight $W = F_{CA}$, the reaction $F_{BC}$ at $J_3$, and the stresses in the members are determined. The notation need not be explained. To determine $F_{BC}$ and $F_{CA}$, the

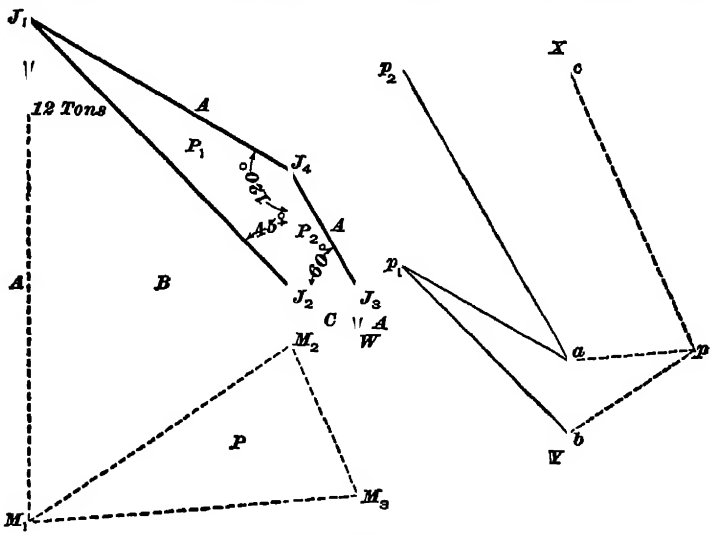


FIG 18

method explained in Art. 17 is used. Having drawn an indefinite vertical line $XY$, the vector $ab = F_{AB} = 12$ tons is laid off anywhere on it, to a convenient scale. A pole $p$ is chosen, and the rays $pa$ and $pb$ are drawn. From any point $M_1$ on $AB$, $M_1M_2$, or $PB$, is drawn parallel to $pb$, meeting $BC$ at $M_2$; $M_1M_3$ is drawn parallel to $pa$, meeting $CA$ at $M_3$ Through $p$, the line $pc$ is drawn parallel to $M_3M_2$, meeting $XY$ at $c$ Then, $bc = F_{BC}$ and $ca = F_{CA} = W$.

Starting now with $J_1$, the force polygon $abp_1a$ is constructed in the usual manner by drawing $bp_1$ and $ap_1$ parallel, respectively, to $BP_1$ and $AP_1$. The construction of the

polygons for the other joints is effected as in Art. 28. The results, which the student should verify, are as follows:

$$W = F_{CA} = 49.2 \text{ tons}, \; F_{BC} = 61.2 \text{ tons}$$
$$S_{BP_1} = +40.2 \text{ tons}, \; S_{P_1A} = -32.8 \text{ tons}$$
$$S_{P_1P_2} = +32.8 \text{ tons}, \; S_{P_2A} = -56.8 \text{ tons}$$
$$S_{P_2C} = +28.4 \text{ tons}.$$

NOTE —Since, in this case, the directions of the members are given by angles, the stresses are independent of the height of the crane.

## EXAMPLE FOR PRACTICE

The derrick represented in Fig 19 supports a load of 12,000 pounds at $J_1$. The dimensions being as given, find the reactions at $J_2$ and $J_3$ and the stresses in the members.

$$\text{Ans.}\begin{cases} R_{AC} = 9{,}240 \text{ lb} ; \; R_{CB} = 18{,}440 \text{ lb}. \\ S_{AP_2} = -9{,}240 \text{ lb} , \; S_{AP_1} - 10{,}000 \text{ lb}. \\ S_{P_2P_1} = +10{,}600 \text{ lb} ; \; S_{P_1B} = +10{,}000 \text{ lb}. \end{cases}$$

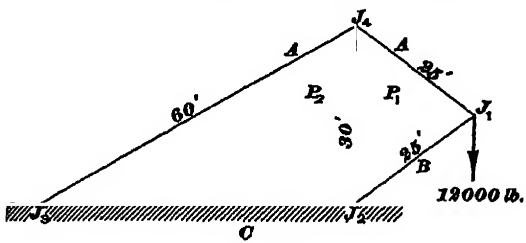


FIG 19

NOTE —In this case, $A P_2$ is the line of action of the reaction at $J_3$ The three external forces acting on the derrick are the load at $J_1$ and the reactions at $J_3$ and $J_2$. Since three forces that are in equilibrium must be concurrent (see *Analytic Statics*, Part 2), the line of action of the reaction at $J_2$ must pass through the point of intersection of $A P_2$ and the vertical $A B$ (line of action of the suspended weight).

# GRAPHIC DETERMINATION OF MOMENTS

## GENERAL CASE

**31. Moment of a Force About a Point.**—Let $F_{AB}$, Fig. 20, be a force whose moment about a point $O$ is required. The shortest and most direct method of obtaining

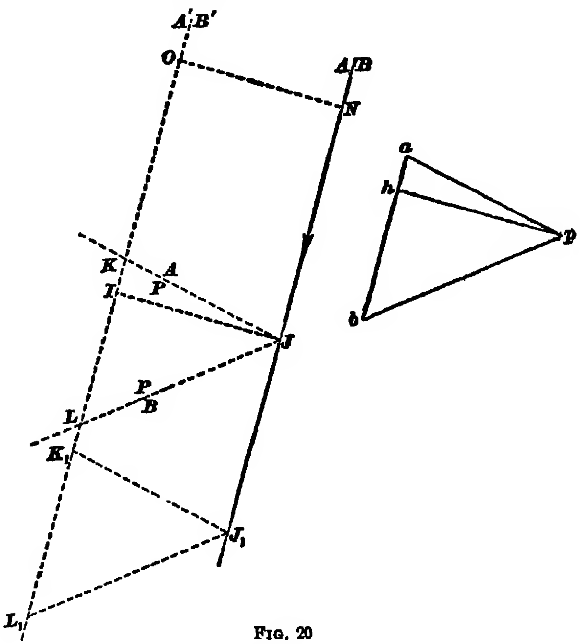


FIG. 20

the required moment in this simple case is to measure the perpendicular $ON$ from the point $O$ to the line $AB$, and multiply it by the magnitude of the force There is, however, another method, which finds its application in cases in which the force itself is not given, but in which its components or its equilibrants along given lines are known.

Draw the vector $ab = F_{AB}$, assume the pole $p$, and draw the rays $pa$ and $pb$; also, $ph$, perpendicular to $ab$. From any point $J$ on $AB$ draw the strings $PB$ and $PA$, and through the given point $O$ draw $A'B'$ parallel to $AB$, meeting $PA$ and $PB$ at $K$ and $L$, respectively. Draw $JI$ perpendicular to $A'B'$ and $AB$, and, therefore, equal to $ON$. If the required moment is denoted by $M$, we may write,

$$M = F_{AB} \times ON = ab \times JI \qquad (a)$$

The similar triangles $JKL$ and $pab$ give

$$\frac{ab}{ph} = \frac{KL}{JI},\ ab \times JI = ph \times KL$$

Comparing the second of these equations with $(a)$, we get

$$M = ph \times KL \qquad (b)$$

It is obvious that, so long as the pole $p$ remains the same, the distance $KL$ is independent of the position of the point $J$ from which the strings are drawn. For, if any other point, as $J_1$, is taken and the strings drawn as shown, intersecting $A'B'$ at $K_1$ and $L_1$, the triangles $J_1K_1L_1$ and $JKL$ are equal, and, therefore, $K_1L_1 = KL$. The distance $KL$ varies according to the position of the pole; but, since $M$ can have but one value, it follows from $(b)$ that the product $ph \times KL$ is constant.

**32. The Intercept.**—The distance $KL$, Fig. 20, is called the *intercept* of the force $F_{AB}$, with respect to the point $O$ and the pole $p$. The following general definition may, therefore, be given: The **intercept** of a force with respect to a given point $O$ in the space diagram, and a given pole $p$ in the force diagram, is the segment that the strings of the force intercept on a line drawn through the given point parallel to the line of action of the force.

**33. The Normal Ray.**—In the triangle $abp$, the vectors $ap$ and $pb$ represent the components of $F_{AB}$ in directions parallel to $AP$ and $PB$. Each of these components can be resolved into two resolutes: one along the line of action of the given force, and one perpendicular to that line. The resolutes of $ap$ are $ah$ and $hp$, and those of $pb$ are $ph$ and $hb$. The line $hp$ represents, irrespectively of its direction, the

magnitude of either of the two resolutes perpendicular to $AB$, and will be called the **normal ray** of the force $F_{AB}$. This term is here introduced as more logical and consistent than the term *pole distance*, used by other writers on graphic statics.

**34. Moment of a Force in Terms of the Intercept and the Normal Ray.**—If the normal ray is denoted by $F_n$, and the intercept by $i$, equation ($b$) of Art. **31** may be written $M = F_n\, i$.

In words, *the moment of a force about a point is equal to the product of the normal ray and the intercept of the force, both referred to the same pole*

The true magnitude of the normal ray is found by multiplying the length of the line $p\,h$ by the scale of forces, and the true length of the intercept $i$ is found by multiplying the length of $KL$ by the scale of distances Thus, if the forces are laid off to a scale of 100 pounds to the inch, and $p\,h$ is found to measure 2.75 inches, then $F_n$ will be $2.75 \times 100 = 275$ pounds. If the scale of distances is 20 feet to the inch, and $KL$ is found to measure $\frac{9}{40}$ inch, then $i$ will be $20 \times \frac{9}{40} = 4.5$ feet. These values in the formula give, $M = 275 \times 4.5 = 1{,}237.5$ foot-pounds.

**35. Resultant Moment of Several Forces.**—Since the resultant moment of several forces about any point is equal to the moment of the resultant of the forces about the same point, it can be found graphically by the method explained above, after the resultant of the forces has been determined. As an example, let it be required to find the resultant moment of the forces $F_{AB}$, $F_{BC}$, $F_{CD}$, and $F_{DE}$, Fig. 21, about the point $O$. The force polygon $a\,b\,c\,d\,e\,a$, constructed in the usual manner, gives the magnitude and direction of the resultant, represented by the vector $a\,e$. The funicular $J_1\,J_2\,J_3\,J_4\,J$ is next constructed. A parallel to $a\,e$ drawn through $J$ would be the line of action of the resultant; but it is not necessary to draw that line. The strings of the resultant are $AP$ and $PE$, and they intercept the segment $KL$ on a line drawn through $O$ parallel to $a\,e$; therefore,

$KL$ is the intercept of the resultant. The normal ray of the resultant is $ph$, perpendicular to $ae$; therefore, the required resultant moment is $ph \times KL$, care being taken before per-

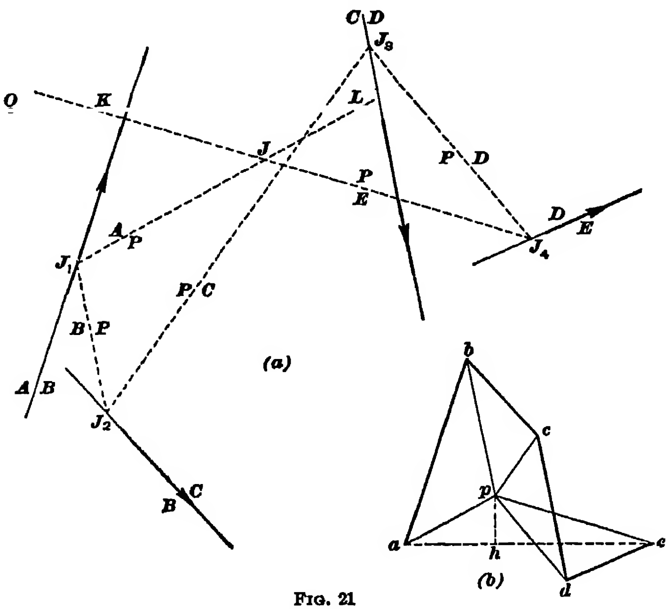


FIG. 21

forming the multiplication to make the proper reductions, according to the scales used.

## PARALLEL FORCES

**36. The Practical Problem.**—The foregoing method finds its most common and useful application in the case of parallel forces, such as the weights acting on a structure. The problem, as it occurs in practice, may be stated in the following general terms:

*Given a balanced system of parallel forces, required the resultant moment, about any point in the plane containing the forces, of all the forces on either side of the point.*

Let $F_{AB}$, $F_{BC}$, etc., Fig. 22, be the given forces; $p\,a\,b\,c\,d$, etc. their force diagram; and $J_1\,J_2\,J_3$, etc., the corresponding

funicular. As the forces are all parallel, they all have the same normal ray $ph$, which is also the normal ray of the resultant of any number of them Let it be required to find the resultant moment, about a point $O$, of all the forces on either side of that point.

The forces being in equilibrium, the resultant moment of

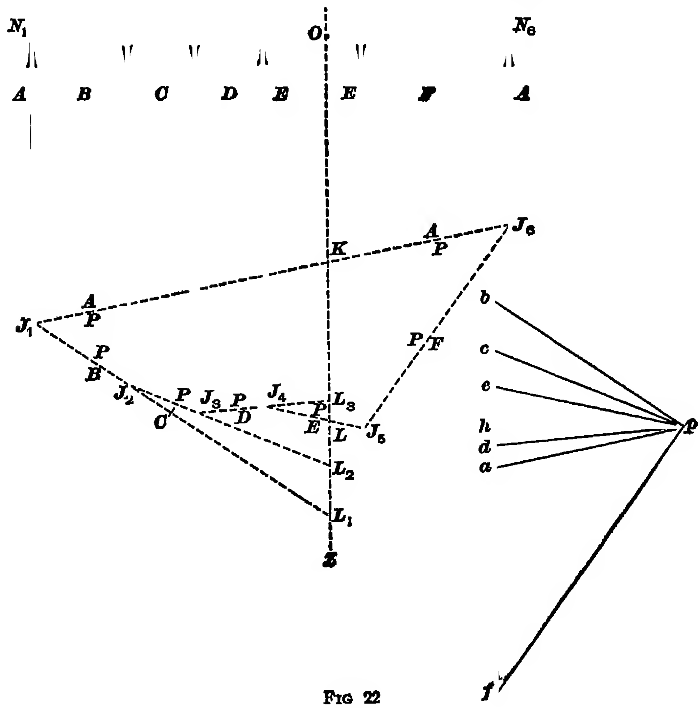


FIG 22

the forces on either side of $O$ is equal in magnitude, but opposite in sign, to the resultant moment of the forces on the other side (see *Analytic Statics*). The resultant moment of the forces acting on the left of $O$ will be determined. Since the resultant of the forces $F_{AB}$, $F_{BC}$, $F_{CD}$, and $F_{DE}$ must be parallel to these forces, the line $OZ$, drawn through $O$ parallel to $AB$, $BC$, etc., is parallel to the line of action of the resultant The funicular for the forces on the left of $O$ has $PA$ for its first string and $PE$ for its last

string; in other words, $PA$ and $PE$ are the strings of the resultant of those forces, and, therefore, $KL$ is the intercept of that resultant. The magnitude of the required moment is, therefore, $ph \times KL$. The magnitude and direction of the resultant of the forces under consideration are given by the vector $a\,e$, which shows that the resultant acts upwards (in the drawing). Its line of action, which passes through the intersection of $PA$ and $PE$, lies, in this case, at the left of $O$. From these conditions, it follows that the resultant moment is right-handed.

The preceding conclusion as to the magnitude and direction of the moment can be directly arrived at as follows: Produce $PB$, $PC$, and $PD$ to their intersections $L_1$, $L_2$, $L_3$ with $OZ$. Then, by definition, the intercepts of $F_{AB}$, $F_{BC}$, $F_{CD}$, and $F_{DE}$ are, respectively, $KL_1$, $L_1L_2$, $L_2L_3$, $L_3L$, and the moment of the resultant of the four forces is (paying due regard to signs),

$$+ph \times KL_1 - ph \times L_1L_2 - ph \times L_2L_3 + ph \times L_3L$$
$$= ph\,(KL_1 - L_1L_2 - L_2L_3 + L_3L) = +ph \times KL$$

as found before, the positive sign indicating that the resultant moment is right-handed.

**37. Determination of the Intercept by the Funicular.**—It follows from the foregoing that, *when several parallel forces are in equilibrium, the intercept, with respect to any point of their plane, of the resultant of all the forces on either side of the point, is the same as the segment intercepted by the funicular on a line drawn through the given point parallel to the common direction of the forces.*

This is a very useful rule, which finds its most important applications in problems relating to the strength of materials.

**38. Selection of the Normal Ray.**—Since the position of the pole in the force diagram is arbitrary, it is convenient so to select it that the normal ray will be a convenient number to multiply by. To accomplish this, it is sufficient to choose any point $h$ in the force polygon (this applies to parallel forces only) and draw $hp$, normal to the vectors of the polygon, making it represent, to the scale of forces

used, a force of 100, 1,000, 10,000 units (pounds, tons, etc.) or some other number easy to be used as a multiplier. Thus, if the scale of forces is 500 pounds to the inch, $p\,h$ may be conveniently made equal to 2 or 4 or 6 inches, so that it will represent a force of 1,000, 2,000, or 3,000 pounds, respectively.

# STRESSES IN BRIDGE TRUSSES

## (PART 1)

## INTRODUCTION

### DEFINITIONS AND GENERAL CONSIDERATIONS

#### BEAMS AND GIRDERS

**1. Wooden and Steel Beams.**—The principles governing the use of beams in supporting loads are fully discussed in *Strength of Materials*, Part 1 For short spans, beams are sometimes made of timber and are of solid rectangular cross-section, the same cross-section being used throughout the span. This is the most economical form when light beams can be used. The rectangular cross-section is not economical, however, when heavy beams are required; it must be made large enough to resist the maximum bending moment, and, as this occurs near the center of the span, some material is wasted by making the cross-section uniform, since no section between the center and the ends has to withstand so great a stress as the section at the center. For very heavy loads and long spans, it is more economical, when beams are used at all, to use steel beams On account of the danger of fire and the frequent cost of renewal attending the employment of timber, steel beams are now almost exclusively used in permanent structures, even though their original cost may be many times that of wooden beams

**2. I Beams.**—Where the bending moment is comparatively small and a low value of the section modulus is

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sufficient, rolled-steel beams of uniform cross-section are used, the material in the section being so distributed as to give a comparatively large value of the section modulus for a given amount of material. Since the top and bottom of the beam are farthest from the neutral axis, as much material as possible is concentrated in these parts. The beam whose cross-section is shown in Fig. 1 fulfills this condition, and, on account of its form, is called an **I beam.** The horizontal part at the top and bottom are called the **flanges**, and the vertical part is called the **web.** I beams are rolled in various sizes up to 24 inches in depth. For lengths not exceeding 25 feet, the I beam is the most economical form of steel beam, but above this length too many rolled beams are required to resist the bending moment, so that a single deeper beam, made as explained in the next article, is more economical.

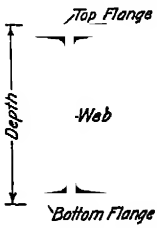

FIG. 1

**3. Plate Girders.**—The name **plate girder** is given to a beam, usually of steel, that has the same general form as an I beam, but is composed of several pieces. The cross-section of such a beam is shown in Fig. 2. The vertical part consists of a plate called the **web**; while the top and bottom parts, which consist of plates and angles, are called the **flanges.** Plate girders are generally used for spans of 25 to 100 feet, and occasionally for greater lengths.

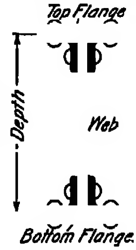

FIG. 2

The **span** of a plate girder is the horizontal distance from center to center of its supports; the **depth** is the vertical distance between the outer surfaces of the angles that form a part of the flanges. Experience shows that the most economical plate girder has a depth of from one-seventh to one-eighth of the span. The usual practice for railroad and highway bridges is to make the depth one-eighth to one-tenth of the span, although the depth is frequently made as small as one-twelfth. These latter proportions require heavier

sections than the former, and are, therefore, very wasteful of material. For spans over 100 feet in length, it is impossible to get sufficient depth for an economical section, and the girder is so heavy that it is difficult to handle. For these reasons, plate girders are unadaptable to spans over 100 feet in length, except under special conditions.

**4.** In the best modern practice, the top and bottom flanges of a plate girder are made parallel throughout the entire length of the girder, and are riveted to the web. The section of the flange is decreased from the center toward the end, and so no material is wasted, as the flanges at any point are just large enough to resist the maximum bending moment that can occur at that point. In the past, plate girders were built with curved flanges. The curve was usually made a parabola, and the flange cross-section was constant from end to end. There is no additional economy in the use of girders with curved or inclined flanges. Conditions, however, sometimes require that the ends be made shallower than the center, in which case one of the flanges is inclined near the end of the girder. The web of a plate girder is very thin and requires to be stiffened at intervals with angles riveted to the sides of the web to prevent it from buckling. This subject will be more fully treated elsewhere.

**5. Lattice Girder.**—The **lattice girder** is sometimes used for the same span length as the plate girder, and resembles the latter in that it has flanges, usually parallel, at the top and bottom, which decrease in size from the center toward the end of the girder Instead of the solid web, however, there is an open-web system, consisting usually of angles running diagonally from top to bottom in both directions, and riveted to vertical plates that project from between the vertical legs of the flange angles. The lattice girder belongs to a special class of structures called *trusses*, which are designed to act as beams for any span length, but are most frequently and economically used for spans over 100 feet in length. Lattice girders, however, are somewhat used for highway bridges for spans much less than 100 feet.

## THE TRUSS

**6. Definition.**—A **truss** may be defined as a framework composed of straight pieces, called **members**, so connected as to act, to a great extent, as a rigid structure. Trusses, like beams, are designed to support loads.

The intersection of two or more members, where they are connected or joined to each other, is called a **joint.**

While the truss as a whole resists the effect of the applied forces in much the same manner as the shear and the bending moment are resisted by a solid beam, each individual member of the truss is subjected only to direct tensile or compressive stresses in the direction of its length In order that this may be the case, the applied forces are resolved into components acting at the joints of the truss.

The simplest form of truss is a triangle, and any truss is merely an assemblage of connected triangles. As the triangle is a rigid figure whose form cannot change so long as the length of each of its sides remains the same, it is the primary and essential element of the truss.

**7. Truss Members.**—The upper members of a truss, such as $BC$, $CD$, etc., Fig. 3, taken together form the

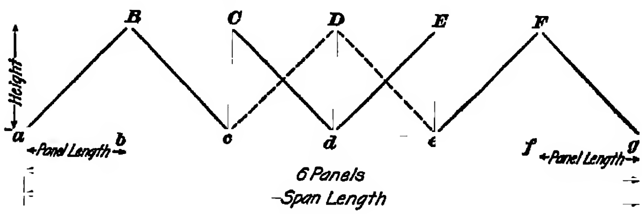


FIG 3

**upper chord** of the truss; the lower members, such as $ab$, $bc$, etc , taken together form the **lower chord.** The other members of the truss, which lie between and connect the upper and lower chords, are called **web members.** The end web members $aB$ and $Fg$ are called **end posts.** The intermediate inclined web members, such as $Bc$ and $Cd$, are called **diagonals.**

**8. Panel, Span, and Height.**—A **panel** is a subdivision of a truss between two consecutive joints in a chord. The joints are often called the **panel points.** The length of a panel is called the **panel length.** In Fig. 3, the points *b*, *c*, *d*, etc. are the joints or panel points, *a b* is the panel length, and the truss is a six-panel truss. Usually, the panel lengths of a truss are equal.

**9.** The **span** of a truss, for purposes of stress computation, is to be taken equal to the horizontal distance between the end panel points, as shown in Fig. 3.

**10.** The **height,** or **depth,** of a truss, for purposes of stress computation, is to be taken equal to the vertical distance between the joints of the chords, as shown in Fig. 3.

**11.** The three dimensions just given—panel length, span length, and depth of truss—bear a relation to each other that, to a certain extent, decides the type of truss to be used. Engineering experience has shown that for the greatest economy the depth of truss should be about one-sixth of the length, and that the diagonal web members should make an angle with the vertical of about 40°. The panel lengths most frequently used lie between 15 and 25 feet. These values need not be strictly adhered to; there may be a reasonable departure from them without seriously increasing the cost of the truss. From the span length, the depth and panel length are so chosen as to satisfy the economical conditions as far as possible.

**12. Kinds of Trusses.**—A **symmetrical truss** is a truss that can be cut at the center into two parts exactly alike. If it could be folded at the center on itself in such a manner that the two ends would come together, all corresponding members in the two halves of the truss would coincide. Nearly all trusses are symmetrical.

**13.** A **simple truss,** like a simple beam, is one that is supported only at the ends. A **continuous truss** and a **cantilever truss** are, likewise, similar to the corresponding forms of beams.

**14.** **Parallel-chord trusses** are trusses in which the upper and lower chords are parallel. In these trusses, the panel lengths are usually equal and all the diagonal web

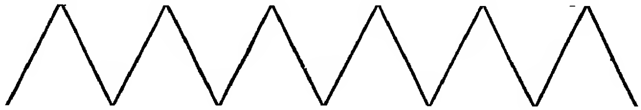

Fig 4

members have the same inclination (see Figs 3 and 4). The parallel-chord truss is especially adapted to short spans.

**15.** In **inclined-chord trusses**, which are used for long spans, the depth at the center and the panel length are so chosen that the web members near the center make an angle with the vertical of less than 40°. The panels are made the same length throughout the bridge, but the depth

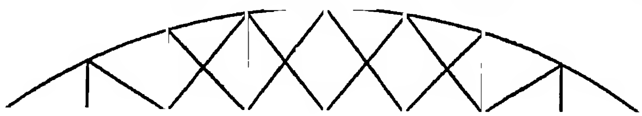

Fig. 5

of the truss is decreased at each panel point from the center toward the end, inclining one or both of the chords, thereby making the web members slope differently, those near the end making an angle with the vertical of more than 40°. This saves material near the end of the truss by shortening the heavy web members, and at the same time allows an economical depth to be used at the center of the truss.

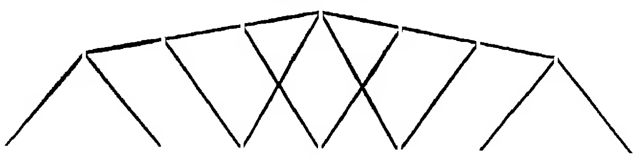

Fig. 6

When the joints of the chord are in a straight inclined line, the chord is called an **inclined chord**, and the truss, an **inclined-chord truss**. When the joints of the chord are on a curve, the chord is called a **curved chord**, and the

truss, a **curved-chord truss.** The chord is not really curved, the members being straight between the joints. An inclined or a curved chord gives a graceful outline to a truss; on this account an inclined- or a curved-chord truss is

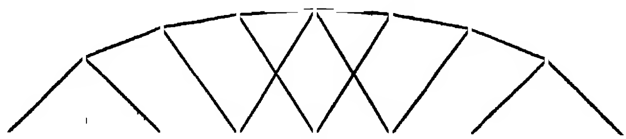

FIG 7

preferable, from an esthetic standpoint, to a parallel-chord truss. Examples of inclined- and curved-chord trusses are shown in Figs. 5, 6, and 7.

**16. Multiple-System Trusses.**—For long spans, the chords are sometimes made parallel and the diagonal web members are continued across two or more panels. Trusses

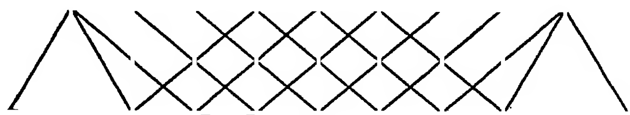

FIG 8

of this kind are called **multiple-intersection,** or **multiple-system, trusses.** When the diagonals extend over two panels, the truss is called a **double-intersection,** or **double-system, truss;** when the diagonals extend over three or four panels, the truss is called a **triple-** or **quadruple-system truss,** and also, to some extent, a **lattice**

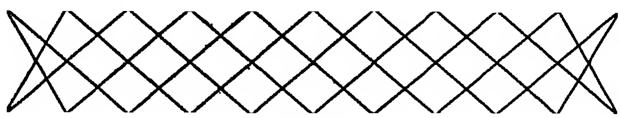

FIG 9

**truss.** The arrangement has also been used in trusses with curved and inclined chords.

The multiple-system truss allows an economical choice of both the center depth and the slope of the diagonals, without making it necessary to increase the length of panel.

Trusses of this type are objectionable, however, because the stresses cannot be found directly by the ordinary conditions

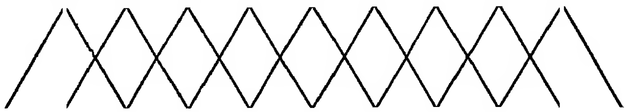

FIG 10

of equilibrium. Examples of multiple-system trusses are shown in Figs. 8, 9, and 10

**17. Subdivided-Panel Trusses.**—Another method of obtaining economy in the depth at the center and in the slope

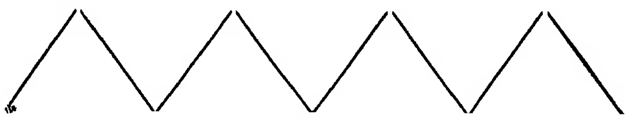

FIG 11

of the diagonals, without increasing the length of the panel, is to subdivide the main panels of the truss by adding secondary verticals, or secondary verticals and diagonals.

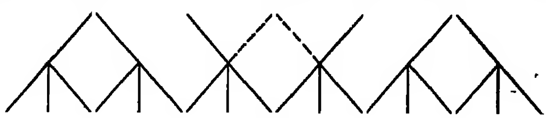

FIG 12

This arrangement of members is made use of to a considerable extent in both parallel- and curved-chord trusses, and is very satisfactory. Any truss with subdivided panels is

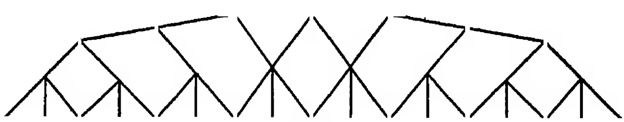

FIG 13

called a **subdivided-panel truss.** Examples of this type are shown in Figs. 11, 12, and 13

**18.** The term **girder bridge,** or **truss bridge,** is the name usually applied to a bridge simply resting on supports at the same level, so that the reactions due to vertical

loads are vertical. This distinguishes the truss bridge from the arch and the suspension bridge, in which the reactions are not vertical.

**19. Pin-connected trusses** are those in which the several members that meet at a joint are connected to each other, and transmit their stresses, by means of an accurately turned pin, somewhat resembling a large bolt, which is made to fit very closely into holes drilled through the ends of the members. This affords a simple and convenient method of connecting the members. As the connection acts, to some extent, like a large hinge, it allows each member to adjust itself in the line of its stress without developing large bending stresses. In this type of truss, each member is practically finished at the shop; and, in erecting the bridge at its site, the members are assembled and connected, and the structure completed, in the shortest possible time and with the minimum amount of field labor. Owing largely to this fact, this type of truss is the standard in America for long spans. The pin-connected truss is well adapted for both highway and railroad bridges over 150 feet in length.

**20. Riveted trusses** are those in which the several members that meet at a joint are riveted to each other, and transmit their stresses by means of connecting plates called **gussets,** to which all the members at the joint are riveted. The tendency of the best modern practice is toward such details as will give great rigidity, and for this reason the riveted truss, which is very rigid, is coming into general use for the shorter spans to which trusses are adapted. It is used for highway and for railroad bridges for spans up to about 150 feet in length.

**21. Line of Action of Stress in a Truss Member.** The stress in each member of a truss is considered to act in a direct line between the centers of the connections. The member itself should, therefore, be straight, and the connections at its extremities should be as nearly as possible on a line passing through the center of gravity of its cross-section. The pins in a pin-connected truss are located on lines passing

nearly through the centers of gravity of the members, and the stresses are therefore all direct stresses. There is some eccentricity in the connections of a riveted truss, due to the fact that the members are connected to large gusset plates; and, although the center lines of all the members at any joint intersect in the same point, the centers of the connections are not coincident; this causes eccentric stress.

**22. Stress Sheet.**—For all purposes relating to the investigation of the stresses in the members of a truss, each member is represented by a straight line indicating the line of action of its stress A **stress sheet** of a bridge is a skeleton drawing in which the members of the bridge are shown by straight lines, which occupy positions on the drawing corresponding to the positions of the members in the bridge On the stress sheet are shown the arrangement of floor and lateral systems, with the spacing of stringers and trusses; the span length, number of panels, panel length, and depth of truss; the lengths of diagonals; the assumed dead, live, and wind loads, and the panel loads and reactions resulting therefrom; and the maximum and minimum stresses in each member due to the assumed loads. The amount of material required in each member, and the different parts that form the cross-section of the member, are sometimes shown on the stress sheet.

## TRUSS AND PLATE-GIRDER BRIDGES

### MAIN PARTS

**23.** The main parts of a truss bridge or of a plate-girder bridge are: (1) two longitudinal vertical trusses or two-plate girders, such as have been described in the foregoing articles; (2) a *floor system,* (3) a *lateral system.*

**24. The Floor System.**—The traffic on a bridge is carried by a floor, which transfers the load to the longitudinal trusses or girders on the sides. In a highway bridge, the floor usually consists of planking resting on longitudinal

**joists, or stringers,** and of cross-girders called **floor-beams,** which support the stringers and transfer the load to the trusses at the panel points. In a railroad bridge, the floor usually consists of wooden rail ties, longitudinal stringers, and floorbeams. In some truss bridges, the floor joists, or the ties, rest directly on the chord members, thus producing bending stresses in these members in addition to the direct stresses. These bending stresses must be separately computed and added to the direct stresses. This subject, however, will be left for subsequent treatment; it will here be assumed that the floor system transfers the load to the trusses at the joints, and that, therefore, the load acts at these points only. The complete analysis of a bridge includes the analysis of the stresses in the various parts of the floor system, as well as in the members of the trusses.

**25. The Lateral System.**—The lateral forces acting on a bridge are resisted by lateral trusses lying in the planes of the chords and connected to the latter; these trusses transmit all lateral forces to the supports. In addition to this, there are transverse braces, or frames, at each panel point, which are made as deep as the conditions will allow. The transverse brace, or frame, at the end is placed in the plane of the end posts and is usually called the **portal brace,** or simply the **portal.**

### CLASSIFICATION OF BRIDGES

**26.** There are several ways in which bridges may be classified. One of the most common is to classify them according to the position of the floor or roadway with respect to the chords. In this classification, bridges are divided into three general types, namely: *through bridges*, *deck bridges*, and *half-through bridges*.

**27. Through bridges** are those that support their floors or loads at or near the level of the bottom chord, and have room for a system of lateral bracing between the top chords without interfering with the traffic. The loads pass between the trusses, or *through* the bridge. Bridges of this type

require very little space below the floor, and, therefore, the locations to which they are adapted are very common. Where trusses are of such a height that vertical transverse frames may be put in between the web members above the overhead clearance line, they are sometimes spoken of as *high-truss bridges.*

**28. Deck bridges** are those that support their floors or loads at or near the level of the upper chord. In bridges of this class, all portions of the structure are entirely below the floor. For long spans, the locations to which they are adapted are not very common. Deck bridges are more economical than any other type of bridge, and are used wherever conditions permit.

**29.** The chord that supports the floor system is called the **loaded chord**; the other, the **unloaded chord.** In a through bridge, the loaded chord is the lower chord; in a deck bridge, the loaded chord is the upper chord.

**30. Half-through bridges** are those that support their floors or loads at some elevation intermediate between the top and the bottom chord, or at the bottom chord, and the trusses of which are not deep enough to allow a system of overhead bracing. When plate girders are used, such a bridge is called a half-through plate-girder bridge. When trusses are used, such a bridge is called a **low-truss,** or **pony-truss, bridge.**

**31.** Bridges are also spoken of as *plate-girder bridges, riveted-truss bridges,* and *pin-connected truss bridges,* according to the type of beam or truss that supports the loads Bridges may be further classified according to the style of truss, giving two general classes; namely, **parallel-chord bridges** and **inclined-chord bridges.** Each of these classes may be subdivided into single-system, multiple-system, and sub divided-panel bridges, as explained in connection with trusses

# LOADS AND REACTIONS

## CLASSIFICATION OF LOADS

**32. External Forces Acting on a Bridge.**—The external forces acting on a bridge consist of: (1) the weight of the structure itself; (2) the weight of whatever the structure is designed to support; (3) the pressure of the wind; and (4) the reactions of the abutments. The first two of these classes of forces are called **loads.** In addition to the forces just mentioned, it is sometimes necessary to consider other applied forces, such as the centrifugal force of a train moving on a curved track, and the horizontal force caused by moving trains.

**33. Kinds of Loads.**—Loads are divided into two general classes, namely. *dead loads* and *live loads.*

The **dead load** consists of the weight of the structure itself, including the track or floor, the floor system, and the girders or trusses. It is the force of gravity acting on every part of the structure, and is, therefore, actually applied at all points. The weight of the floor system is transferred to the joints of the loaded chord by the floor-beams The weight of the truss is transferred to the joints of the loaded and unloaded chords by the members themselves Methods of estimating in advance the approximate weight of floor systems and trusses will be given elsewhere. The dead load is usually assumed to be a uniform amount per linear foot of structure. For short spans, up to about 125 feet, the direct stresses due to dead load are found by assuming all the load to be applied at the joints of the loaded chord. For longer spans, a part of the load, usually one-third, is assumed to act at the joints of the unloaded chord. This assumption leads to results that are very close to the actual stresses.

**34.** The **live load** is the load due to the traffic. It is therefore a **moving** load, and is often so called. For highway bridges, the live load is assumed to be a specified amount per square foot, uniformly distributed over the roadway. For bridges that are subject to the passage of heavy loads concentrated on wheels, the live load is assumed to be a certain arrangement of concentrated loads, or wheel loads, which may be taken by themselves or in connection with the uniform load. For railroad bridges, the live load usually consists of a system of concentrated wheel loads representing a type of locomotive, or of certain uniform loads that will give an approximately equivalent effect The calculation of stresses due to uniform and to concentrated loads require separate consideration. This subject will be fully treated elsewhere. The amount of live load that a bridge is designed to carry is sometimes called the **capacity** of the bridge.

**35. Wind Pressure.**—The wind pressure, sometimes called the **wind load,** is the force exerted by the wind on all surfaces exposed to it. These surfaces consist of the sides of the members and the exposed surfaces of the moving loads that cross the structure. In highway bridges, the surface of the loads is usually very small compared to the exposed surface of the bridge. In railroad bridges, it is necessary to consider the pressure of the wind against the side of a train, in addition to the pressure against the exposed surface of the structure. It is customary to assume the wind pressure as a uniformly distributed force, and express it in pounds per square foot of exposed surface, or in pounds per linear foot of the loaded or unloaded chord.

**36. Centrifugal Force.**—The centrifugal force considered in bridges is the pressure exerted by a train of cars moving on a curved track; it acts radially outwards, and is transferred by the floor system to the joints of the loaded chord. It is usually expressed as a percentage of the live load and depends on the degree of curvature of the track, ard on the weight and speed of the train.

**37.** The **longitudinal thrust** is the force exerted on a structure by a train of cars crossing the structure while the brakes are set This force is a maximum when the brakes are set hard enough to prevent the wheels from turning, in which case they slide on the rails

The **tractive force** is the force exerted on a structure by the friction of the driving wheels of a locomotive drawing a train of cars; it is usually less than the longitudinal thrust, and may be neglected if the latter is taken into consideration.

## PANEL LOADS

**38. Definition.**—The amount of load that is transferred to a joint of the loaded chord is called a **panel load, or panel concentration.**

**39. Dead Panel Loads**—As the weight of the floor system is a large part of the dead load, the dead panel loads of the loaded chord are larger than those of the unloaded chord. If all the dead load were considered as applied along the loaded chord, each panel load would equal the uniform load per linear foot multiplied by the length of one panel and divided by 2 (there being two trusses to the bridge). It is convenient to compute first this panel load, and then, if considered necessary, assume an approximate distribution of it between the loaded and the unloaded chord, as explained in Art. **42.**

**40. Live Panel Loads.**—All the live load is applied along the loaded chord. In highway bridges, the load per linear foot is equal to the specified load per square foot on the floor, multiplied by the clear width of the roadway, plus a like product for the sidewalks, if any. The panel load is then equal to one-half this total uniform load multiplied by a panel length Some highway bridges have only one sidewalk, which is sometimes supported outside of the truss. In this case, the load on and stresses in each truss must be computed separately. In railway bridges, if the load is a uniform load per linear foot, the panel load is found as just

stated; if it consists of a series of wheel loads, the panel loads will vary in amount. The treatment of this class of loads is considered in a subsequent Section.

**41. Wind Panel Load.**—The wind pressure per linear foot, assumed or computed for either chord, multiplied by the panel length gives the wind panel load for that chord.

**42. Illustrative Example.**—As an example, let a through highway bridge, such as is shown in Fig. 14, contain seven equal panels, each 20 feet long, and have a clear width of roadway of 16 feet Suppose the dead load to be 600 pounds per linear foot, and the live load to be 100 pounds

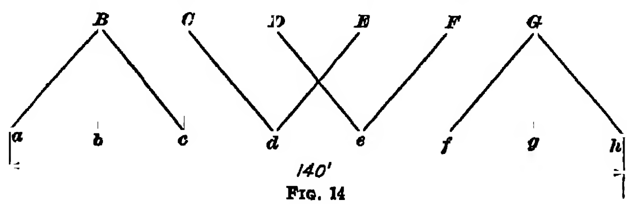


FIG. 14

per square foot of roadway. The dead panel load will then be

$$\frac{600 \times 20}{2} = 6{,}000 \text{ pounds}$$

and the live panel load,

$$\frac{100 \times 16 \times 20}{2} = 16{,}000 \text{ pounds}$$

Assuming all the dead load to be applied at the joints of the loaded chord, each of the joints $b, c, d, e, f$, and $g$ will be loaded with 6,000 pounds dead load; while, if the bridge sustains a live load throughout its length, each of the joints will also have a load of 16,000 pounds At the points $a$ and $h$, there is a half-panel load consisting of 3,000 pounds dead load and 8,000 pounds live load. These half-panel loads are carried directly by the supports and do not affect the trusses, and hence can be omitted in the calculation of stresses. A seven-panel truss would thus be considered as loaded with six intermediate panel loads; a six-panel truss would likewise have five intermediate panel loads; etc. If it is desired to distribute the dead load between the lower and

the upper chord, one-third of the load, or 2,000 pounds, may be assumed as acting at each of the upper joints, and 4,000 pounds at each of the lower joints, except at *a* and *h*, each of which would still carry the half-panel load of 3,000 pounds

### EXAMPLES FOR PRACTICE

1. A bridge 99 feet long is designed to sustain a live load of 100 pounds per square foot on a roadway 16 feet wide, clear width The trusses are divided into 6 panels What is. (*a*) the live load per linear foot? (*b*) the panel live load? Ans $\begin{cases} (a) & 1{,}600 \text{ lb.} \\ (b) & 13{,}200 \text{ lb.} \end{cases}$

2 An eight-panel bridge of 120 feet span carries a roadway 18 feet wide, clear width The live load assumed for the trusses is 96 pounds per square foot of roadway. What is (*a*) the live load per linear foot? (*b*) the panel live load? Ans $\begin{cases} (a) & 1{,}728 \text{ lb.} \\ (b) & 12{,}960 \text{ lb.} \end{cases}$

3. If for the bridge of example 2 the wind load per linear foot is assumed as 300 pounds for the lower chord and 150 pounds for the upper chord, what is the panel wind load. (*a*) for the lower chord? (*b*) for the upper chord? Ans $\begin{cases} (a) & 4{,}500 \text{ lb.} \\ (b) & 2{,}250 \text{ lb.} \end{cases}$

4 Suppose, for the bridge described in example 2, that the dead load is assumed to be 760 pounds per linear foot What will the panel dead load be? Ans. 5,700 lb.

### REACTIONS

**43.** In all bridge trusses, one end is free to move horizontally in a longitudinal direction, so that the reactions due to the dead and live loads will be vertical. The effect of wind pressure and other horizontal forces will receive separate consideration.

**44. Dead-Load Reactions.**—By the principles of statics, the reactions exerted by the supports must hold in equilibrium the applied loads. Let Fig. 15 represent any truss acted on by the panel loads $W$ as shown. Let $R_1$ and $R_2$ be the reactions. The loads and reactions together comprise all the external forces acting on the structure, and any of the conditions of equilibrium may be applied to these forces. The object being to determine the values of the reactions $R_1$ and $R_2$, it will be convenient to take moments

of the forces about the point $f$. The following equation therefore obtains

$$\Sigma M_f = R_1 \times 100 - W \times 80 - W \times 60 - W \times 40 - W \times 20 = 0$$

whence $$R_1 = \frac{W \times 200}{100} = 2\,W$$

In like manner, by taking moments about $a$, the reaction $R_2$

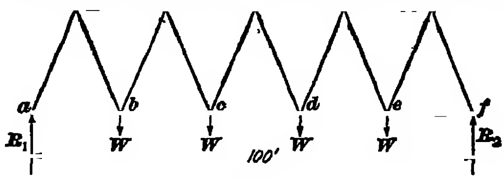


FIG 15

is found to be $2\,W$. Or, by putting the sum of the vertical forces equal to zero,

$$R_1 - 4\,W + R_2 = 0;$$

whence, as before,

$$R_2 = 4\,W - R_1 = 4\,W - 2\,W = 2\,W$$

In this case, where the load is symmetrical, it is evidently unnecessary to write out these equations, since the reactions must be equal, and each must be equal to one-half the total load, or to $2\,W$

**45. Live-Load Reactions.**—If the truss is only partly loaded, as in Fig. 16, the reactions are not equal, and it is necessary to calculate them by applying the principles of

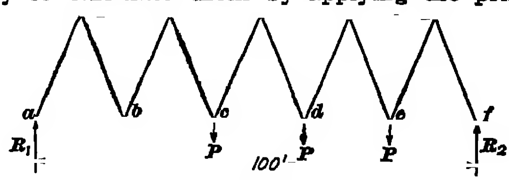


FIG 16

statics as just stated. In the truss shown, with $f$ as a center of moments, we have

$$\Sigma M_f = R_1 \times 100 - P \times 60 - P \times 40 - P \times 20 = 0; \qquad (1)$$

whence $$R_1 = \frac{P \times 120}{100} = 1.2\,P$$

Also, $$R_1 + R_2 - 3\,P = 0;$$

whence $$R_2 = 3\,P - R_1 = 3\,P - 1.2\,P = 1.8\,P$$

This method of calculation is perfectly general and applies as well when the panel loads or panel lengths are unequal.

**46.** Where the panel lengths are equal, as is usually the case, the reactions for a partial load, as in Fig. 16, can be found more readily by writing equation (1), Art. **45**, in a little different form. Taking the panel length as the unit of length, and changing the order of arrangement, we have

$$R_1 \times 5 - P \times 1 - P \times 2 - P \times 3 = 0;$$

whence $$R_1 \times 5 = 1P + 2P + 3P$$

and $$R_1 = \tfrac{1}{5}P + \tfrac{2}{5}P + \tfrac{3}{5}P$$

Thus, it is seen that the left reaction is equal to one-fifth the load at *e*, plus two-fifths the load at *d*, plus three-fifths the load at *c*. This statement can be written at once by inspection; for, as regards the load at *e*, it is known, from the principle of moments, that one-fifth will be carried by the left support, and four-fifths by the right; likewise, two-fifths of the load at *d* will be carried at *a* and three-fifths at *f*, etc. This method of getting reactions for partial loads, *when the panel lengths are equal*, is very convenient and will be frequently employed hereafter.

EXAMPLE —(*a*) Assuming the truss shown in Fig 17 to be loaded at each of the lower chord joints with a load of 5,000 pounds, to calculate the reactions. (*b*) Assuming the joints *d*, *e*, and *f* only, to be loaded with a load of 5,000 pounds each, to calculate the reactions

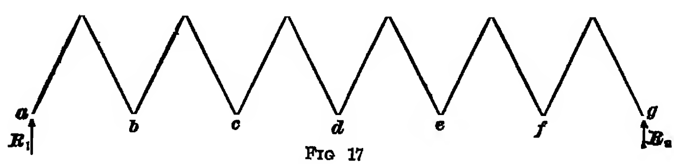

FIG 17

SOLUTION.—(*a*) In this case, the load is symmetrical; therefore,

$$R_1 = R_2 = \frac{5 \times 5{,}000}{2} = 12{,}500 \text{ lb.} \quad \text{Ans.}$$

(*b*) Taking moments about *g*,

$$R_1 \times 6 - 5{,}000 \times 1 - 5{,}000 \times 2 - 5{,}000 \times 3 = 0;$$

whence

$$R_1 = \frac{5{,}000 \times 1 + 5{,}000 \times 2 + 5{,}000 \times 3}{6} = 5{,}000 \text{ lb.} \quad \text{Ans}$$

Taking moments about $a$

$$-R_2 \times 6 + 5{,}000 \times 3 + 5{,}000 \times 4 + 5{,}000 \times 5 = 0;$$

whence

$$R_2 = \frac{5{,}000 \times 3 + 5{,}000 \times 4 + 5{,}000 \times 5}{6} = 10{,}000 \text{ lb. Ans.}$$

EXAMPLES FOR PRACTICE

1. If the truss shown in Fig 17 carries a panel load of 6,000 pounds at each of the lower chord joints, what are the two reactions?

$$\text{Ans} \begin{cases} R_1 = 15{,}000 \text{ lb.} \\ R_2 = 15{,}000 \text{ lb.} \end{cases}$$

2. If the truss shown in Fig. 17 carries a panel load of 6,000 pounds at each of the joints $b$, $c$, and $d$, what are the two reactions?

$$\text{Ans} \begin{cases} R_1 = 12{,}000 \text{ lb.} \\ R_2 = 6{,}000 \text{ lb.} \end{cases}$$

# SHEARS AND MOMENTS

## MAXIMUM MOMENTS AND SHEARS IN BEAMS

### BENDING MOMENTS

**47.** It is desirable here to review some of the principles explained in *Strength of Materials*, Part 1, in connection with beams, and to add sufficient further discussion to make complete the analysis of maximum moments and shears in simple beams and trusses for fixed and for moving uniform loads. In designing a beam, it is necessary to know the maximum bending moments and the maximum shears at various sections. From the former, the flange or fiber stresses are found; and from the latter, the shearing or web stresses. It will be sufficient here to deal with this subject in a general manner, stating the fundamental principles. The application of these principles to the calculation of the actual flange and web stresses will be taken up in connection with design, where it can be more conveniently treated

**48. Bending Moments Due to a Uniform Dead Load.** As explained in *Strength of Materials*, Part 1, the bending moment at any section of a beam is the sum of the

moments of all the external forces to the left or right of the section about the neutral axis of the section. In Fig. 18 ($a$), $AB$ is a beam of length $l$, loaded with a uniform load of

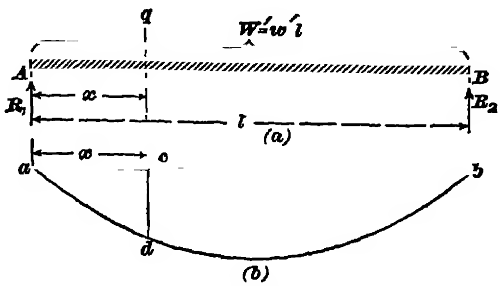

FIG 18

$w'$ units of weight per unit of length. The total load is $w' l$, and each reaction is equal to $\frac{w' l}{2}$. The bending moment $M'$ at any section $q$, distant $x$ from the left end, is given by the formula

$$M' = R_1 x - w' x \times \frac{x}{2};$$

or, substituting the value of $R_1$, and factoring,

$$M' = \frac{w'}{2}(l - x)\, x \qquad (1)$$

This moment is a maximum at the center, where its value is given by the formula

$$\text{Max. } M' = \frac{w' l^2}{8} \qquad (2)$$

The bending moments at all points of the beam due to this uniform load are represented graphically in Fig. 18 ($b$) by the moment curve $a d b$, which is a parabola and shows how the moment varies along the beam.

**49. Bending Moment Due to a Uniform Live Load.** The maximum bending moment at any section of a simple beam, caused by a uniform live load, will occur when the load extends entirely across the span. The beam being fully loaded for maximum moments, these moments will be given by formula 1 of the preceding article, except that $w'$

should be replaced by the live load $w''$ per unit of length. If, therefore, the maximum live-load moment at the point $q$ is denoted by $M''$, then

$$M'' = \frac{w''}{2}(l - x)\,x \qquad (1)$$

From the preceding article,

$$M' = \frac{w'}{2}(l - x)\,x$$

Dividing the first by the second of these two equations, the result is

$$\frac{M''}{M'} = \frac{w''}{w'};$$

whence

$$M'' = M' \times \frac{w''}{w'} \qquad (2)$$

When, therefore, the dead-load moment at any section of the beam has been determined, the maximum live-load moment at the same section is obtained by multiplying the dead-load moment by the ratio $\frac{w''}{w'}$ of the live load per unit of length to the dead load per unit of length.

If the total or resultant bending moment at $q$ is denoted by $M$, then

$$M = M' + M'';$$

or, substituting the values of $M'$ and $M''$,

$$M = \frac{w' + w''}{2}(l - x)\,x \qquad (3)$$

EXAMPLE.—A beam 40 feet long supports a dead load of 500 pounds per foot and a live load of 1,800 pounds per foot. What are the dead-load and maximum live-load bending moments at points 10 feet apart along the beam?

SOLUTION —The moments will be found by means of formula 1, Art 48, and formula 2, Art 49. Here, $l = 40$, $w' = 500$ lb, $w'' = 1{,}800$ lb., and

$$\frac{w''}{w'} = \frac{1{,}800}{500} = 3.6$$

The values of $x$ for the several points are 0, 10, 20, 30, and 40. The moments, in foot-pounds, are as follows:

For $x = 0$, $M' = \frac{500}{2}(40 - 0)\,0 = 0$, $M'' = 0$.

For $x = 10$, $M' = 250(40 - 10)\,10 = 75{,}000$, $M'' = 75{,}000 \times 3\,6 = 270{,}000$.

For $x = 20$, $M' = 250(40 - 20)20 = 100{,}000$, $M'' = 100{,}000 \times 3.6 = 360{,}000$

For $x = 30$, $M' = 250(40 - 30)30 = 75{,}000$, $M'' = 75{,}000 \times 3.6 = 270{,}000$.

For $x = 40$, $M' = 250(40 - 40)40 = 0$, $M'' = 0$.

### EXAMPLES FOR PRACTICE

1. A beam 100 feet long supports a dead load of 600 pounds per foot What is the dead-load moment: (*a*) at the center? (*b*) at a point 75 feet from the left end?

Ans { (*a*) 750,000 ft -lb
(*b*) 562,500 ft -lb.

2. A beam 40 feet long supports a live load of 750 pounds per foot What is the live-load moment at a point. (*a*) 10 feet from the left support? (*b*) 20 feet? (*c*) 30 feet?

Ans { (*a*) 112,500 ft -lb.
(*b*) 150,000 ft -lb
(*c*) 112,500 ft -lb.

3. A beam 80 feet long supports a dead load of 450 pounds per foot and a live load of 1,200 pounds per foot. What is (*a*) the dead-load bending moment, and (*b*) the live-load bending moment at a point 30 feet from the left support?

Ans { (*a*) 337,500 ft -lb
(*b*) 900,000 ft -lb

4 In example 3, what is the total bending moment at the center of the beam?

Ans. 1,320,000 ft -lb

### SHEARS

**50. Shear and Its Sign.**—The shear at any section of a beam is defined as the sum of all the vertical forces acting on the beam to the left or right of the section. The correct

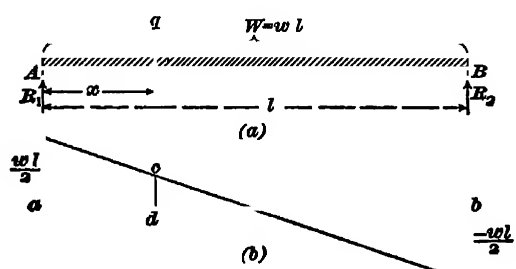


FIG 19

sign will be given by considering the forces on the left and taking upward forces as positive and downward forces as

negative. If the load is a uniform load $w$ per unit of length, Fig. 19 ($a$), the shear at any point whose distance from the left end is $x$ is given by the formula

$$V = R_1 - w\,x = \frac{w\,l}{2} - w\,x$$

At the left end of the beam, where $x = 0$, the shear is equal to $R_1$, or $\frac{w\,l}{2}$; at the center, where $x = \frac{l}{2}$, the shear is 0; and at the right end, where $x = l$, the shear is $-\frac{w\,l}{2}$, or $-R_2$. The shear diagram for dead load is shown in Fig. 19 ($b$).

It is to be noted that the shears on the right of the center are negative, and those on the left are positive, but that for two points equidistant from the center the shears are of the same numerical value.

**51.** The meaning of positive and negative shears will be made clearer by a study of Fig. 20, which shows how the forces acting tend to shear the beam at any given section. *Positive shear* at any section $q$ occurs when the external forces tend to move the left-hand portion upwards and the right-hand portion downwards; *negative shear* occurs when the external forces tend to move the left-hand portion downwards and the right-hand portion upwards.

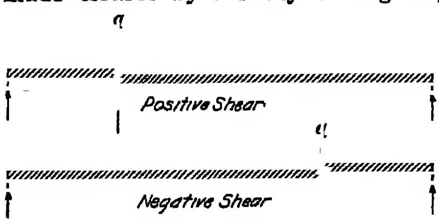


FIG. 20

**52. Live-Load Shears.**—In order to arrive at a general rule for determining the maximum live-load shear at any section of a beam, it will be convenient to consider first the effect of a single load $W$, Fig. 21, placed anywhere on the beam. Let $q$ be the section at which the shear is required. Considering the weight $W$ first at any point on the *right* of the section, it is seen that the only external force acting on the left of the section will be the reaction. The shear at the section will then be equal to this reaction, and will be *positive*

If the load $W$ is placed anywhere to the *left* of the section, the shear will be equal to the left reaction *minus* the load $W$; and, as the reaction is always less than the load, the resultant of the reaction and the load will act *downwards*, and the shear will be *negative*.

It is thus seen that a load placed anywhere on the right of a section will cause positive shear in that section, and if placed

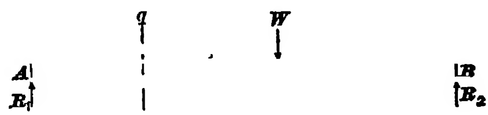


FIG 21

anywhere on the left of the section, it will cause negative shear. From this it follows that, if a number of loads is to be placed on a beam so as to cause the maximum positive shear at a given section, as many loads as possible should be placed on the right of the section, and none on the left. For maximum negative shear, the reverse would be true. If the given load is a uniform live load, the maximum positive shear at any section obtains when the beam is loaded on the right of the section only, and the maximum negative shear, when the beam is loaded on the left.

**53.** Applying the rule just given to any beam $AB$, Fig. 22 $(a)$, the maximum positive live-load shear at any

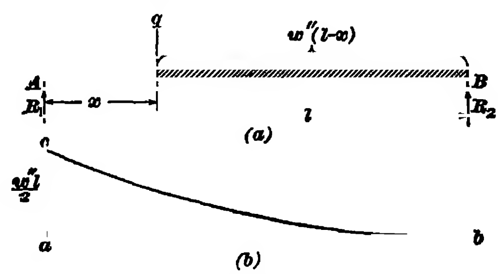


FIG 22

section $q$, distant $x$ from the left end, due to a uniform load of $w''$ per unit of length, obtains when the load covers all the part of the beam to the right of $q$. The shear, which will be denoted by $V''$, will be equal to the left reaction. Taking

moments about $B$, and noting that the total load on the beam is equal to $w''(l-x)$, and that its lever arm about $B$ is equal to $\frac{l-x}{2}$, the following equation obtains:

$$R_1 \times l - w''(l-x) \times \frac{l-x}{2} = 0;$$

whence
$$R_1 = \frac{w''(l-x)\left(\frac{l-x}{2}\right)}{l} = \frac{w''}{2l}(l-x)^2$$

As the maximum live-load shear is equal to $R_1$, then

$$V'' = \frac{w''}{2l}(l-x)^2$$

If the load covers the whole beam, $x = 0$, and

$$V'' = \frac{w'' l^2}{2l} = \frac{w'' l}{2}$$

If the load covers half the beam, $x = \frac{l}{2}$, and, therefore,

$$V'' = \frac{w''}{2l}\left(l - \frac{l}{2}\right)^2 = \frac{w''}{2l}\left(\frac{l}{2}\right)^2 = \frac{w''}{2l} \times \frac{l^2}{4} = \frac{w'' l}{8}$$

In Fig. 22 ($b$), the ordinates to the curve $cb$ represent the

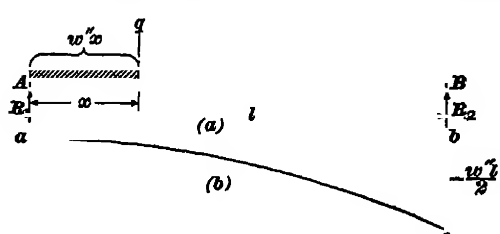


FIG 23

maximum positive shears in the beam at all points, obtained by giving to $x$ values between $l$ and 0. This curve is a parabola with its vertex at $b$, and may be constructed by the methods explained in *Rudiments of Analytic Geometry.*

**54.** The maximum live-load negative shear at any point in the beam will occur when the beam is loaded on the left at that point, as shown in Fig 23 ($a$). The maximum shear at section $q$ will equal the left reaction minus the load $w''x$.

The left reaction $R_1$ is found by taking moments, as before, about the point $B$:

$$R_1 = \frac{w'' x\left(l - \frac{x}{2}\right)}{l} = w'' x - \frac{w'' x^2}{2l}$$

and the maximum negative shear is

$$V'' = R_1 - w'' x = -\frac{w'' x^2}{2l}$$

This value is equal to the right reaction, and might have been found by taking moments about $A$, without previously computing $R_1$. Fig. 23 ($b$) shows the shear diagram for negative shears.

By examining the values of the negative shears, beginning at the right end and passing toward the left, it will be observed that the maximum positive shear at any point is numerically equal to the maximum negative shear at a point in the opposite end of the beam equidistant from the center.

**55. Illustrative Example.**—Let it be required to determine the dead-load shears at sections 5 feet apart in a beam 60 feet long, Fig. 24. Let the dead load be 400 pounds

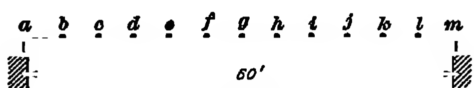

FIG 24

($= w'$) per linear foot, and the live load 1,200 pounds ($= w''$) per linear foot.

1. *Dead-Load Shears.*—The reactions are each equal to

$$\frac{400 \times 60}{2} = 12{,}000 \text{ pounds}$$

The shears will be found by the formula of Art. **50**, substituting $w'$, or 400, for $w$.

At $a$, $x = 0$, and $V' = R_1 = +12{,}000$ pounds.

At $b$, $x = 5$, and $V' = 12{,}000 - 400 \times 5 = +10{,}000$ pounds.

At $c$, $x = 10$, and $V' = 12{,}000 - 400 \times 10 = +8{,}000$ pounds.

In like manner, the shear for each of the other given sections is found. The results are given in the second column of the accompanying table.

TABLE OF SHEARS

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| | | Maximum Live-Load Shear | | Combined Shear | |
| Section | Dead-Load Shear | Positive | Negative | Dead-Load and Maximum Positive Live-Load Shear | Dead-Load and Maximum Negative Live-Load Shear |
| *a* | +12,000 | +36,000 | 0 | +48,000 | +12,000 |
| *b* | +10,000 | +30,250 | − 250 | +40,250 | + 9,750 |
| *c* | + 8,000 | +25,000 | − 1,000 | +33,000 | + 7,000 |
| *d* | + 6,000 | +20,250 | − 2,250 | +26,250 | + 3,750 |
| *e* | + 4,000 | +16,000 | − 4,000 | +20,000 | 0 |
| *f* | + 2,000 | +12,250 | − 6,250 | +14,250 | − 4,250 |
| *g*(center) | 0 | + 9,000 | − 9,000 | + 9,000 | − 9,000 |
| *h* | − 2,000 | + 6,250 | −12,250 | + 4,250 | −14,250 |
| *i* | − 4,000 | + 4,000 | −16,000 | 0 | −20,000 |
| *j* | − 6,000 | + 2,250 | −20,250 | − 3,750 | −26,250 |
| *k* | − 8,000 | + 1,000 | −25,000 | − 7,000 | −33,000 |
| *l* | −10,000 | + 250 | −30,250 | − 9,750 | −40,250 |
| *m* | −12,000 | 0 | −36,000 | −12,000 | −48,000 |

It is to be noted that the shears beyond *g*, the center section, may be written out at once from the values to the left of this section.

2. *Live-Load Shears.*—The maximum positive live-load shears are obtained from the formula of Art. **53.** At *a*, where $x = 0$,

$$V'' = R_1 = \frac{1{,}200 \times 60}{2} = +36{,}00 \text{ pounds}$$

At *b*, $x = 5$, and

$$V'' = \frac{1{,}200}{2 \times 60}(60 - 5)^2 = +30{,}250 \text{ pounds}$$

At *c*, $x = 10$, and

$$V'' = \frac{1{,}200}{2 \times 60}(60 - 10)^2 = +25{,}000 \text{ pounds}$$

and so on. These shears are given in the third column of the above table. The maximum negative live-load shears are found in like manner, by the use of the formula in Art. 54. The fifth and the sixth column of the table contain the combined shears, found as explained below.

3. *Combined Shears.*—Since the dead load is always present, the maximum total shear at any section of the beam will be found by combining the dead-load shear with one or the other of the maximum live-load shears. Combining with the maximum positive shears, the fifth column of the preceding table is obtained; while the sixth column is obtained by combining the dead-load shears with the maximum negative live-load shears. By comparing these two columns, it will be noticed that, for sections to the left of the center, the maximum shears are the positive shears of column 5; while to the right of the center they are the negative shears of column 6. For two sections equidistant from the center, the maximum shears are numerically equal.

It is to be further noted that in column 5 there is a small positive shear at the sections $h$, zero shear at $i$, and negative shears beyond. These negative shears, which are smaller than the dead-load shears, are equal to the difference between the dead-load negative and the live-load positive shears. They are in fact the least negative shears that can occur in the beam. In the same way, column 6 gives, in the upper part, the least positive shears down as far as $e$, then the maximum negative shears for the remainder of the beam. Taking the two columns together, the greatest *range* of shear is obtained that can possibly occur in the beam at the several sections for any positions of the live load. Thus, at section $a$, the shear may be as high as + 48,000 and as low as + 12,000; at $b$, the limits are + 40,250 and + 9,750; at $e$, + 20,000 and 0; at $f$, + 14,250 and − 4,250; and at $g$, + 9,000 and − 9,000. On the right end, the limits are the same numerically at corresponding sections, but are of opposite sign. From this discussion, it is plain that only positive shears can exist from $a$ to $e$; both kinds are possible from $e$ to $i$; and only negative shears beyond $i$. This table should

be carefully studied, as the relations here illustrated are of great importance in the analysis of trusses.

4. *Summary of Results.*—Summarizing the results of the foregoing analysis, the following statement may be made regarding a beam subjected to both a dead and a uniform live load: (1) The *maximum shear* at any section to the left of the center will be *positive*, and may be found by adding to the dead-load shear the maximum positive live-load shear, the beam being loaded to the *right* of the section. (2) The *minimum shear* at any section to the left of the center may be found by combining the dead-load shear with the maximum negative live-load shear, the beam being loaded to the *left* of the section Near the end of the beam, these minimum shears will be positive, but near the center, where the live-load negative shears exceed numerically the dead-load positive shears, the resulting values will be negative For sections on the right of the center, exactly the reverse of this holds true: the shears are numerically equal to, but of opposite sign from, those on the left of the center.

**56.** In the design of beams and plate girders, the sign of the shear is usually immaterial, and the maximum numerical value is all that is needed. In the case of trusses, however, it is essential to know the sign of the shear and of the stresses resulting therefrom. Furthermore, in many cases it will be necessary to find not only the maximum shear and stress but also the minimum values, so that the greatest range of stress to which the member is subjected may be known.

## MAXIMUM MOMENTS AND SHEARS IN TRUSSES

**57. Notation—Reactions.**—So far, it has been assumed that the load was applied at every point along the beam. In the case of trusses and plate-girder bridges with floor systems, the load is applied along the stringers and transmitted to the trusses or girders by the floorbeams A truss, for example, is subjected to loads acting at the joints of the chords. It is necessary to find the maximum moments

at the joints and the maximum shears in the panels. In what follows, $l$ will denote the length of the span; $p$, the length of one panel; $n$, the number of panels; $w'$, the dead load per unit of length; $w''$, the live load per unit of length; $W'$, the panel dead load; $W''$, the panel live load; $R_1$, the left reaction; and $R_2$, the right reaction. When necessary to distinguish between reactions due to the dead and to the live load, the former will be denoted by one accent—$R_1'$, $R_2'$; the latter, by two—$R_1''$, $R_2''$.

**58.** The loads $W'$, $W''$ are taken as acting at the joints. By referring to Fig. 25, where $n = 6$, it will be observed that the number of panel points (the end joints are not counted)

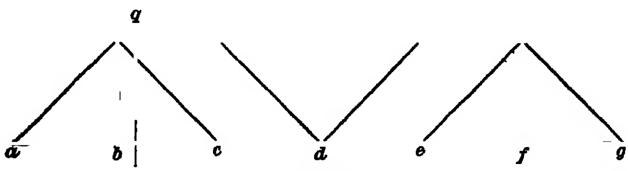

FIG 25

is 5, or $6 - 1$. In general, the number of loaded joints is $n - 1$, or one less than the number of panels. This being the case, the total dead load on the truss is $(n - 1) W'$, and the total live load, when all the truss is loaded, is $(n - 1) W''$. Therefore, in this case,

$$R_1' = \tfrac{1}{2}(n - 1) W' = \tfrac{1}{2}(n - 1) p w'$$
$$R_1'' = \tfrac{1}{2}(n - 1) W'' = \tfrac{1}{2}(n - 1) p w''$$

It should be kept in mind that $w'$ and $w''$ are, in this discussion, loads per unit of length for *one truss*, not for the *bridge*.

**59. Dead-Load Moments and Shears.**—The dead-load moments and shears are determined as for a beam loaded at various points In the case of a truss, those points are the panel points, each of which carries a load equal to $W'$ Thus, the moment at $b$, Fig. 25, is $R_1' p$; at $c$, $R_1' \times 2p - W'p$; at $d$, $R_1' \times 3p - W' \times 2p - W'p$; etc. The shear between $a$ and $b$ is $R_1'$; between $b$ and $c$, $R_1' - W'$; between $c$ and $d$, $R_1' - 2W'$; etc. The shear between $a$ and $b$

is referred to as the shear in the panel $ab$; that between $b$ and $c$, as the shear in the panel $bc$; etc.

**60. Live-Load Moments.**—The maximum live-load bending moment at any joint occurs when the whole truss is loaded; that is, when a load equal to $W''$ is applied at every joint of the loaded chord, and is calculated in the same manner as the dead-load moment. Thus, the maximum live-load bending moment at $d$ is equal to

$$R_1'' \times 3p - W'' \times 2p - W'' \times p$$

**61. Live-Load Shears.**—For the maximum positive live-load shear at any section in a beam, it has been shown that the beam should be fully loaded up to that section from the right, and should have no load on the left of the section. In any panel $bc$ of the truss represented in Fig. 25, the positive shear on any section $q$ equals the left reaction minus the load, if any, at the joint $b$. For a maximum positive shear, the live load should extend up to joint $c$ at least, for up to this point the reaction is the only force acting on the left of the section, and this increases as the load is moved from the right until it reaches $c$. If the load advances beyond $c$, the joint $b$ will begin to receive some load, and the shear will then be equal to the left reaction minus the load

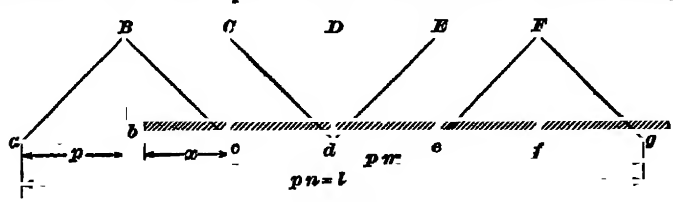


FIG 26

at $b$. As the load continues to move to the left, the reaction is increased, but the load at $b$ is also increased, so that the shear in the panel may be increased or decreased by such movement, according to the relative increase in the left reaction and the load at $b$. There is a certain point in each panel to which the load should extend in order that the shear in that panel may be a maximum. That point is determined as explained in the next article.

**62.** Let $bc$, Fig. 26, be any panel in which the maximum live-load shear is to be determined. Let the load, whose amount per unit of length will be denoted by $w''$, cover all the panels on the right of $c$, and extend a distance $x$ to the left. The number of panels in the truss will be denoted by $n$, and the number of panels on the right of $c$, by $m$.

It can be shown that the greatest shear in $bc$ occurs when

$$x = \frac{pm}{n-1} \qquad (1)$$

and that the value of the shear, for one truss, is then given by the formula

$$V'' = W'' \frac{m^2}{2(n-1)} \qquad (2)$$

NOTE.—The derivation of formulas 1 and 2 is as follows: For the reaction $R_1''$, found by the usual methods, we have

$$R_1'' = \frac{w''(x+pm)^2}{2l}$$

The load on the left of $c$ is $w''x$, whose lever arm about $c$ is $\frac{x}{2}$. This load is to be resolved into two components, one acting at $b$ and the other at $c$ Let $W_b''$ be the component at $b$, or the partial panel load at the latter point. Taking moments about $c$,

$$W_b''p = w''x \times \frac{x}{2} = \frac{w''x^2}{2},$$

whence

$$W_b'' = \frac{w''x^2}{2p}$$

The live-load shear $V_{bc}''$ in the panel $bc$ is equal to $R_1'' - W_b''$; or, substituting the values of $R_1''$ and $W_b''$,

$$\begin{aligned}
V_{bc}'' &= \frac{w''(x+pm)^2}{2l} - \frac{w''x^2}{2p} \\
&= \frac{w''}{2} \times \frac{px^2 + 2p^2mx + p^2m^2 - lx^2}{lp} \\
&= \frac{w''}{2} \times \frac{p^2m^2 + 2p^2mx - (l-p)x^2}{lp}
\end{aligned}$$

or, since $l = np$, and, therefore, $l - p = np - p = (n-1)p$,

$$\begin{aligned}
V_{bc}'' &= \frac{w''}{2} \times \frac{p^2m^2 + 2p^2mx - (n-1)px^2}{lp} \\
&= \frac{w''}{2} \times \frac{p^2m^2 + 2pmx - (n-1)x^2}{l} \\
&= \frac{w''}{2} \times \frac{p^2m^2 + (n-1)\left(\frac{2pm}{n-1}x - x^2\right)}{l} \\
&= \frac{w''}{2} \times \frac{p^2m^2 + (n-1)\left[\left(\frac{pm}{n-1}\right)^2 - \left(\frac{pm}{n-1} - x\right)^2\right]}{l}
\end{aligned}$$

This expression evidently has its greatest value when $\frac{p\,m}{n-1} - x = 0$ Hence, for the position of maximum live-load shear,

$$x = \frac{p\,m}{n-1},$$

which is formula **1.**

Making $\left(\frac{p\,m}{n-1} - x\right)^2 = 0$, and writing $p\,n$ for $l$, the resulting value of the maximum shear becomes

$$V_{bc}'' = \frac{w''}{2} \times \frac{p^2 m^2 + (n-1)\frac{p^2 m^2}{(n-1)^2}}{p\,n}$$

$$= \frac{w''}{2} \times \frac{p\,m^2 + \frac{p\,m^2}{n-1}}{n}$$

$$= \frac{w''}{2} \times \frac{p\,m^2(n-1) + p\,m^2}{n(n-1)} = \frac{w'' p\,m^2}{2(n-1)}$$

or, since $w''p$ is the value $W''$ of one panel load,

$$\text{Max } V_{bc}'' = W'' \frac{m^2}{2(n-1)},$$

which is formula **2.**

EXAMPLE —Referring to Fig 26, suppose that the panel length is 15 feet, and that the live load is 1,200 pounds per linear foot Required· (*a*) the distance $x$ that the load should extend to the left of $c$, in order that the live-load shear in the panel $b\,c$ may be a maximum, (*b*) the value of that maximum shear for one truss

SOLUTION —(*a*) To apply formula **1**, we have $p = 15$, $m = 4$, and $n = 6$. Substituting in the formula,

$$x = \frac{15 \times 4}{6-1} = 12 \text{ ft. Ans.}$$

(*b*) To apply formula **2**, we have, in addition to the values just stated,

$$W'' = \frac{1{,}200 \times 15}{2} = 9{,}000 \text{ lb.}$$

Substituting in the formula,

$$V_{bc}'' = 9{,}000\left(\frac{4^2}{2(6-1)}\right) = 14{,}400 \text{ lb. Ans.}$$

## EXAMPLES FOR PRACTICE

1 A five-panel bridge whose panel length is 14 feet carries a live load of 1,440 pounds per foot Required. (*a*) the distance $x$ that the load should extend to the left of the third joint (counted from the left end of the bridge) in order that the live-load shear in that panel may be a maximum; (*b*) the value of the maximum shear in that panel, for one truss.

Ans $\begin{cases} (a) & 10\ 5 \text{ ft.} \\ (b) & 11{,}340 \text{ lb.} \end{cases}$

2 A seven-panel bridge whose panel length is 16 feet carries a live load of 1,500 pounds per foot Required· (*a*) the distance $x$ that the load should extend to the left of the fourth joint (counted from the left end of the bridge) in order that the live-load shear in that panel may be a maximum, (*b*) the value of the maximum shear in that panel, for one truss.

Ans $\begin{cases} (a) & 10\frac{2}{3} \text{ ft} \\ (b) & 16{,}000 \text{ lb.} \end{cases}$

3 (*a*) In the bridge in example 2, how far should the load extend to the left of the third joint (counted from the right end of the bridge) in order that the live-load shear in that panel should be a maximum? (*b*) What is the value of that shear for one truss?

Ans $\begin{cases} (a) & 5\frac{1}{3} \text{ ft.} \\ (b) & 4{,}000 \text{ lb.} \end{cases}$

**63. Approximate Method of Calculation.**—In calculating the maximum live-load shear in any panel, it is customary to assume that all panel points on one side of the panel are loaded with one full-panel load and that those on the other side have no load. Thus, the maximum live-load shear in *b c*, Fig. 26, is found by assuming each of the joints *f*, *e*, *d*, and *c*, to carry a full-panel load, and the joint *b* to be unloaded. The shear *b c* is then taken equal to the left reaction corresponding to this assumed distribution of the load. It is, however, obviously impossible fully to load one panel point of a truss by means of a uniform load placed on the floor, without also causing a half-panel load at the next panel point. For example, the point *c* cannot be fully loaded unless the floor in the panels *b c* and *c d* is loaded, and this gives a half-panel load at the point *b*. If the load at *b* is neglected, and it is assumed that there is a full-panel load at *c*, the computed shear in the panel *b c* will be too large. This approximate method, therefore, gives results larger than the actual shears, but, as the error is not great, it is usually neglected. The error is greatest near the center of the truss, and zero at the end—a result not altogether undesirable, as it tends to increase the size of the small web members near the center.

# GENERAL METHODS OF CALCULATING STRESSES

## INTRODUCTION

**64. Conditions of Equilibrium.**—The stresses in the members of a structure are determined by applying the general conditions of equilibrium established and illustrated in *Analytic Statics*, Part 2, and in *Graphic Statics*. As there stated, when a structure or any part of it is in equilibrium under the action of forces, the forces fulfil the following conditions: (1) the algebraic sum of their vertical components is equal to zero; (2) the algebraic sum of their horizontal components is equal to zero; (3) the algebraic sum of their moments about any point or line is equal to zero.

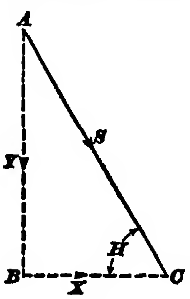

Fig. 27

If $S$, Fig. 27, represents the stress in any member; $H$, the angle that the member makes with a horizontal line; $X$, the horizontal component of $S$; and $Y$, the vertical component; then, any two of the quantities $S, X, Y$ can be found from the right triangle $ABC$ when the other quantity and the angle $H$ are known. Thus,

$X = S \cos H$, $Y = S \sin H = X \tan H$, $S = X \sec H$, etc.

The functions of the angle $H$ can usually be determined from the dimensions of the truss, in which the lengths of the horizontal and of the vertical members are generally known. Thus, for calculating the stresses in $Bc$, Fig. 29 ($a$), we have

$$\tan H = \tan Bcb = \frac{Bb}{bc};$$

$$\sec H = \frac{Bc}{bc} = \frac{\sqrt{\overline{Bb}^2 + \overline{bc}^2}}{bc} = \sqrt{1 + \left(\frac{Bb}{bc}\right)^2}, \text{ etc.}$$

As usual, forces acting upwards will be treated as positive, and those acting downwards as negative.

For a vertical member, the angle $H$ is 90°, the horizontal component is zero, and the vertical component is equal to the stress in the member, for a horizontal member, the angle $H$ is zero, the horizontal component is equal to the stress in the member, and the vertical component is zero.

As explained in *Analytic Statics*, Part 2, the three conditions of equilibrium are thus expressed algebraically:

$$\Sigma X = \Sigma S \cos H = 0$$
$$\Sigma Y = \Sigma S \sin H = 0$$
$$\Sigma M = 0$$

**65. Free-Body Method.**—In applying the foregoing principles to the calculation of the stresses in the members of a truss, the truss is considered as cut into two parts by a surface that cuts several members, among them the member or members whose stresses are sought. Either of the two parts is treated as a free body held in equilibrium by the loads, if any, that act on that part of the truss, and by external forces applied to the members cut by the surface, those external forces being numerically equal to the stresses in the corresponding members. The unknown stresses are then determined by applying the general conditions of equilibrium to the system of forces acting on that part of the truss which is treated as a free body. In order that those conditions can be applied, the cutting surface should not intersect more members in which the stresses are unknown than there are equations of equilibrium.

This subject will be fully illustrated in subsequent articles.

**66. Indeterminate Stresses.**—There are cases in which it is impossible to cut the truss by a surface without cutting more members than there are equations of equilibrium The stresses in some of the members cannot then be found directly by applying the principles of statics to the members cut, and it is necessary to calculate one or more of the stresses independently, or to make an assumption regarding the distribution of stress among them. A truss

in which the stresses cannot be found by the principles of statics is said to be statically **undetermined.** The use of such trusses should be avoided as much as possible.

**67. Total Stresses.**—The total or resultant stress in any member is found by adding together the stress due to dead load and that due to live load. This resultant is sometimes called the **combined stress.**

**68. Analytic and Graphic Methods.**—Stresses may be determined either analytically or graphically. In some cases, the former method is preferable, in other cases, the latter. The analytic method gives exact results, is very easily applied to simple types of structures, such as parallel-chord trusses, and is used to a great extent in trusses that support live loads. The graphic method gives close approximations to the real stresses, is easily applicable to all types of structures, and is especially useful in trusses that support only fixed loads, particularly those trusses that have curved or inclined chords. In practice, it is customary to use the method that will give the desired result with the least amount of work. The stresses found by one method may be checked by applying the other method.

## ANALYTIC METHODS

### METHOD OF SECTIONS

**69. General Description of the Method.**—When the members cut by a surface—usually a plane—are non-concurrent, as when a plane $PQ$, Fig 28, cuts two chord members and a web member, the stresses in one or two of the members cut may be found by applying the equations of equilibrium to the part on either side—customarily the left-hand side—of the cutting surface, that

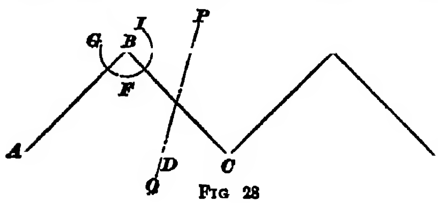


FIG 28

part being treated as a free body. This method of determining stresses is called the **method of sections.** Some of the stresses may be determined by applying the equations of moments ($\Sigma M = 0$), and others by applying the equations of components ($\Sigma X = 0$, $\Sigma Y = 0$). The method is called also the **method of moments and shears.** When the equation of moments is used to determine the stress in one of three members cut, as $AC$ (or $AD$), the origin of moments should be taken at the intersection (in this case $B$) of the other two.

This method, being well adapted to all cases, is used more than any other, but is of special value for the determination of horizontal and vertical stresses.

**70. Special Case.**—The application of the method will be understood by considering a special case. Fig 29 ($a$) shows a truss loaded with three equal loads $W$. The reactions $R_1$ and $R_2$ are each equal to $\frac{3W}{2}$ It is proposed to find the stresses in $BD$, $Bc$, and $bc$. The truss being cut by a plane $PQ$, the part on the left is treated as a free body held in equilibrium by the reaction $R_1$, by the load $W$ at $b$, and by the forces $S_1$, $S_2$, and $S_3$, equal, respectively, to the stresses in $BD$, $Bc$, and $bc$, these forces acting at $K$, $L$, and $N$, as shown in Fig. 29 ($b$). The directions in which $S_1$, $S_2$, and $S_3$ act are not known, but may be assumed: if the value of a stress comes out positive, the assumed direction is correct; if negative, the stress acts in a direction opposite to the one assumed In general, the true direction can be ascertained by inspection.

*Member* $Bc$.—The vertical forces acting on the part $aBKLN$, Fig. 29 ($b$), are $R_1$, $-W$, and $-S_2 \sin H$. Therefore, the equation $\Sigma S \sin H = 0$ gives

$$R_1 - W - S_2 \sin H = 0;$$

whence $$S_2 \sin H = R_1 - W = \frac{3W}{2} - W = \frac{W}{2}$$

and $$S_2 = \frac{W}{2 \sin H} = \frac{W}{2} \csc H = \frac{W}{2} \frac{\sqrt{\overline{bc}^2 + \overline{Bb}^2}}{Bb}$$

The value of $S_2$ is positive; therefore, the assumed direction is correct. If the force $S_2$ is applied to $BL$ at $L$, the joint $B$ being supposed to be fixed, its effect will be to lengthen $BL$; hence, the stress in $BL$, and therefore in $Bc$, is tension. If the value of $S_2$ had come out negative, this would have indicated that the direction of $S_2$ was opposite to

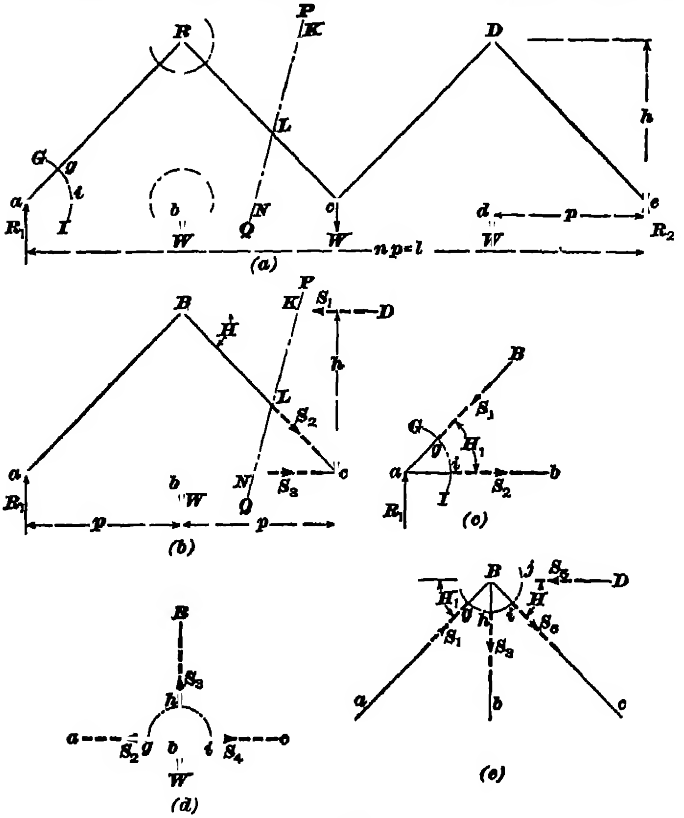

FIG. 29

that assumed, and that the stress in $Bc$ was compression instead of tension.

It will be noticed that the vertical component of the stress in $Bc$ is the shear on the truss at the section $PQ$. For this reason, the method of determining stresses by applying the

equation $\Sigma S \sin H = O$ is called the **method of shears.** This method is especially useful in finding web stresses.

*Member BD.*—From the equation $\Sigma M = 0$, taking the point $c$ as the center of moments and assuming that $S_1$ acts toward the point $B$, we have, since the lever arms and, therefore, the moments of $S_2$ and $S_3$ about $c$ are each zero,

$$R_1 \times 2p - W \times p - S_1 \times h = 0;$$

whence

$$S_1 = \frac{R_1 \times 2p - W \times p}{h} = \frac{\frac{3W}{2} \times 2p - W \times p}{h} = 2W\frac{p}{h}$$

The value of $S_1$ is positive, which shows that the assumed direction is correct, and that the stress in $BD$ is compression.

*Member bc.*—From the equation $\Sigma M = 0$, taking the point $B$ as the center of moments, and assuming that $S_3$ acts away from $N$, we have, since the lever arms and, therefore, the moments of $W$, $S_2$, and $S_1$ about $B$ are each zero,

$$R_1 \times p - S_3 \times h = 0;$$

whence $$S_3 = \frac{R_1 \times p}{h} = \frac{\frac{3W}{2} \times p}{h} = \frac{3p}{2h}W$$

The value of $S_3$ being positive, the assumption that $S_3$ acts away from the point $b$ is correct, and the stress in $bc$ is tension.

## METHOD OF JOINTS

**71. General Description of the Method.**—When the truss is cut by a curved surface, as $GFI$, Fig. 28, in such a manner that all the members cut are concurrent, the stresses in one or two of these members can be determined by considering as a free body that part of the truss which contains the joint (in this case $B$) common to the members cut. This is called the **method of joints**, or the **method of resolution of forces.** The equations best adapted to this method are the equations of components $\Sigma S \sin H = 0$, and $\Sigma S \cos H = 0$.

The method of joints is especially useful for finding stresses in parallel-chord trusses that support fixed loads; it

is not well adapted to trusses with curved or inclined chords, nor to those that support live loads

**72. Illustrative Example.**—Let it be required to calculate the stresses in the members of the truss shown in Fig. 29 ($a$). Since there are, in this case, only two equations of equilibrium, it is necessary to begin with the joint $a$, where only two stresses—that in $a\,b$ and that in $a\,B$—are unknown. The truss is cut by the surface $G\,I$, and the part containing the joint $a$ is treated as a free body, as shown in Fig. 29 ($c$). The forces acting on this part are $R_1$, $S_1$, and $S_2$, the last two being numerically equal to the stresses in $a\,B$ and $a\,b$, respectively In this case, $H_1$ is the angle made by $a\,B$ with the horizontal. The functions of $H_1$ (or of $B\,a\,b$) can be found from the triangle $B\,a\,b$, Fig 29 ($a$).

The vertical component of $S_1$ is $-S_1 \sin H_1$; that of $S_2$ is 0; therefore, the equation $\Sigma\,Y = 0$ gives

$$R_1 - S_1 \sin H_1 = 0,$$

whence

$$S_1 = \frac{R_1}{\sin H_1} = R_1 \csc H_1$$

The horizontal component of $S_1$ is equal to

$$-S_1 \cos H_1 = -\frac{R_1}{\sin H_1} \cos H_1 = -R_1 \cot H_1$$

The equation $\Sigma X = 0$ gives, since the reaction $R_1$, being vertical, has no $X$ component,

$$S_2 - R_1 \cot H_1 = 0,$$

whence

$$S_2 = R_1 \cot H_1$$

It is necessary to consider joint $b$ next before going to joint $B$. The part containing the joint $b$ is shown in Fig. 29 ($d$) as a free body. The forces acting are $S_2$, already determined, $W$, $S_3$, and $S_4$. As neither $S_3$ nor $W$ has a horizontal component, the forces $S_2$ and $S_4$ must be numerically equal; and, as neither $S_2$ nor $S_4$ has a vertical component, $S_3$ must be numerically equal to $W$. This determines the stresses in $b\,B$ and $b\,c$.

The joint $B$ is shown in Fig. 29 ($e$) as a free body acted on by the forces $S_1$ and $S_3$, already determined, and by the unknown forces $S_5$ and $S_6$, which are equal, respectively, to

the stresses in $B D$ and $B c$. Since the force $S_8$, being horizontal, has no vertical component, the equation $\Sigma Y = 0$ takes the following form:

$$S_1 \sin H_1 - S_3 - S_6 \sin H_1 = 0;$$

whence
$$S_6 = \frac{S_1 \sin H_1 - S_3}{\sin H_1}$$

The horizontal component of $S_6$ can now be found, and putting the algebraic sum of this component, that of $S_1$, and that of $S_8$ (which is equal to $S_8$ itself) equal to zero, the value of $S_8$ can be found.

## GRAPHIC METHODS

**73. Method of Moments and Shears.**—In the graphic method, the moments and shears are found by means of the force polygon and the funicular, or equilibrium polygon. The stresses are then found in the same way as in the analytic method of sections. This graphic method (of moments and shears) is as a rule no shorter than the analytic method for finding the stresses in bridge trusses, and its use is not recommended for the ordinary cases of loading. There are special cases of loading, which will be discussed later, in which it is sometimes employed.

Fig 30 (*a*) shows a truss with five panel loads $W$. The force polygon for the external forces, including the reactions, is shown in Fig 30 (*c*), and the equilibrium polygon is shown in Fig. 30 (*b*). As explained in *Graphic Statics*, the moment at any point, such as $c$, is equal to the intercept $c' c''$ multiplied by the normal ray $N$. The shear in any panel, such as $b c$, is equal to $R_1 - W$, which in the force polygon is $(0\text{-}1) - (1\text{-}2) = 0\text{-}2$.

When the truss is cut by the plane $P Q$, Fig. 30 (*a*), the moment of the stress in $B C$ about $c$ is numerically equal to the moment of the external forces on one side of $P Q$ about $c$ (see Art. 70), and therefore the stress in $B C$ is obtained by dividing that moment by the lever arm $C c$, which is the depth of the truss The vertical component in $B c$ is equal to *minus* the shear in the panel $b c$; that is, to *minus* the algebraic sum of the external forces on the left of $P Q$. This

follows from the fact that, if $V$ is the shear on $PQ$ and $Y$ is the vertical component of the stress in $B\,c$, then

$$Y + V = 0;\ \text{whence}\ Y = -V$$

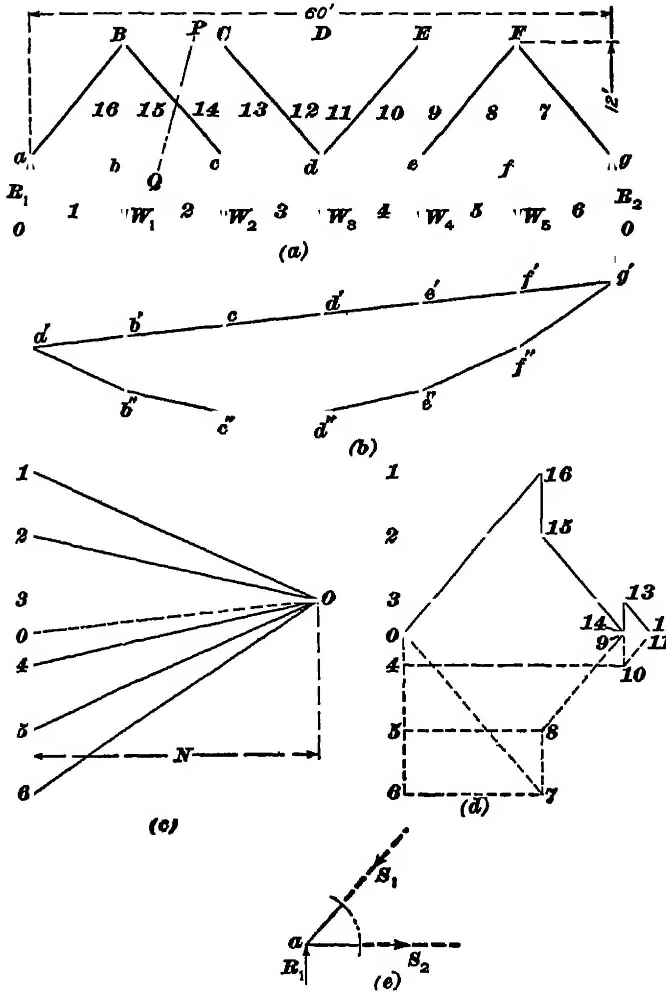


Fig. 36

**74. Method by the Stress Diagram.**—The stresses in the members may also be found by the stress diagram in the

same way as by the method of joints. The stress diagram is first drawn for a joint at which the number of unknown stresses does not exceed two, then for the next joint at which the number of unknown stresses does not exceed two, and so on until the desired stresses are found. This is called the **method by the stress diagram**, and is very well adapted to trusses that support a fixed load, especially those having curved or inclined chords. When the graphic method is shorter than the analytic, the stress diagram gives the result with less work than the graphic method of moments and shears, and is, therefore, preferable.

Fig. 30 (*d*) is the stress diagram for the truss shown in Fig. 30 (*a*). The portion of the diagram in full lines gives the stresses in all the members to the left of the center; the portion in dotted lines gives the stresses in all the members to the right of the center, and is drawn simply as a check, the truss and loading being symmetrical.

The character of the stress in any member may be found from the sense of the vector that represents the stress in the member. As explained in *Graphic Statics*, if the sense of one of the vectors of a force polygon representing a balanced system is known, the senses of all the others may be found by taking them in cyclic order with the known vector. For example, at joint $a$, Fig. 30 ($a$), there are three forces, as shown in Fig. 30 ($e$): $R_1$, $S_1$, and $S_2$. The sense of $R_1$ is upwards and is represented by the vector *0-1* in the stress diagram; the sense of $S_2$ is given by taking *1-16*, and of $S_1$ by taking *16-0* in cyclic order with *0-1*. Then, $S_2$ is a pull, as the direction of *1-16* is toward the right, away from joint $a$; and $S_1$ is a push, as the direction of *16-0* is downwards to the left toward joint $a$. Therefore, the stress in $a\,b$ is tension, and that in $a\,B$ is compression. In like manner, the character of the stress in any other member may be determined.

**EXAMPLES FOR PRACTICE**

1. Using the analytic method of sections, calculate the stresses in the members $bc$, $bC$, and $BC$ of the truss shown in Fig. 31, assuming

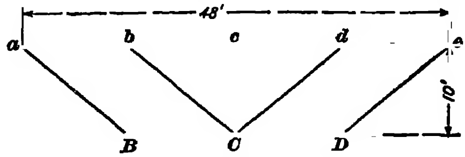


Fig. 31

a load of 5,000 pounds at each of the joints $b$, $c$, and $d$.

Ans. { Stress in $bc$ = 12,000 lb compression
Stress in $bC$ = 3,900 lb tension
Stress in $BC$ = 9,000 lb tension

2. Using the analytic method of joints, calculate the stresses in the members that meet at joint $b$ of the truss shown in Fig 32.

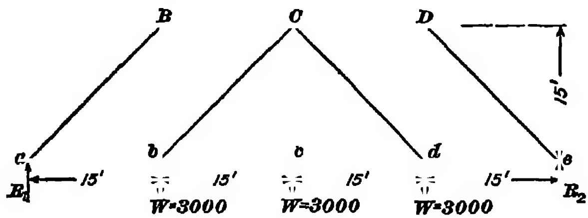


Fig. 32

Ans. { Stress in $ab$ = 4,500 lb tension
Stress in $Bb$ = 4,500 lb tension
Stress in $bC$ = 2,120 lb compression
Stress in $bc$ = 6,000 lb tension

3. Using the stress diagram, determine the stresses in all the members of the left half of the truss shown in Fig 32, assuming a load of 4,500 pounds at each of the joints $b$, $c$, and $d$

Ans. { Stress in $aB$ = 9,550 lb compression
Stress in $ab$ = 6,750 lb tension
Stress in $Bb$ = 6,750 lb tension
Stress in $BC$ = 6,750 lb compression
Stress in $bC$ = 3,180 lb compression
Stress in $bc$ = 9,000 lb tension
Stress in $Cc$ = 4,500 lb tension

# STRESSES IN BRIDGE TRUSSES

(PART 2)

## PARALLEL-CHORD TRUSSES

### THE SINGLE-SYSTEM WARREN TRUSS

#### INTRODUCTION

**1. Description.**—The **Warren truss**, Fig. 1, is a simple type of truss with parallel chords, in which the web members are all inclined and make the same angle with the vertical, giving the truss the appearance of a series of connected isosceles triangles; it is sometimes called the *triangular*

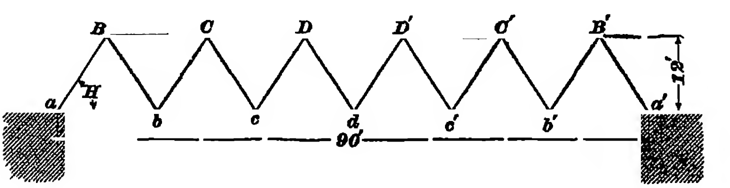


FIG 1

*truss*. The Warren truss is used in deck, through, and half-through bridges, is more frequently built as a riveted than as a pin-connected truss, and is especially adapted to the shorter spans for which trusses are used. For spans up to about 100 feet, it is frequently spoken of as a *lattice girder*. For longer spans, it is sometimes built with subdivided panels, or with multiple systems of web members.

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**2. Methods of Calculation.**—The stresses in the members of the simple type of Warren truss can be readily found, either graphically or analytically, by applying the general conditions of equilibrium. The work of calculation by the analytic methods is so simple that the graphic method is seldom used in practice for this type of truss.

The analytic methods are illustrated in the following articles, which contain the calculations of the maximum and minimum stresses in all the members of the six-panel truss shown in Fig. 1. This truss has a span of 90 feet and a height of 12 feet; the dead load is taken as 600 pounds, and the live load as 1,600 pounds, per linear foot of the bridge, all the dead load is assumed to be applied at the joints of the loaded chord, and the truss is assumed to support one-half the entire load on the bridge.

## METHOD OF SECTIONS

**3. Panel Loads and Reactions.**—The dead panel load $W'$ for one truss is equal to $\frac{600}{2} \times 15 = 4{,}500$ pounds. As explained in *Stresses in Bridge Trusses*, Part 1, the number of panel loads considered in determining the reactions is one less than the number of panels in the truss. In this case, the number of panels in the truss is six; therefore, only five panel loads are taken into account in determining the reactions. The reactions $R_1'$ and $R_2'$, Fig. 2 (*a*), due to the dead load are each equal to

$$\frac{4{,}500 \times 5}{2} = 11{,}250 \text{ pounds}$$

The live panel load $W''$ for one truss is equal to $\frac{1{,}600}{2} \times 15 = 12{,}000$ pounds; and the reactions $R_1''$ and $R_2''$ for a fully loaded truss are each equal to

$$\frac{12{,}000 \times 5}{2} = 30{,}000 \text{ pounds}$$

**4. Chord Stresses in General.**—Chord stresses may be conveniently determined by the method of sections

explained in *Analytic Statics*, Part 2, and in *Stresses in Bridge Trusses*, Part 1. The dead loads and reactions are shown in Fig. 2 ($a$). The method will be illustrated by determining the dead-load stresses in $CD$ and $cd$. The truss may be considered cut by a plane $q$ intersecting the members $CD$, $cD$, and $cd$. The portion of the truss to the left of section $q$ is shown in Fig. 2 ($b$), the external forces being $R_1'$ at $a$,

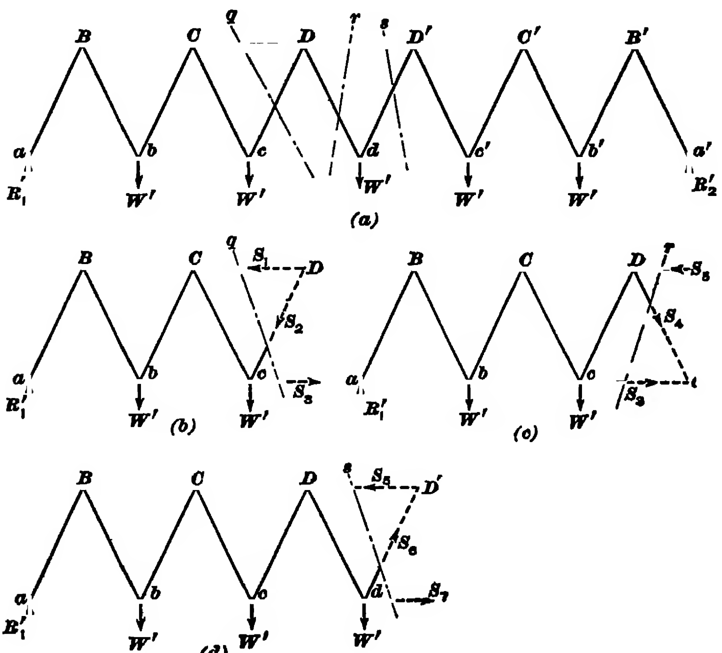



FIG. 2

$W'$ at $b$, $W'$ at $c$, and the forces $S_1$, $S_2$, and $S_3$, equal numerically to the stresses in the members cut.

Assuming that the stress in $CD$ is compression, $S_1$ will be directed toward the left, and its magnitude may be computed by taking moments of all the forces about $c$, the point of intersection of $S_2$ and $S_3$. The moments of the load at $c$ and of $S_2$ and $S_3$ are each equal to zero, since their lever arms are zero. Writing the equation for the

moments of all the forces shown in Fig. 2 (*b*) about the point *c*, we have

$$R_1' \times 30 - W' \times 15 - S_1 \times 12 = 0;$$

whence

$$S_1 = \frac{R_1' \times 30 - W' \times 15}{12} = \frac{\frac{5\,W'}{2} \times 30 - W' \times 15}{12}$$

$$= \frac{4\,W' \times 15}{12}$$

As the value of $S_1$ comes out positive, the assumption that the stress in $CD$ is compression is correct. The advantage of taking *c* as a center of moments is that, by so doing, an equation is obtained that contains but one unknown force, namely, $S_1$; the other two, $S_2$ and $S_3$, do not appear in the equation, since the moment of each is zero. The center of moments may be taken at any point, whether it is on the structure or not, but it is better, if possible, to take it at the intersection of two of the members, thereby eliminating the stresses in those members from the equation of moments.

To find $S_3$, the center of moments may be taken at the intersection $D$ of $S_1$ and $S_2$. Assuming the stress in $c\,d$ to be tension, $S_3$ will be directed toward the right Taking moments about $D$,

$$R_1' \times 37\,5 - W' \times 22.5 - W' \times 7.5 - S_3 \times 12 = 0;$$

whence $S_3 = \dfrac{R_1' \times 37\,5 - W' \times 22.5 - W' \times 7.5}{12}$

If this comes out positive, the assumption that the stress in $c\,d$ is tension is correct; if negative, the stress is compression, but its numerical value will be that determined by the last equation.

In a similar manner, if the stress in $D\,D'$ is required, the truss may be considered cut by a plane *r*, intersecting $D\,D'$, $D\,d$, and $c\,d$, or by a plane *s*, intersecting $D\,D'$, $d\,D'$, and $d\,c'$. The portion to the left of section *r* is shown in Fig 2 (*c*), the portion to the left of section *s* is shown in Fig. 2 (*d*) The proper center of moments is *d*. The stress in $D\,D'$ will be assumed as compression; then, $S_4$ will be

directed toward the left. Writing the expression for the moment at $d$,

$$R_1' \times 45 - W' \times 30 - W' \times 15 - S_6 \times 12 = 0;$$

whence $$S_6 = \frac{R_1' \times 45 - W' \times 30 - W' \times 15}{12}$$

In Fig. 2 ($c$), the lever arms of $S_3$ and $S_4$ are each zero; in Fig 2 ($d$), the lever arms of $S_6$ and $S_7$ and of the load at $d$ are each zero, hence, these do not appear in the equation of moments.

The values of the other chord stresses can be found in a similar manner. All upper-chord members will be in compression and all lower-chord members in tension.

**5.** It will be seen that the numerator of the expression for the stress in any chord member is, in each case, the sum of the moments of the panel loads and reactions at the left of the section, about the joint opposite the member. This sum is the bending moment on the truss at that point. The denominator is the height of the truss. We may, therefore, state the following general principle:

*The stress in any chord member of a simple Warren truss is equal to the bending moment on the truss, at the joint opposite the member considered, divided by the height of the truss.*

**6. Dead-Load Chord Stresses.**—Applying the principle just stated to the determination of the dead-load chord stresses, the following values are found:

$$\text{Stress in } a\,b = \frac{\text{moment at } B}{\text{height}} = \frac{11{,}250 \times 7.5}{12}$$

$$= 7{,}030 \text{ pounds, tension.}$$

$$\text{Stress in } b\,c = \frac{\text{moment at } C}{\text{height}} = \frac{11{,}250 \times 22.5 - 4{,}500 \times 7.5}{12}$$

$$= 18{,}280 \text{ pounds, tension.}$$

$$\text{Stress in } c\,d = \frac{\text{moment at } D}{\text{height}}$$

$$= \frac{11{,}250 \times 37.5 - 4{,}500 \times 22.5 - 4{,}500 \times 7.5}{12}$$

$$= 23{,}910 \text{ pounds, tension.}$$

$$\text{Stress in } BC = \frac{\text{moment at } b}{\text{height}} = \frac{11{,}250 \times 15}{12}$$
$$= 14{,}060 \text{ pounds, compression.}$$

$$\text{Stress in } CD = \frac{\text{moment at } c}{\text{height}} = \frac{11{,}250 \times 30 - 4{,}500 \times 15}{12}$$
$$= 22{,}500 \text{ pounds, compression.}$$

$$\text{Stress in } DD' = \frac{\text{moment at } d}{\text{height}}$$
$$= \frac{11{,}250 \times 45 - 4{,}500 \times 30 - 4{,}500 \times 15}{12}$$
$$= 25{,}310 \text{ pounds, compression}$$

As the truss is symmetrical, the stresses in the members on the right of the center are equal to those in the corresponding members on the left That is, the stress in $D'C'$ is equal to the stress in $CD$; the stress in $C'B'$ is equal to the stress in $BC$; etc.

**7. Live-Load Chord Stresses.**—The maximum bending moments, and, therefore, the maximum chord stresses, due to a moving load, occur when the truss is fully loaded. This condition of loading is similar to the dead loading, each panel load being now 12,000 pounds, and each reaction 30,000 pounds, in place of 4,500 and 11,250 pounds, respectively. The chord stresses may be found in precisely the same way as for dead loads; thus,

$$\text{stress in } ab = \frac{30{,}000 \times 7.5}{12} = 18{,}750 \text{ pounds}$$

and so on. The results are (using the minus sign for tension and the plus sign for compression):

| Member | Stress, in Pounds |
|---|---|
| $ab$ | $-18{,}750$ |
| $bc$ | $-48{,}750$ |
| $cd$ | $-63{,}750$ |
| $BC$ | $+37{,}500$ |
| $CD$ | $+60{,}000$ |
| $DD'$ | $+67{,}500$ |

**8.** As all the dead load is assumed as being applied at the joints of the loaded chord, the live-load stresses in the

chords may be obtained from the dead-load stresses by multiplying the latter by the ratio of the live to the dead load per linear foot, which ratio is $\frac{1,600}{600}$, or $\frac{8}{3}$. For example, the dead-load stress in $bc$ is $-18,280$, and the live-load stress is

$$-18,280 \times \frac{8}{3} = -48,750 \text{ pounds}$$

**9. Maximum and Minimum Chord Stresses.**—Since the live-load stress in any chord member is of the same sign as the dead-load stress, the maximum stress in the member is equal to the sum of the two stresses; and the minimum stress is equal to the dead-load stress.

**10. Web Stresses in General.**—The stresses in the web members may be found by the method of shears, explained in *Stresses in Bridge Trusses*, Part 1. For example, to determine the stress in the web member $cD$, Fig. 2 ($a$), the portion of the structure to the left of section $q$ may be considered as a free body, as shown in Fig. 2 ($b$). Any of the conditions of equilibrium may be applied to the forces shown. It is desirable, if possible, to use an equation that contains the web force $S_2$, to be determined, but which does not involve either of the two forces $S_1$ and $S_3$. As $S_1$ and $S_3$ are horizontal, they will not appear in the equation $\Sigma Y = \Sigma S \sin H = 0$. Assuming the stress in $cD$ to be compression, $S_2$ will act downwards to the left. Writing the expression for $\Sigma Y = \Sigma S \sin H = 0$ gives

$$\Sigma Y = R_1' - W' - W' - \text{vertical component of } S_2 = 0;$$

that is, denoting the angle $D c S_2$ by $H$,

$$R_1' - 2 W' - S_2 \sin H = 0;$$

whence $$S_2 \sin H = R_1' - 2 W'$$

and $$S_2 = \frac{R_1' - 2 W'}{\sin H} = (R_1' - 2 W') \csc H$$

The term $R_1' - 2 W'$ is the shear on the section $q$; therefore, the vertical component $S_2 \sin H$ of $S_2$ is numerically equal to the shear on the plane of section that cuts $cD$. In general, the following principle may be stated.

*For single-system parallel-chord trusses, the vertical component of the stress in any web member is numerically equal to the shear*

*on the plane of section cutting that web member and the two chord members between which the web member lies, and the stress in the same web member is numerically equal to the shear just referred to, multiplied by the cosecant of the angle that the member makes with the horizontal.*

**11. Character of Web Stresses.**—If the shear on section $q$, Fig. 2 ($b$), is positive, the resultant of the external vertical forces on the left of the section acts upwards; then $S_3$ must act downwards, and the stress in $c\,D$ is compression. If the shear is negative, $S_3$ acts upwards, and the stress in $c\,D$ is tension. If the shear on section $r$, Fig. 2 ($c$), is positive the resultant of the external forces on the left acts upwards, $S_4$ acts downwards, and the stress in $D\,d$ is tension. If the shear is negative, $S_4$ acts upwards, and the stress in $D\,d$ is compression. These conclusions may be stated as a general principle thus:

*In those web members inclining downwards toward the left or upwards toward the right, positive shear causes compression, and negative shear tension, in those web members inclining upwards toward the left or downwards toward the right, positive shear causes tension, and negative shear compression.*

**12. Dead-Load Shears and Web Stresses.**—In order to calculate the stresses in the web members due to dead load, it will be convenient first to find the shears on the sections cut by the planes $o, p, q'$, etc., Fig 3 ($a$). They are as follows:

| MEMBER | SECTION | SHEAR, IN POUNDS |
|---|---|---|
| $a\,B$ | $o$ | + 11,250 |
| $B\,b$ | $p$ | + 11,250 |
| $b\,C$ | $q'$ | + 6,750 |
| $C\,c$ | $r'$ | + 6,750 |
| $c\,D$ | $s'$ | + 2,250 |
| $D\,d$ | $t$ | + 2,250 |
| $d\,D'$ | $u$ | − 2,250 |

From symmetry, the shears to the right of the center $d$ will be equal and of opposite sign to the corresponding shears on the left. For example, the shear on section

$t$ is + 2,250 pounds, and that on section $u$ is − 2,250 pounds. The shears on the sections $o$ and $p$ are equal, as are also those on $q'$ and $r'$, and those on $s'$ and $t$; because, in each case, the two planes are passed between the same two panel loads, that is, in the same panel of the loaded chord; and, as in the present case there are no loads applied at the joints of the unloaded chord, the shears on all sections in any panel are equal. As all shears to the left of $d$ are positive, the stresses in the members $aB$, $bC$, and $cD$ that incline downwards toward the left are compression, and the stresses in the members $Bb$, $Cc$, and $Dd$ that incline upwards toward the left are tension. On the right of the center, the shears are negative; and the stresses in the members that incline downwards toward the left are tension, while the stresses in those that incline upwards toward the left are compression. In the present case, the stresses in $aB$ and $Bb$ are equal and of opposite signs, as are also those in $bC$ and $Cc$, etc.

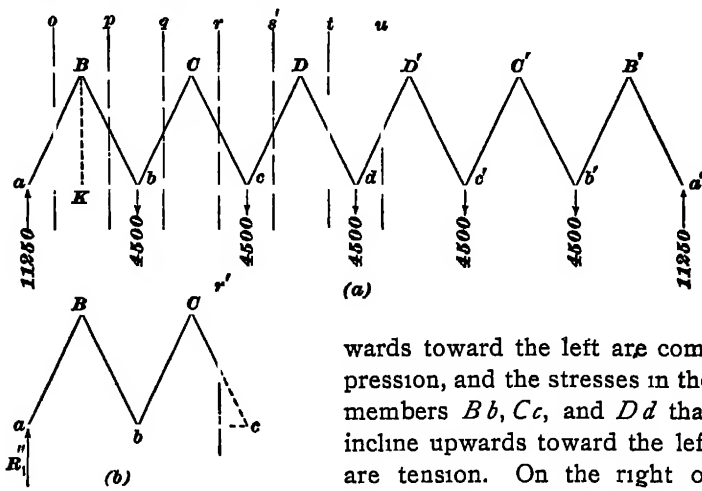



FIG 3

Referring to Fig. 3 ($a$), and applying the general principle given in Art. **10**, we have

$$\csc H = \csc BaK = \frac{Ba}{BK} = \frac{\sqrt{\overline{BK}^2 + \overline{aK}^2}}{BK}$$

$$= \frac{\sqrt{\overline{BK}^2 + \left(\frac{ab}{2}\right)^2}}{BK} = \frac{\sqrt{12^2 + 7.5^2}}{12} = 1.18$$

The web stresses can now be computed. They are as follows:

| Member | Stress, in Pounds |
|---|---|
| $aB$, $a'B'$ | $11{,}250 \times 1.18 = +13{,}280$ |
| $Bb$, $B'b'$ | $11{,}250 \times 1.18 = -13{,}280$ |
| $bC$, $b'C'$ | $6{,}750 \times 1.18 = +\ 7{,}970$ |
| $Cc$, $C'c'$ | $6{,}750 \times 1.18 = -\ 7{,}970$ |
| $cD$, $c'D'$ | $2{,}250 \times 1.18 = +\ 2{,}660$ |
| $Dd$, $D'd$ | $2{,}250 \times 1.18 = -\ 2{,}660$ |

**13. Live-Load Shears and Web Stresses.**—The stresses caused in the web members by the live load may be found from the shears. As the maximum stresses are desired, the truss must be so loaded as to cause the maximum shear for each case. The approximate method of loading explained in *Stresses in Bridge Trusses*, Part 1, will be used. The maximum positive shear in any panel occurs when all joints to the right of the panel are loaded; the maximum negative shear occurs when all joints to the left are loaded. Thus, in member $Cc$, Fig. 3 ($a$), the maximum tension occurs when all joints from $c$ to $b'$ are loaded; and the maximum compression occurs when the joint $b$ is loaded. When joints $c$ to $b'$ are loaded, the left reaction is

$$\frac{12{,}000 \times (1 + 2 + 3 + 4)}{6} = 20{,}000 \text{ pounds}$$

As there is no load at $b$, the only force acting on the portion of the truss to the left of $r'$ is the left reaction. Then, the shear in the panel $bc$ is equal to the left reaction, Fig. 3 ($b$), or 20,000 pounds. The stress in $Cc$ is equal to the shear in panel $bc$ multiplied by csc $H$, or,

$$\text{stress in } Cc = 20{,}000 \times 1.18 = 23{,}600 \text{ pounds, tension}$$

In like manner, the stress in any other member may be found. The maximum positive live shears are as follows:

| Panel | Load | Shear, in Pounds |
|---|---|---|
| $ab$ | From $b$ to $b'$ | 30,000 |
| $bc$ | From $c$ to $b'$ | 20,000 |
| $cd$ | From $d$ to $b'$ | 12,000 |
| $dc'$ | At $c'$ and $b'$ | 6,000 |
| $c'b'$ | At $b'$ | 2,000 |

In the panel $b' a'$ there can be no positive shear.

The maximum negative shear in any panel is numerically equal to the maximum positive shear in the corresponding panel at the other end of the truss The maximum and minimum stresses in the members can now be found by multiplying the respective shears by csc $H$. These stresses are given in the following table:

| Panel | Member | Positive Shear Pounds | Stress Due to Positive Shear Pounds | Negative Shear Pounds | Stress Due to Negative Shear Pounds |
|---|---|---|---|---|---|
| *a b* | *a B* | 30,000 | + 35,400 | | |
| *a b* | *B b* | 30,000 | − 35,400 | | |
| *b c* | *b C* | 20,000 | + 23,600 | 2,000 | − 2,360 |
| *b c* | *C c* | 20,000 | − 23,600 | 2,000 | + 2,360 |
| *c d* | *c D* | 12,000 | + 14,160 | 6,000 | − 7,080 |
| *c d* | *D d* | 12,000 | − 14,160 | 6,000 | + 7,080 |

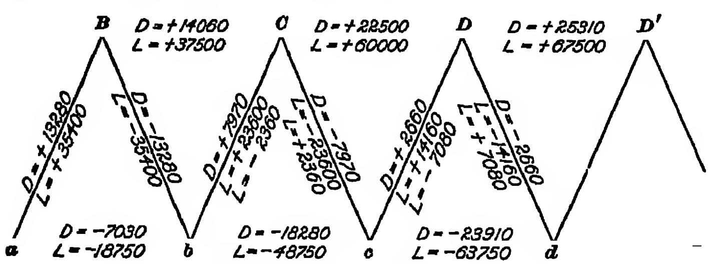
**14. Combined Shears and Web Stresses.**—The maximum and minimum stresses caused in the members on the left of the center by combined dead and live loads may be found by multiplying the maximum and minimum shears, respectively, by csc $H$. The maximum shear in any panel is equal to the sum of the positive dead-load and the positive live-load shear in the panel; the minimum shear is equal to the algebraic sum of the positive dead-load and the negative live-load shear in the panel. In columns 3 and 5 of the following table are given the maximum and minimum shears, respectively; while in columns 4 and 6 are given the maximum and minimum stresses, respectively, each stress being obtained by multiplying the corresponding shear by csc $H$.

In the members *a B* and *B b*, the minimum stresses are equal to the dead-load stresses, as there can be no negative live-load shear in the panel *a b*. The minimum stresses in *c D* and *D d* are of opposite sign to the maximum, because

in the panel *cd* the negative live-load shear exceeds the positive dead-load shear. Under the special conditions here assumed, the combined shear is positive when the joints to the right of *d* are loaded, and negative when those to the

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| Panel | Member | Maximum Shear Pounds | Maximum Stress Pounds | Minimum Shear Pounds | Minimum Stress Pounds |
| *ab* | *aB* | + 41,250 | + 48,680 | + 11,250 | + 13,280 |
| *ab* | *Bb* | + 41,250 | − 48,680 | + 11,250 | − 13,280 |
| *bc* | *bC* | + 26,750 | + 31,570 | + 4,750 | + 5,610 |
| *bc* | *Cc* | + 26,750 | − 31,570 | + 4,750 | − 5,610 |
| *cd* | *cD* | + 14,250 | + 16,820 | − 3,750 | − 4,420 |
| *cd* | *Dd* | + 14,250 | − 16,820 | − 3,750 | + 4,420 |

left of *d* are loaded. This is an important point, and shows that, in the present case, the members *cD* and *Dd* are sometimes in tension and sometimes in compression, according to the position of the live load. The two values of the stress given for each member are the extreme values that can




*Dead and Live Load Stresses*

FIG 4

exist in that member for the given loads. In each of the members *aB*, *Bb*, *bC*, and *Cc*, the stress may have any value between the extreme values, and such stresses will always be of the same kind, that is, tension or compression. In each of the members *cD* and *Dd*, the stress may have

any value between the positive and the negative value given. The stress in each member will reverse when the combined shear changes from positive to negative.

The stresses in all the members are shown in Figs. 4 and 5, which should be carefully studied. In Fig. 4, $L$ represents

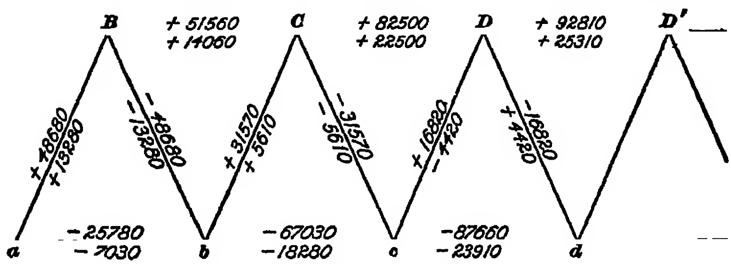



FIG. 5

the live-load stress, and $D$ the dead-load stress. In Fig. 5, the maximum is placed above and the minimum below the line representing the member. Notice how the maximum and minimum stresses in Fig. 5 are obtained by addition from the stresses in Fig. 4.

## METHOD OF JOINTS

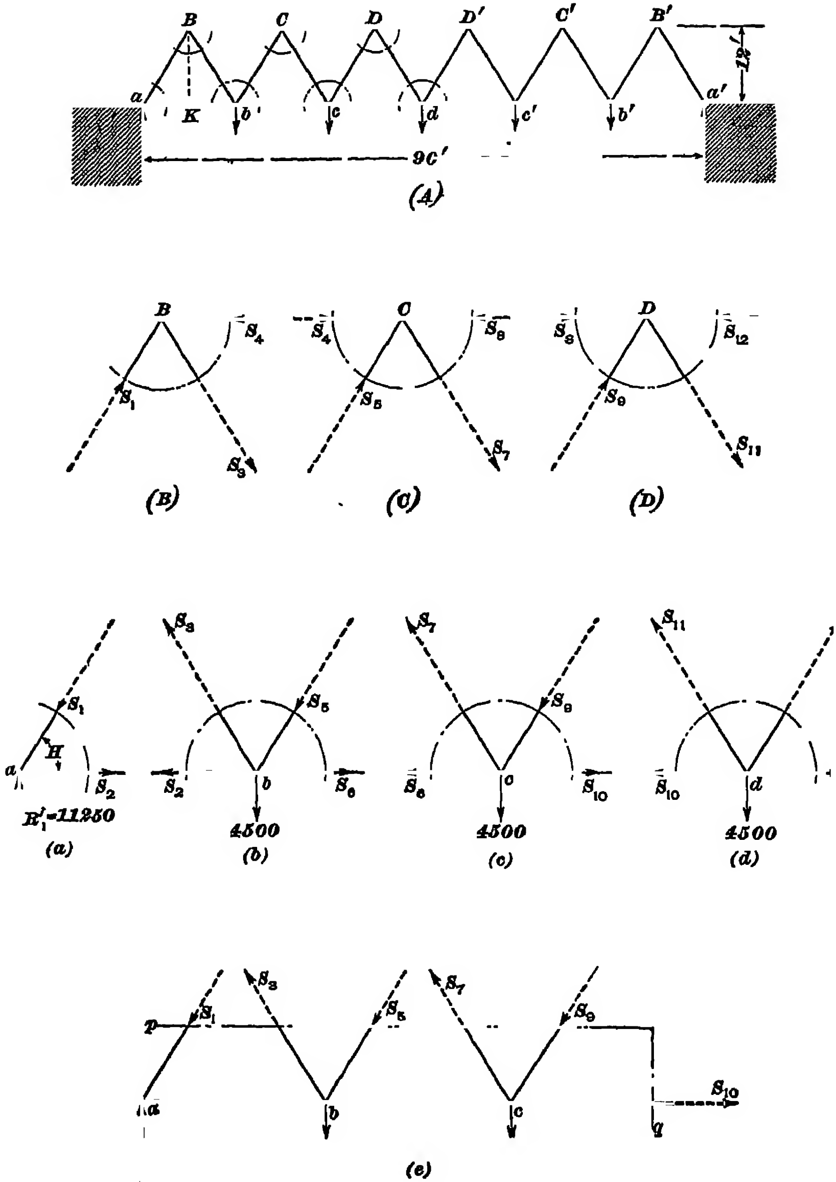
**15.** For purposes of comparison, the maximum and minimum stresses in the example of Art. **2** will be calculated by the method of joints. The truss is represented in Fig. 6 ($A$), the dead panel load being 4,500 pounds, and the live panel load, 12,000 pounds. The figure gives

$$\cot H = \cot B a K = \frac{aK}{BK} = \frac{7.5}{12} = .625$$

and, as before (Art. **12**), $\csc H = 1.18$.

**16. Dead-Load Stresses.**—It will be convenient to start at joint $a$, as there are only two unknown stresses at that joint

*Joint a.*—This joint is represented as a free body in Fig. 6 ($a$), the forces acting on it being the reaction $R_1' = 11{,}250$ pounds, and the forces $S_1$ and $S_2$, the last two



B
C
D
D′
C′
B′
12′
a
K
b
c
d
c′
b′
a′
90′
(A)
$S_1$
$S_3$
$S_4$
$S_5$
$S_7$
$S_8$
$S_9$
$S_{11}$
$S_{12}$
(B)
(C)
(D)
H
$S_2$
$S_6$
$S_{10}$
$R_1'=11250$
4500
(a)
(b)
(c)
(d)
p
q
(e)


Fig 6

representing the stresses in $aB$ and $ab$, respectively. The stress in $aB$ will be assumed as compression, and that in $ab$ as tension. Then, $S_1$ will act downwards to the left, and $S_2$ horizontally to the right. From the general conditions of equilibrium, we have, since the vertical component of $S_2$ is zero,

$$\Sigma Y = R_1' - S_1 \sin H = 0;$$

whence $S_1 \sin H = R_1' = 11{,}250$ pounds

and

$$S_1 = 11{,}250 \csc H = 11{,}250 \times 1.18$$
$$= 13{,}280 \text{ pounds, compression in } aB$$

Likewise, since $R_1'$ has no horizontal component,

$$\Sigma X = S_1 \cos H - S_2 = 0;$$

whence

$$S_2 = S_1 \cos H = 11{,}250 \csc H \cos H$$
$$= 11{,}250 \frac{\cos H}{\sin H}$$
$$= 11{,}250 \cot H = 11{,}250 \times .625$$
$$= 7{,}030 \text{ pounds, tension in } ab$$

*Joint B*.—This joint is represented in Fig. 6 ($B$), the forces acting on it being $S_1$, $S_3$, and $S_4$, which represent the stresses in $aB$, $Bb$, and $BC$, respectively. The force $S_1$ is known and acts upwards to the right, the stress in $Bb$ will be assumed as tension, and that in $BC$ as compression. Then, $S_3$ will act downwards to the right, and $S_4$ horizontally to the left. From the conditions of equilibrium,

$$\Sigma Y = S_1 \sin H - S_3 \sin H = 0;$$

whence $S_3 \sin H = S_1 \sin H = 11{,}250$ pounds

and $S_3 = 11{,}250 \csc H = 13{,}280$ pounds, tension in $Bb$

Likewise,

$$\Sigma X = S_1 \cos H + S_3 \cos H - S_4 = 0;$$

whence

$$S_4 = (S_1 + S_3) \cos H = (11{,}250 \csc H + 11{,}250 \csc H) \cos H$$
$$= 22{,}500 \csc H \cos H = 22{,}500 \cot H$$
$$= 14{,}060 \text{ pounds, compression in } BC$$

*Joint b*.—This joint is represented in Fig. 6 ($b$), the forces acting on it being $S_2$, $S_3$, $S_5$, and $S_6$, which represent the stresses in $ab$, $Bb$, $bC$, and $bc$, respectively; and the panel

load of 4,500 pounds. The latter acts vertically downwards, $S_2$ acts horizontally to the left, and $S_3$ acts upwards to the left; $S_5$ and $S_6$ are unknown. The stress in $bC$ will be assumed as compression, and that in $bc$ as tension. Then, $S_5$ will act downwards to the left and $S_6$ horizontally to the right.

$$\Sigma Y = S_3 \sin H - 4{,}500 - S_5 \sin H = 0;$$

whence

$$S_5 \sin H = S_3 \sin H - 4{,}500 = 11{,}250 - 4{,}500$$
$$= 6{,}750 \text{ pounds}$$

and

$$S_5 = 6{,}750 \csc H = 7{,}970 \text{ pounds, compression in } bC$$

Likewise,

$$\Sigma X = S_2 + S_3 \cos H + S_5 \cos H - S_6 = 0;$$

whence

$$S_6 = S_2 + S_3 \cos H + S_5 \cos H$$
$$= 11{,}250 \cot H + 11{,}250 \csc H \cos H + 6{,}750 \csc H \cos H$$
$$= 29{,}250 \cot H = 18{,}280 \text{ pounds, tension in } bc$$

*Joint C.*—This joint is represented in Fig. 6 ($C$), the forces acting on it being $S_4$, $S_5$, $S_7$, and $S_8$, which represent the stresses in $BC$, $bC$, $Cc$, and $CD$, respectively; $S_4$ acts horizontally to the right, $S_5$ upwards toward the right, while $S_7$ and $S_8$ are unknown. The stress in $Cc$ will be assumed as tension, and that in $CD$ as compression. Then, $S_7$ will act downwards to the right, and $S_8$ horizontally to the left.

$$\Sigma Y = S_5 \sin H - S_7 \sin H = 0;$$

whence

$$S_7 \sin H = S_5 \sin H = 6{,}750 \text{ pounds}$$

and

$$S_7 = 6{,}750 \csc H = 7{,}970 \text{ pounds, tension in } Cc$$

Likewise,

$$\Sigma X = S_4 + S_5 \cos H + S_7 \cos H - S_8 = 0;$$

whence

$$S_8 = S_4 + S_5 \cos H + S_7 \cos H$$
$$= 22{,}500 \cot H + 6{,}750 \csc H \cos H + 6{,}750 \csc H \cos H$$
$$= 36{,}000 \cot H = 22{,}500 \text{ pounds, compression in } CD$$

*Joint c.*—This joint is represented in Fig 6 ($c$), the forces acting on it being $S_6$, $S_7$, $S_9$, and $S_{10}$, which represent the

stresses in $b\,c$, $C\,c$, $c\,D$, and $c\,d$, respectively; $S_6$ acts horizontally to the left, and $S_7$ upwards to the left; while $S_9$ and $S_{10}$ are unknown. The stress in $c\,D$ will be assumed as compression, and that in $c\,d$ as tension. Then, $S_9$ will act downwards to the left, and $S_{10}$ horizontally to the right.

$$\Sigma Y = S_7 \sin H - S_9 \sin H - 4{,}500 = 0;$$

whence

$$S_9 \sin H = S_7 \sin H - 4{,}500 = 6{,}750 - 4{,}500 = 2{,}250 \text{ pounds}$$

and

$$S_9 = 2{,}250 \csc H = 2{,}660 \text{ pounds, compression in } c\,D$$

Likewise,

$$\Sigma X = S_6 + S_7 \cos H + S_9 \cos H - S_{10} = 0:$$

whence

$$\begin{aligned} S_{10} &= S_6 + S_7 \cos H + S_9 \cos H \\ &= 29{,}250 \cot H + 6{,}750 \csc H \cos H + 2{,}250 \csc H \cos H \\ &= 38{,}250 \cot H = 23{,}910 \text{ pounds, tension in } c\,d \end{aligned}$$

*Joint D.*—This joint is represented in Fig. 6 ($D$), the forces acting on it being $S_8$, $S_9$, $S_{11}$, and $S_{12}$, which represent the stresses in $C\,D$, $c\,D$, $D\,d$, and $D\,D'$, respectively, $S_8$ acts horizontally to the right, and $S_9$ upwards to the right, while $S_{11}$ and $S_{12}$ are unknown. The stress in $D\,d$ will be assumed as tension, and that in $D\,D'$ as compression. Then, $S_{11}$ will act downwards to the right, and $S_{12}$ horizontally to the left.

$$\Sigma Y = S_9 \sin H - S_{11} \sin H = 0;$$

whence

$$S_{11} \sin H = S_9 \sin H = 2{,}250 \text{ pounds}$$

and

$$S_{11} = 2{,}250 \csc H = 2{,}660 \text{ pounds, tension in } D\,d$$

Likewise,

$$\Sigma X = S_8 + S_9 \cos H + S_{11} \cos H - S_{12} = 0;$$

whence

$$\begin{aligned} S_{12} &= S_8 + S_9 \cos H + S_{11} \cos H \\ &= 36{,}000 \cot H + 2{,}250 \csc H \cos H + 2{,}250 \csc H \cot H \\ &= 40{,}500 \cot H = 25{,}310 \text{ pounds, compression in } D\,D' \end{aligned}$$

*Joint d.*—This joint is represented in Fig 6 ($d$). It is evident at once that $S_{13}$ is numerically equal to $S_{11}$, and that $S_{14}$ is numerically equal to $S_{10}$. Therefore, the stress in $d\,D'$ is 2,660 pounds tension, and that in $d\,d'$ is 23,910 pounds tension

It is unnecessary to proceed further than joint $d$, as the stresses in the members at the right end are the same as those in the corresponding members at the left end.

From the preceding discussion, it may be seen that the stress in any web member is equal to the algebraic sum of all the vertical forces that act on the truss on the left of the member considered, that is, to the shear in the panel in which the member is located, multiplied by $\csc H$ (see Art. **10**).

**17.** For the chord members, it is convenient to refer again to the stress in one of the members, such as $cd$, Fig. 6 ($c$) (*joint* $c$). The equation $\Sigma X = 0$ gives

$$S_{10} = S_6 + S_7 \cos H + S_9 \cos H$$

Substituting for $S_6$ its value $S_2 + S_3 \cos H + S_5 \cos H$,

$$S_{10} = S_2 + S_3 \cos H + S_5 \cos H + S_7 \cos H + S_9 \cos H$$

Likewise, substituting for $S_2$ its value $S_1 \cos H$,

$$S_{10} = S_1 \cos H + S_3 \cos H + S_5 \cos H + S_7 \cos H + S_9 \cos H$$

Letting $Y_1$, $Y_3$, etc. represent the vertical components of the stresses $S_1$, $S_3$, etc., and substituting for $S_1$, $S_3$, etc their values $Y_1 \csc H$, $Y_3 \csc H$, etc., respectively, we have

$$\begin{aligned} S_{10} &= Y_1 \csc H \cos H + Y_3 \csc H \cos H + Y_5 \csc H \cos H \\ &\quad + Y_7 \csc H \cos H + Y_9 \csc H \cos H \\ &= Y_1 \cot H + Y_3 \cot H + Y_5 \cot H + Y_7 \cot H \\ &\quad + Y_9 \cot H = (Y_1 + Y_3 + Y_5 + Y_7 + Y_9) \cot H \end{aligned}$$

Now, $Y_1 \cot H$, $Y_3 \cot H$, etc. are the horizontal components of the stresses in $aB$, $Bb$, etc., respectively, and the sum of these components from $Y_1 \cot H$ to $Y_9 \cot H$ is the algebraic sum of the horizontal components of the stresses in all the web members that connect with the lower chord at the left of $cd$ In like manner, it may be shown that the stress in $DD'$ is equal to the algebraic sum of the horizontal components of the stresses in all the web members that connect with the upper chord to the left of $DD'$. In general,

*The stress in any portion of either chord is equal to the algebraic sum of the horizontal components of the stresses in all the web members that connect with the chord at the left of the portion considered.*

**18.** In the present case, the web members all make the same angle with the horizontal; that is, $H$ is constant, and the stress $S_{10}$ in a chord member, such as $c\,d$, is equal to $(Y_1 + Y_3 + Y_5 + Y_7 + Y_9)$ cot $H$. Letting $\Sigma Y$ represent the sum of the vertical components in all the web members that connect with a chord at the left of any portion, then, the stress in that portion may be found by the formula

$$S = \Sigma Y \times \cot H$$

In applying this formula to the determination of the stress in a chord member, care must be taken that the horizontal components are given the proper signs. For example, for $c\,d$, the truss may be considered cut by the section $p\,q$, Fig. 6 ($e$), and the portion below and to the left of this section treated as a free body The horizontal forces that act on this portion are the horizontal components of $S_1$, $S_3$, etc., and the stress in $c\,d$. $S_1$, $S_5$, and $S_9$ act downwards to the left, and $S_3$ and $S_7$ act upwards to the left; therefore, all the horizontal components of these stresses act to the left, and in finding $S_{10}$ the vertical components of the stresses from $S_1$ to $S_9$ must be *added* numerically to find $\Sigma Y$

From the foregoing, the following general rule is derived

*To find the stress in any web member of a single-system Warren truss by the method of joints, multiply the shear in the panel in which the member is located by csc H, to find the stress in any chord member, multiply by cot H the algebraic sum of all the shears used in obtaining the stresses in all the web members that connect with the chord at the left of the member considered.*

The application of this rule can be greatly simplified by constructing a diagram, as shown in Fig. 7. A sketch of the truss is drawn (not necessarily to scale) and on the upper side of each web member is written, with its proper sign and as the coefficient of csc $H$, the shear in the panel in which the member is located. On the upper side of each chord member is written, as the coefficient of cot $H$, the algebraic sum of the shears that have been written on all the web members that connect with the chord at the left of the member considered. On the under side of each member is written the stress obtained by performing the indicated multiplication.

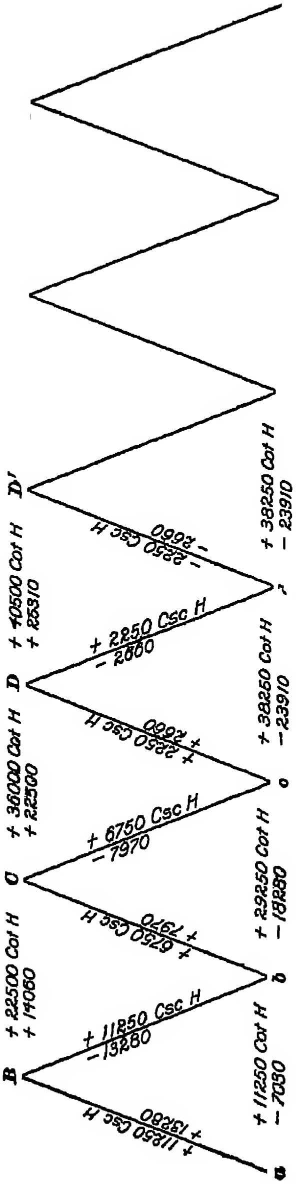

B
C
D
D'
a
b
c
+ 22500 Cot H
+ 14060
+ 36000 Cot H
+ 22500
+ 40500 Cot H
+ 25310
+ 11250 Cot H
− 7030
+ 29250 Cot H
− 18280
+ 38250 Cot H
− 23910
+ 38250 Cot H
− 23910
+ 11250 Csc H
+ 13280
+ 11250 Csc H
− 13280
+ 6750 Csc H
+ 7970
+ 6750 Csc H
− 7970
+ 2250 Csc H
+ 2660
+ 2250 Csc H
− 2660
− 2250 Csc H
− 2660

Fig 7

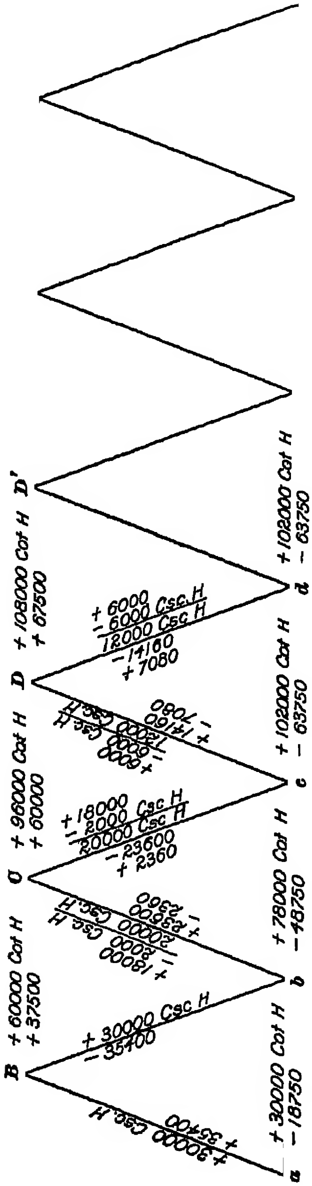

B
C
D
D'
a
b
c
d
+ 60000 Cot H
+ 37500
+ 96000 Cot H
+ 60000
+ 108000 Cot H
+ 67500
+ 30000 Cot H
− 18750
+ 78000 Cot H
− 48750
+ 102000 Cot H
− 63750
+ 102000 Cot H
− 63750
+ 30000 Csc. H
+ 35400
+ 30000 Csc H
− 35400
+ 18000 Csc H
− 2000 Csc H
20000 Csc H
− 23600
+ 2360
+ 6000 Csc.H
− 6000 Csc H
12000 Csc H
− 14160
+ 7080

Fig 8

The plus and minus signs written before the stresses indicate, as usual, compression and tension, respectively. Thus, in Fig. 7, the coefficient +6,750 of csc $H$ on $cC$ is the shear in the panel $bc$; the product 6,750 csc $H$ gives 7,970, which is the numerical value of the stress in $cC$. The coefficient +29,250 of cot $H$ on $bc$ is the algebraic sum of the shears written on the members $ab$, $Bb$, and $bC$; the product 29,250 cot $H$ gives 18,280, which is the numerical value of the stress in $bc$.

**19. Live-Load Chord Stresses.**—As the chord stresses are greatest when the truss is fully loaded, it is necessary first to find the shears due to a full live load. They are as follows:

| Panel | Shear, in Pounds |
|---|---|
| $ab$ | 30,000 |
| $bc$ | 18,000 |
| $cd$ | 6,000 |

These values are written on the upper side of the web members, as shown in Fig 8, and the coefficients of cot $H$ for the chord stresses are found by adding the shears as explained in Art. 18. Then, the stresses in the chord members are obtained by performing the multiplications indicated, and the results written on the under sides of the members.

**20. Live-Load Web Stresses.**—The shears that were found in Art. 19 are those due to full live load, the shear in any panel being the difference between the left reaction and the sum of all the panel loads between the left reaction and the panel under consideration. For example, the shear in panel $cd$ due to full live load is

$$\frac{12{,}000\,(1+2+3+4+5)}{6} - (12{,}000+12{,}000)$$

or,

$$\left[\frac{12{,}000\,(1+2+3)}{6}\right] + \left[\frac{12{,}000\,(4+5)}{6} - (12{,}000+12{,}000)\right]$$

$$= \left[\frac{12{,}000\,(1+2+3)}{6}\right] - \left[\frac{12{,}000\,(1+2)}{6}\right]$$

The expression contained in the left-hand brackets of the last member of this equation is the left reaction that would be caused by loads at $d$, $c'$, and $b'$, if they were the only loads on the truss; it was explained in Art. **13** that this is the maximum positive live-load shear in panel $c\,d$ Also, the expression contained in the right-hand bracket is the maximum negative live-load shear in panel $c\,d$. From this the following principle is obtained:

*The shear in any panel of a truss due to full live load is equal to the algebraic sum of the maximum live-load positive shear and the maximum live-load negative shear that can occur in that panel.*

Let $V''$ = shear in any panel due to full live load;

$V_p''$ = maximum positive live-load shear that can occur in that panel;

$V_n''$ = maximum negative live-load shear that can occur;

then,
$$V'' = V_p'' + V_n'',$$
whence
$$V_p'' = V'' - V_n'';$$
that is, the maximum positive live-load shear in any panel may be found by subtracting algebraically the maximum negative live-load shear that can occur in the panel from the shear due to full live load. This principle is of special value in finding live-load web stresses by the method of joints; the shear in each panel due to full load is found in connection with the chord stresses; the maximum negative shear in each panel is then found in order to get the minimum stresses in the members; then, the maximum positive shear in any panel may be found by subtracting algebraically the maximum negative shear from the shear in the panel due to full load.

In panel $a\,b$ there can be no negative shear. Then, in this panel the maximum live-load stresses occur when the truss is fully loaded; csc $H$ may be written after the shear that has been written on $a\,B$ and $B\,b$ (30,000 pounds), Fig. 8, and the stresses found by multiplying that shear by csc $H$ (1.18). The results are written on the other side of the lines that represent $a\,B$ and $B\,b$. On the other web members,

directly under the values of the shear due to full load, are written, as coefficients of csc $H$, the maximum negative live-load shear and the maximum positive live-load shear, the latter being obtained by subtracting, algebraically, the negative shear from the shear due to full load. The maximum and minimum stresses are obtained by performing the multiplications indicated, and the results are written on the under side of the members. As stated in Art **18**, the plus and minus signs written before the stresses represent compression and tension, respectively. Thus, in panel $b\,c$, Fig. 8, the shear due to full load (+18,000 pounds) is written on $b\,C$ and $C\,c$; the maximum negative shear (−2,000 pounds) is written under +18,000, and is the coefficient of csc $H$ for the minimum live-load stresses in $b\,C$ and $C\,c$ The algebraic difference between the shear due to full load and the maximum negative shear, +18,000−(−2,000) = +20,000 pounds, is then written under −2,000 csc $H$, as the coefficient of csc $H$, for the maximum live-load stresses in $b\,C$ and $C\,c$ These stresses are obtained by performing the multiplications, and their values are written on the under sides of the members. Thus, for the member $b\,C$, the maximum live-load stress is 20,000 csc $H$, or +23,600 pounds; the minimum live-load stress is −2,000 csc $H$, or −2,360 pounds.

From the foregoing, the following general rule is obtained

*To find the maximum and minimum live-load stresses in the web members of a single-system Warren truss, when the shears due to full live load are known, write on each member the maximum negative live-load shear in the panel in which the member is located, and multiply it by csc H for the minimum stress, subtract, algebraically, the maximum negative shear from that due to full load, and multiply the result for each member by csc H for the maximum stresses.*

The combined stresses are found in the same way as in Art **14**.

EXAMPLE —The truss represented in Fig 9 is a seven-panel through Warren truss, with dimensions as shown The dead load is 1,000 pounds, and the live load, 2,000 pounds per linear foot of bridge Assuming that one-third of a dead panel load is applied at each of the

joints of the upper chord, and two-thirds at the joints of the lower chord, find, by the method of joints, the maximum and minimum stresses in all the members

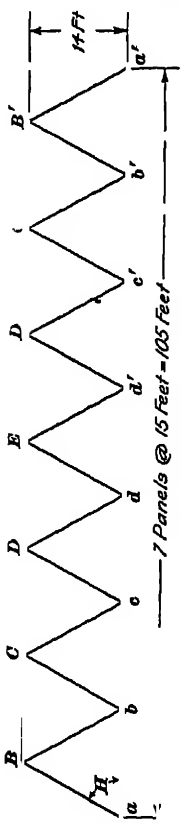


Fig. 9

SOLUTION.—Each dead panel load is equal to

$$\frac{1{,}000 \times 15}{2} = 7{,}500 \text{ pounds}$$

of which 5,000 pounds is applied at the joints of the lower chord, and 2,500 pounds at the joints of the upper chord. It will be noticed that there are six joints in the lower chord and seven in the upper chord. It is customary to assume that one-third of a panel load is applied at each of the joints of the upper chord. The dead-load reaction for one truss is equal to

$$\frac{5{,}000 \times 6}{2} + \frac{2{,}500 \times 7}{2} = 23{,}750 \text{ pounds}$$

Each live panel load is equal to

$$\frac{2{,}000 \times 15}{2} = 15{,}000 \text{ pounds}$$

and the live-load reaction for one truss fully loaded is equal to

$$\frac{15{,}000 \times 6}{2} = 45{,}000 \text{ pounds}$$

As a portion of the dead load is applied at the upper-chord joints, which lie midway between the joints of the lower chord, the dead-load shear in any panel of the lower chord is not constant. For example, in the panel $b\,c$, the dead-load shear from $b$ to $C$ is equal to

$$23{,}750 - (5{,}000 + 2{,}500) = 16{,}250 \text{ pounds}$$

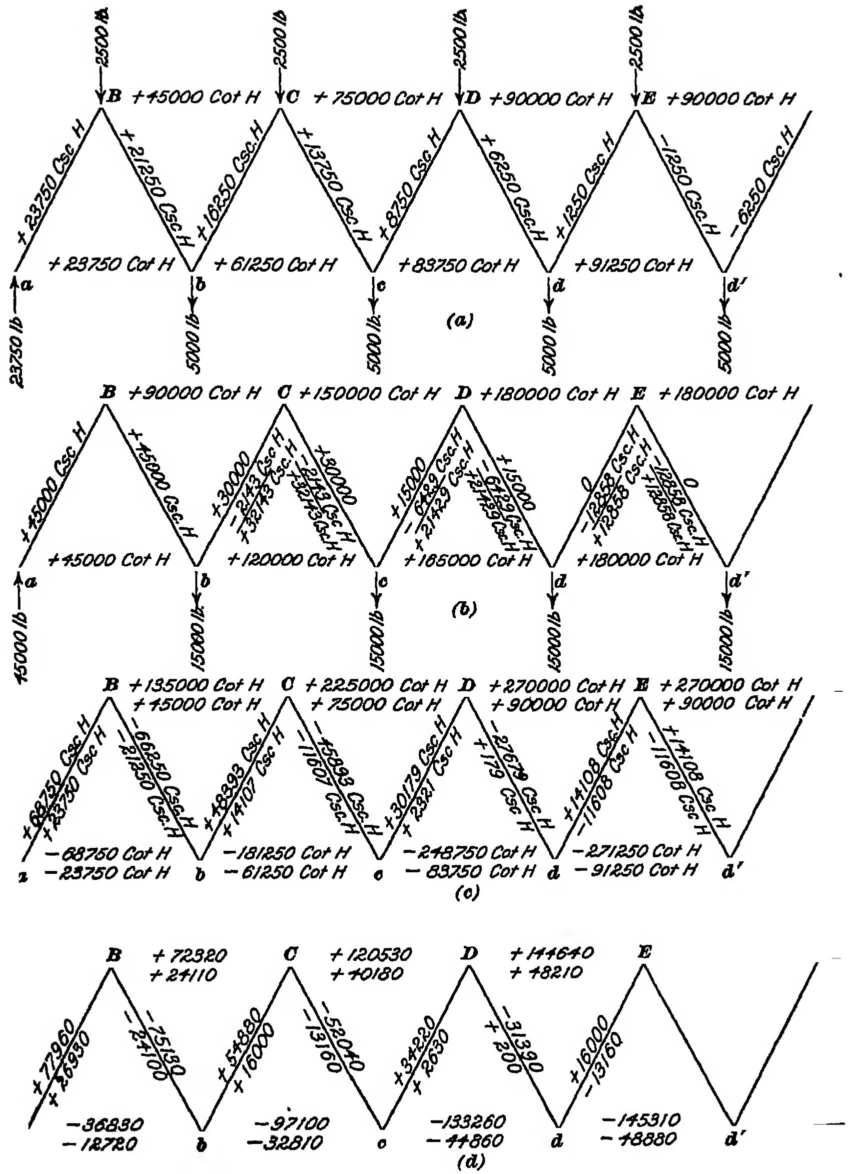
while from $C$ to $c$ it is

$$23{,}750 - (5{,}000 + 2{,}500 + 2{,}500), \text{ or } 13{,}750 \text{ pounds}$$

The figure gives

$$\csc H = \frac{\sqrt{14^2 + 7.5^2}}{14} = 1.134; \ \cot H = \frac{7.5}{14} = .5357$$

The dead-load stresses, found by the rule given in Art. **18**, are indicated in Fig. 10 ($a$); the live-load stresses, found by the rule given in Art. **20**, are indicated in Fig. 10 ($b$). As the dead-load and live-load stresses are not required separately, the work will be shortened in the present case by combining the coefficients of $\csc H$ and $\cot H$, respectively, and indicating the maximum and minimum stresses, as represented in Fig. 10 ($c$). There remains now simply the operation of multiplying these coefficients by $\csc H$ and $\cot H$, respectively, to get the maximum and the minimum combined stresses, as represented in Fig. 10 ($d$). The student should verify the values of these



2500 lb
B +45000 Cot H
C +75000 Cot H
D +90000 Cot H
E +90000 Cot H
+23750 Csc H
+21250 Csc H
+16250 Csc H
+13750 Csc H
+8750 Csc H
+6250 Csc H
+1250 Csc H
−1250 Csc H
−6250 Csc H
+23750 Cot H
+61250 Cot H
+83750 Cot H
+91250 Cot H
a
b
c
d
d′
23750 lb
5000 lb
(a)
B +90000 Cot H
C +150000 Cot H
D +180000 Cot H
E +180000 Cot H
+45000 Csc H
+45000 Csc H
+45000 Cot H
+120000 Cot H
+165000 Cot H
+180000 Cot H
15000 lb
(b)
B +135000 Cot H
+45000 Cot H
C +225000 Cot H
+75000 Cot H
D +270000 Cot H
+90000 Cot H
E +270000 Cot H
+90000 Cot H
−68750 Cot H
−23750 Cot H
−181250 Cot H
−61250 Cot H
−248750 Cot H
−83750 Cot H
−271250 Cot H
−91250 Cot H
(c)
B +72320
+24110
C +120530
+40180
D +144640
+48210
E
+77960
+26930
−75180
−24100
+54880
+16000
−52040
−13160
+34220
+2630
−31390
+200
+16000
−13160
−36830
−12720
−97100
−32810
−133260
−44860
−145310
−48880
(d)


FIG. 10

stresses The signs of the coefficients given in Figs 10 (*a*) and (*b*) are the signs of the shears, the minus and plus signs in Fig 10 (*c*) and (*d*) represent tension and compression, respectively

### THE DECK WARREN TRUSS

**21.** When used in a deck bridge, the Warren truss may be supported either as shown in Fig 11 or as shown in Fig. 12 The live load is supported at the joints of the upper chord, the dead load may be assumed to be applied at

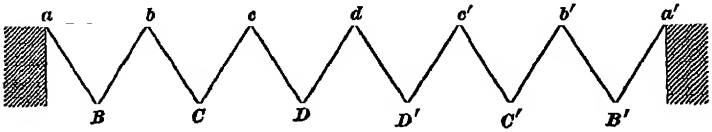



FIG 11

the joints of the upper chord, or one-third of it at the joints of the lower chord In calculating the stresses, the same methods and rules are used as for the through truss

In Fig. 11, each panel load is a full load. In Fig 12, the loads are supported between $A$ and $B$, and between $B'$ and $A'$,

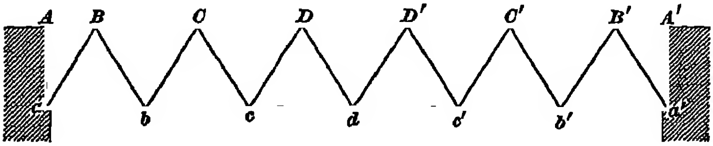



FIG 12

by the end stringers, one end of which rests on the abutments, and the other end connects with the floorbeam at $B$ or $B'$. As the distances $AB$ and $B'A'$ are each equal to a half panel, each of the joints $B$ and $B'$ supports three-quarters of a panel load, and this value must be used at these joints in the calculation of reactions and stresses.

### EXAMPLES FOR PRACTICE

1 Suppose that the truss represented in Fig 9 has a span of 112 feet, and a height of 16 feet, if the dead load is equal to 800 pounds, all of which is applied at the joints of the loaded chord, and the live load is 1,800 pounds per linear foot of bridge, find· (*a*) the maximum and minimum combined stresses in the members $bc$, $CD$, and $dd'$, using

the method of joints, (*b*) the maximum and minimum combined stresses in the members $a\,B$, $b\,C$, and $d\,D$, using the method of sections.

| | Member | Stress, in Pounds Maximum | Stress, in Pounds Minimum |
|---|---|---|---|
| Ans. (*a*) | $b\,c$ | − 83,200 | − 25,600 |
| | $C\,D$ | + 104,000 | + 32,000 |
| | $d\,d'$ | − 124,800 | − 38,400 |
| (*b*) | $a\,B$ | + 69,800 | + 21,500 |
| | $b\,C$ | + 48,900 | + 12,000 |
| | $d\,D$ | − 30,200 | − 250 |

2 Let Fig 9 represent a seven-panel deck Warren truss having the same loads and dimensions as in example 1 and supported in a manner similar to that shown in Fig 12 What are the maximum and minimum stresses due to combined dead and live load (*a*) in the members $b\,c$, $C\,D$, and $d\,d'$, using the method of sections? (*b*) in the members $B\,b$, $b\,C$, and $D\,d$, using the method of joints?

| | Member | Stress, in Pounds Maximum | Stress, in Pounds Minimum |
|---|---|---|---|
| Ans. (*a*) | $b\,c$ | − 85,800 | − 26,400 |
| | $C\,D$ | + 101,400 | + 31,200 |
| | $d\,d'$ | − 127,400 | − 39,200 |
| (*b*) | $B\,b$ | − 59,000 | − 17,000 |
| | $b\,C$ | + 59,000 | + 17,000 |
| | $D\,d$ | − 21,700 | + 6,500 |

## THE WARREN TRUSS WITH SUBVERTICALS

**22. Description.**—The simple type of Warren truss can be used for span lengths up to about 125 feet. For longer spans, it is impossible to fulfil the economical conditions of height, panel length, and slope of diagonals If the proper height of truss is used and the diagonals are given an economical inclination, the panels will be too long, and it is advisable to subdivide them This may be accomplished in several ways, one of which is to use a Warren truss with vertical members attached to the joints of the unloaded chord, dividing each panel of the loaded chord into two equal panels. The truss is then called the **Warren truss with subverticals.** The vertical members are tension members in a through truss and compression members in a deck truss. All the other members correspond in every way to those in the through Warren truss in Fig. 1. The method of calculation is the same as for the single-system Warren truss.

In Fig. 13 (*a*) is represented a twelve-panel through Warren truss with subverticals, having a span of 180 feet and a height of 24 feet. Each panel load will be denoted by $W$, and the reactions, as usual, by $R_1$ and $R_2$.

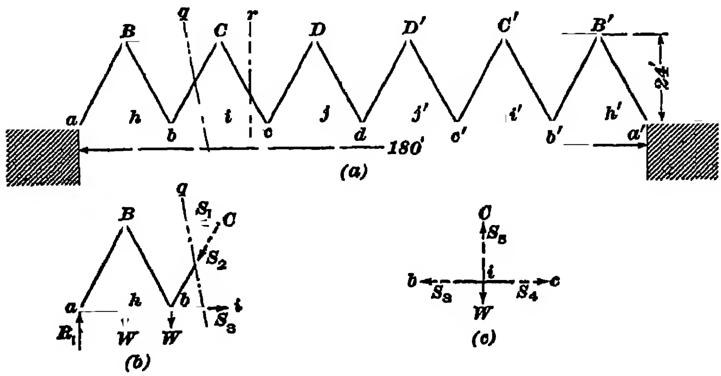



FIG 13

**23. Chord Stresses.**—The stresses in the upper-chord members may be found by dividing the bending moments at the opposite joints $b, c, d$, etc. by the height of the truss; the stresses in the lower-chord members may be found by means of the bending moments at $B, C, D$, etc. (see Art. 5). Thus, for the stresses in $B\,C$ and $b\,i$, the truss may be considered cut by a plane $q$. The portion to the left of section $q$ is shown in Fig. 13 (*b*), $S_1$, $S_2$, and $S_3$ representing the stresses in the members $B\,C$, $b\,C$, and $b\,i$, respectively. The stress in $B\,C$ is compression, and so $S_1$ will act horizontally to the left; the stress in $b\,i$ is tension, and so $S_3$ will act horizontally to the right. For the stress in $B\,C$, the center of moments is taken at $b$. Then,

$$\Sigma M = R_1 \times 30 - W \times 15 - S_1 \times 24 = 0;$$

whence

$$S_1 = \frac{R_1 \times 30 - W \times 15}{24} = \frac{\text{bending moment at } b}{24}$$

For the stress in $b\,i$, the center of moments is taken at $C$. Then,

$$\Sigma M = R_1 \times 45 - W \times 30 - W \times 15 - S_3 \times 24 = 0,$$

whence

$$S_3 = \frac{R_1 \times 45 - W \times 30 - W \times 15}{24} = \frac{\text{bending moment at } C}{24}$$

For the stress in $i c$, the joint $i$ is treated as a free body, as shown in Fig. 13 ($c$). The only horizontal forces are $S_4$ and $S_3$; therefore, they are equal and opposite, and the stress in $i c$ is equal to the stress in $b i$. In like manner, the stress in $a h$ is equal to the stress in $h b$; the stress in $c j$ is equal to the stress in $j d$, etc. Other chord stresses may be determined in the same way as those here explained.

**24. Web Stresses.**—The stress in each vertical is tension and equal to the load applied at the foot of the vertical. This is evident when the equation $\Sigma Y = 0$ is applied to the forces acting on such a joint as $i$, Fig. 13 ($c$). The only vertical forces being $S_5$ and the panel load $W$, they must be equal and opposite. Therefore, the stress in each vertical is equal to a panel load The other web stresses may be found by the method of shears already explained (Art. **10**).

**25. Deck Bridge.**—If the through truss in Fig 13 is inverted and used as a deck truss, as shown in Fig. 14, the maximum stresses in members having the same letters in

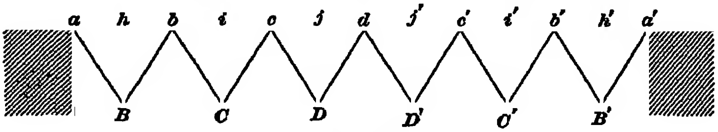



FIG 14

the two figures will be numerically equal, but of opposite characters. If the truss is supported as shown in Fig. 15, the stresses in all the members but the verticals will be of the

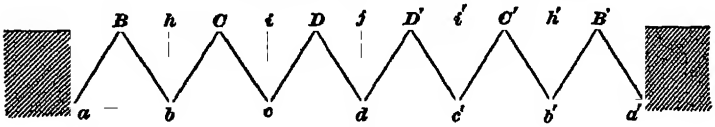



FIG 15

same characters and have the same numerical values as those in the corresponding members in the through truss in Fig. 13. The verticals will be in compression.

## EXAMPLES FOR PRACTICE

1 Suppose that, in the bridge shown in Fig 13, the dead load is 1,000 pounds, and the live load, 2,200 pounds, per linear foot Assume that all the dead load is applied at the joints of the loaded chord What are the maximum and minimum stresses, due to the combined dead and live load, in all the members?

Ans

| Member | Stress, in Pounds Maximum | Stress, in Pounds Minimum |
|---|---|---|
| $aB$, $a'B'$ | + 155,700 | + 48,600 |
| $BC$, $B'C'$ | + 150,000 | + 46,900 |
| $CD$, $C'D'$ | + 240,000 | + 75,000 |
| $DD'$ | + 270,000 | + 84,400 |
| $ah$, $hb$, $a'h'$, $h'b'$ | − 82,500 | − 25,800 |
| $bi$, $ic$, $b'i'$, $i'c'$ | − 202,500 | − 63,300 |
| $cj$, $jd$, $c'j'$, $j'd$ | − 262,500 | − 82,000 |
| $Bh$, $Ci$, $Dj$, $D'j'$, $C'i'$, $B'h'$ | − 24,000 | − 7,500 |
| $Bb$, $B'b'$, | − 129,000 | − 38,200 |
| $bC$, $b'C'$ | + 104,000 | + 26,000 |
| $Cc$, $C'c'$ | − 80,500 | − 12,400 |
| $cD$, $c'D'$ | + 58,700 | − 2,900 |
| $Dd$, $D'd$ | − 38,500 | + 19,900 |

2. Let Fig 16 be a ten-panel deck bridge having a span length of 150 feet and a height of 20 feet If the dead load is 900 pounds, and the live load, 2,000 pounds, per linear foot, and it is assumed that all

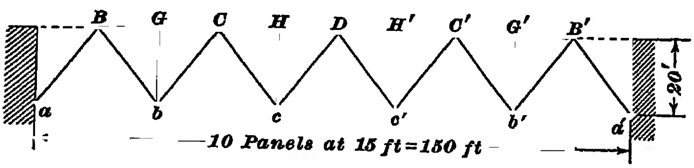



Fig 16

the dead load is applied at the joints of the loaded chord, find· (a) the maximum and minimum stresses in the members $ab$, $Bb$, $Gb$, and $bC$ due to combined dead and live load, (b) the maximum and minimum stresses in the members $bc$, $HD$, $cD$, and $cc'$ due to combined dead and live load

Ans

| | Member | Stress, in Pounds Maximum | Stress, in Pounds Minimum |
|---|---|---|---|
| (a) | $ab$ | − 73,400 | − 22,800 |
| | $Bb$ | − 97,000 | − 27,700 |
| | $Gb$ | + 21,800 | + 6,800 |
| | $bC$ | + 73,600 | + 15,500 |
| (b) | $bc$ | − 171,300 | − 53,200 |
| | $HD$ | + 195,800 | + 60,800 |
| | $cD$ | + 32,300 | − 14,500 |
| | $cc'$ | − 203,900 | − 63,300 |

### THE DOUBLE-INTERSECTION WARREN TRUSS

**26. Description.**—Fig. 17 (*a*) shows another type of Warren truss with subdivided panels, which was extensively used in the past and is used to some extent at the present time. The simple Warren truss is shown in full lines, the panels being subdivided by the addition of the web members shown in dotted lines parallel to the full-line members and

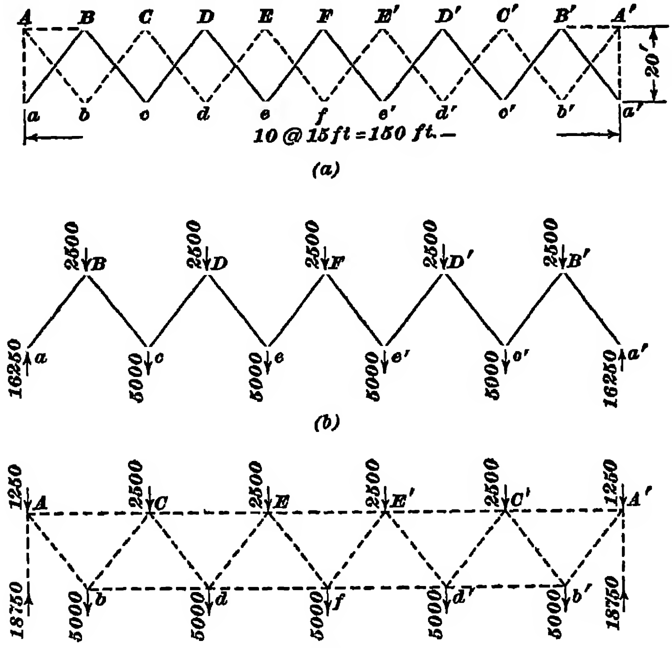



FIG 17

half way between them The two sets of web members are called the *two systems of web.* Such a truss is called a **double system, or double intersection, Warren truss,** and sometimes simply a **double Warren truss.** The joints of one system are in each case vertically opposite the joints of the other. As the end diagonals of the dotted system slope upwards, it is necessary to provide a vertical member, called **the vertical end post,** and produce the top chord at each end.

**27. Methods of Calculation.**—If the truss shown in Fig. 17 (*a*) is considered cut by a plane that intersects two chord members, that plane will cut also two web members, and there will be four unknown stresses. As there are only three equations of equilibrium, they are not sufficient for the determination of the four unknown stresses unless some assumption is made regarding the distribution of stress among the several members cut by the plane. It is customary to assume that the two systems of web members act independently, or, in other words, that they are the web members of two independent trusses lying in the same plane, the top and bottom chords being common to both trusses. The stresses in the web members of each system may be found from the loads that come on the system; and the chord stresses may be found by properly combining the chord stresses of the two systems. This will be made clearer by studying Figs. 17 (*b*) and (*c*). The system shown as a truss in Fig. 17 (*b*) is assumed to support the loads at $c, e, e', c', B, D, F, D'$, and $B'$. The web stresses due to these loads are the actual web stresses in the corresponding members of the truss shown in Fig. 17 (*a*); and the chord stresses are partial or component chord stresses. The system shown as a truss in Fig. 17 (*c*) is assumed to support the loads at $b, d, f, d', b', A, C, E, E', C'$, and $A'$. The web stresses due to these loads are the actual web stresses; the chord stresses are component chord stresses. The actual chord stresses may be found by adding the stresses found in the two systems.

The double-intersection Warren truss may be used in a deck or in a through bridge. The stresses are calculated in the same way for the two kinds. As the analytic method of calculation is shorter than the graphic, the latter will not be considered. The method of calculation can best be illustrated by an example. For this purpose, the dead-load stresses in the truss shown in Fig 17 (*a*) will be determined. The truss has ten panels, the span length is 150 feet, and the height 20 feet. The dead load will be taken as 1,000 pounds per linear foot of bridge, one-third of which

is supposed to be applied at the unloaded chord. The method of joints is best adapted to this case.

**28. Panel Loads and Reactions.**—The truss may be divided into the two systems shown in Fig. 17 (*b*) and (*c*). For convenience of reference, the system shown in full lines in Fig. 17 (*b*) may be called the *primary system*, and that shown in dotted lines in Fig. 17 (*c*), the *secondary system.* The dead panel load is equal to

$$\frac{1{,}000 \times 15}{2} = 7{,}500 \text{ pounds}$$

of which 2,500 is supported at each of the top joints, and 5,000 at each of the bottom joints. The primary system supports four loads of 5,000 and five loads of 2,500 pounds. Therefore, the reaction for the primary system is

$$\frac{4 \times 5{,}000 + 5 \times 2{,}500}{2} = 16{,}250 \text{ pounds}$$

The secondary system supports five loads of 5,000 and four loads of 2,500 pounds. Therefore, the reaction for the secondary system is

$$\frac{5 \times 5{,}000 + 4 \times 2{,}500}{2} = 17{,}500 \text{ pounds}$$

In addition to this, there is a half-panel load of 1,250 pounds at each of the end joints of the top chord. Then, the total reaction for the secondary system is equal to 18,750 pounds. The loads and reactions for the primary systems are shown in Fig. 17 (*b*); those for the secondary system, in Fig. 17 (*c*). As in previous cases,

$$\cot H = \cot Bab = \frac{15}{20} = .75; \ \csc H = \frac{\sqrt{20^2 + 15^2}}{20} = 1.25$$

**29. Web Stresses.**—The stress in the vertical end post is equal to the reaction of the secondary system. The vertical components of the web stresses in each system may be written directly by finding the shears, and the stresses found from them by multiplying by csc $H$. It should be borne in mind that each system is treated as an independent truss loaded as shown in Figs. 17 (*b*) and (*c*); also, that, in determining the shear on any section, both the lower- and the upper-chord loads should be taken into account. Thus, the shear

on a plane cutting $DF$, $De$, and $ce$, Fig 17 ($b$), is 16,250 – (2,500 + 5,000 + 2,500), or the algebraic sum of the external forces acting at $a$, $B$, $c$, and $D$

The web stresses, whose values should be verified by the student, are:

| Member | Stress, in Pounds |
|---|---|
| $aA$ | + 18,750 |
| $aB$ | 16,250 × 1.25 = + 20,300 |
| $Ab$ | 17,500 × 1.25 = – 21,900 |
| $bC$ | 12,500 × 1.25 = + 15,600 |
| $Bc$ | 13,750 × 1.25 = – 17,200 |
| $cD$ | 8,750 × 1.25 = + 10,900 |
| $Cd$ | 10,000 × 1.25 = – 12,500 |
| $dE$ | 5,000 × 1.25 = + 6,250 |
| $De$ | 6,250 × 1.25 = – 7,800 |
| $eF$ | 1,250 × 1.25 = + 1,600 |
| $Ef$ | 2,500 × 1.25 = – 3,100 |

**30. Chord Stresses.**—As explained in Art. **17**, the stress in any chord member of a single-system Warren truss is equal to the algebraic sum of the horizontal components of the stresses in all the web members that connect with the chord on the left (or right) of the member in question. For example, the stress in $DF$, Fig. 17 ($b$), is equal to the sum of the horizontal components in $aB$, $Bc$, $cD$, and $De$; the stress in $EE'$, Fig. 17 ($c$), is equal to the sum of the horizontal components in $Ab$, $bC$, $Cd$, $dE$, and $Ef$. The stress in $EF$, Fig. 17 ($a$), equals the sum of the stresses in $DF$, Fig. 17 ($b$), and $EE'$, Fig. 17 ($c$). Therefore, the stress in $EF$ equals the sum of the horizontal components in $Ab$, $aB$, $Bc$, $bC$, $Cd$, $cD$, $De$, $dE$, and $Ef$. In general,

*The stress in any chord member of a double Warren truss is equal to the algebraic sum of the horizontal components of the stresses in all the web members that connect with the chord on the left (or right) of the member considered.*

Keeping in mind that the horizontal component of the stress in any web member is equal to the vertical component multiplied by cot $H$, the chord stresses may be written as follows:

| Member | Stress, in Pounds |
|---|---|
| $AB$ | $17{,}500 \times 75 = +13{,}100$ |
| $BC$ | $(17{,}500 + 16{,}250 + 13{,}750) \times 75 = +35{,}600$ |
| $CD$ | $(17{,}500 + 16{,}250 + 13{,}750 + 12{,}500 + 10{,}000) \times 75 = +52{,}500$ |
| $DE$ | $(17{,}500 + 16{,}250 + 13{,}750 + 12{,}500 + 10{,}000 + 8{,}750 + 6{,}250) \times 75 = +63{,}750$ |
| $EF$ | $(17{,}500 + 16{,}250 + 13{,}750 + 12{,}500 + 10{,}000 + 8{,}750 + 6{,}250 + 5{,}000 + 2{,}500) \times .75 = +69{,}400$ |
| $FE'$ | $(17{,}500 + 16{,}250 + 13{,}750 + 12{,}500 + 10{,}000 + 8{,}750 + 6{,}250 + 5{,}000 + 2{,}500 + 1{,}250 - 1{,}250) \times .75 = +69{,}400$ |
| $ab$ | $16{,}250 \times 75 = -12{,}200$ |
| $bc$ | $(16{,}250 + 17{,}500 + 12{,}500) \times .75 = -34{,}700$ |
| $cd$ | $(46{,}250 + 13{,}750 + 8{,}750) \times .75 = -51{,}600$ |
| $de$ | $(68{,}750 + 10{,}000 + 5{,}000) \times .75 = -62{,}800$ |
| $ef$ | $(83{,}750 + 6{,}250 + 1{,}250) \times .75 = -68{,}400$ |
| $fe'$ | $(91{,}250 + 2{,}500 - 2{,}500) \times .75 = -68{,}400$ |

**31. Live-Load Stresses.**—The live-load stresses may be found in precisely the same way as the dead-load stresses, by separating the truss into two systems. For the maximum chord stresses, each system should be fully loaded, and the stresses in the members added together to get the combined or actual stresses. For the maximum web stresses, the portion of each system that will give the maximum shear (positive or negative) in the various panels must be loaded; the stresses found from the shears will be the actual maximum and minimum live-load stresses in the web members.

## EXAMPLE FOR PRACTICE

If the live load on the bridge described in Art. 26 and illustrated in Fig. 17 is 2,200 pounds per linear foot, determine: (*a*) the maximum and minimum combined stresses in the members $EF$, $Ef$, $eF$, and $ef$, using the dead-load stresses found in the preceding pages; (*b*) the maximum and minimum combined stresses in the members $BC$, $Bc$, $bC$, and $bc$.

| | | Member | Stress, in Pounds Maximum | Minimum |
|---|---|---|---|---|
| Ans. | (*a*) | $EF$ | + 224,100 | + 69,400 |
| | | $Ef$ | − 21,700 | + 5,100 |
| | | $eF$ | + 13,900 | − 10,800 |
| | | $ef$ | − 216,900 | − 68,400 |
| | (*b*) | $BC$ | + 116,000 | + 35,600 |
| | | $Bc$ | − 58,400 | − 17,200 |
| | | $bC$ | + 48,600 | + 13,600 |
| | | $bc$ | 109,000 | − 34,700 |

### THE DOUBLE WARREN TRUSS WITH SUBVERTICALS

**32. Description.**—Fig. 18 (*a*) shows the double Warren truss with subverticals that subdivide each panel of the loaded chord into two equal panels. In this truss, the loads at the intermediate joints *b*, *d*, *f*, etc. act on both systems at the intersections *B*, *D*, *F*, etc. of the web members, and on this account it is impossible to separate the truss into two

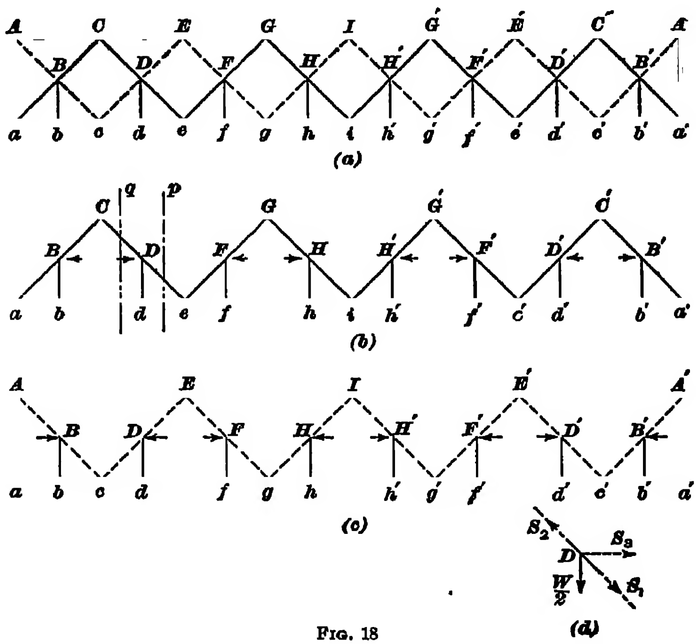



FIG. 18

independent systems. However, an assumption is usually made that is probably very close to the actual distribution of stresses. As in the case of the truss described in the preceding articles, one of the two systems formed by the inclined members is called the primary; the other, the secondary.

**33. Method of Calculation.**—The stress in each subvertical is evidently tension, and equal to the panel load at

its lower joint. In finding the stresses in the other members, it is customary to assume that each system carries one-half of the load transmitted by the subverticals to the joints $B, D, F$, etc. Thus, in Figs. 18 (*b*) and (*c*), which shows the primary and secondary systems, together with the subverticals, each of the joints $b$, $d$, $f$, etc. is supposed to carry one-half of a panel load to each system. The stresses are found in almost the same manner as in the double Warren truss without subverticals, except that, in treating each system as an independent truss, external forces, representing the action of the other system on the system under consideration, must be introduced at the joints $D$, $F$, etc., as will be explained presently.

**34. Web Stresses.**—If the general equation of equilibrium $\Sigma Y = \Sigma S \sin H = 0$ is applied to all the external forces acting on one side of a plane of section that cuts a web member and two chord members of either system, such as section $q$, Fig. 18 (*b*), it will be seen that the vertical component of the stress in the web member is equal to the shear on the section, and the stress is equal to the shear multiplied by csc $H$. For the maximum or minimum stress in any member, the system in which the member occurs should be loaded on the right or left of the member, in the same way as in a single-system Warren truss.

Consider the sections $q$ and $p$, Fig. 18 (*b*). Denoting the load at each lower-chord joint by $W$, the vertical component of the stress in $CD$ is $R_1' - \frac{W}{2}$, and that in $De$ is $R_1' - \frac{W}{2} - \frac{W}{2}$; then, the horizontal component of the stress in $CD$ is equal to $\left(R_1' - \frac{W}{2}\right) \cot H$, and in $De$, to $\left(R_1' - \frac{W}{2} - \frac{W}{2}\right) \cot H$. Writing the expression for the sum of the horizontal forces at joint $D$ of the primary system, shown in Fig. 18 (*d*), we have

$$\Sigma X = S_2 \cos H - S_1 \cos H$$

$$= \left(R_1' - \frac{W}{2}\right) \cot H - \left(R_1' - \frac{W}{2} - \frac{W}{2}\right) \cot H = \frac{W}{2} \cot H,$$

from which it will be seen that there is an unbalanced force,

equal to $\frac{W}{2}$ cot $H$, acting horizontally to the left, which must be held in equilibrium by the force $S_3$, equal to $\frac{W}{2}$ cot $H$, acting horizontally to the right. It may be shown that at the joint $D$ of the secondary system there is also an unbalanced force equal to $\frac{W}{2}$ cot $H$, acting horizontally to the left, which holds in equilibrium the unbalanced force at joint $D$ of the primary system. This force, which may be called $S_3$, is exerted at each joint ($B, D, F$, etc.) of each system by the other system, and may be considered as an external force in writing equations.

**35. Chord Stresses.**—The maximum chord stresses obtain when there is a full live load, the minimum, when there is no live load on the truss. The stress in any member may be found by properly combining the partial stresses in the two systems. When all the stresses are desired, the method of joints is the shorter; when only one or two stresses are desired, the method of moments is shorter.

For example, the stress in $EG$, Fig. 18 ($a$), is equal to the stress in $CG$, Fig 18 ($b$), plus the stress in $EI$, Fig. 18 ($c$). By the method of joints, the stress in $CG$, Fig 18 ($b$), is equal to the sum of the horizontal components of the stresses in $BC$ and $CD$; the stress in $EI$ is equal to the sum of the horizontal components of the stresses in $AB, DE$, and $EF$.

If it is desired to calculate the stress in any chord member by the method of moments, it is necessary to take into account the moments of the horizontal forces $S_3$ As these forces are alternately opposite in direction, it is convenient to pass the planes of section through the truss in such a way that there will be an even number of intermediate joints on the portion of the truss considered. Then, the moment of the forces $S_3$ on one side of the section about the center of moments will be zero (since there will be an even number whose resultant is zero), and they need not be considered in the equation of moments. In the present case, the planes

may be passed between $a$ and $b$, $d$ and $f$, $h$ and $h'$, etc. Thus, for the stress in $a\,b$, the truss may be cut by a vertical plane in panel $a\,b$, or in panel $d\,e$, and the center of moments taken at $B$ or $C$. With the center of moments at $B$, the stress in $a\,b$ is

$$\frac{R_1' \times p}{\frac{h}{2}} = \frac{R_1 \times 2p}{h}$$

and, with the center of moments at $C$ and section $p$, the stress in $a\,b$ is, as before,

$$\frac{R_1 \times 2p - \frac{W}{2} \times p + \frac{W}{2} \times p}{h} = \frac{R_1 \times 2p}{h}$$

In this case, the load at $D$, being on the right of the center of moments, has a positive, or right-handed, moment, and similar cases must be carefully treated in order to get the signs correct. The stress in any chord member may be found in a manner similar to that just given.

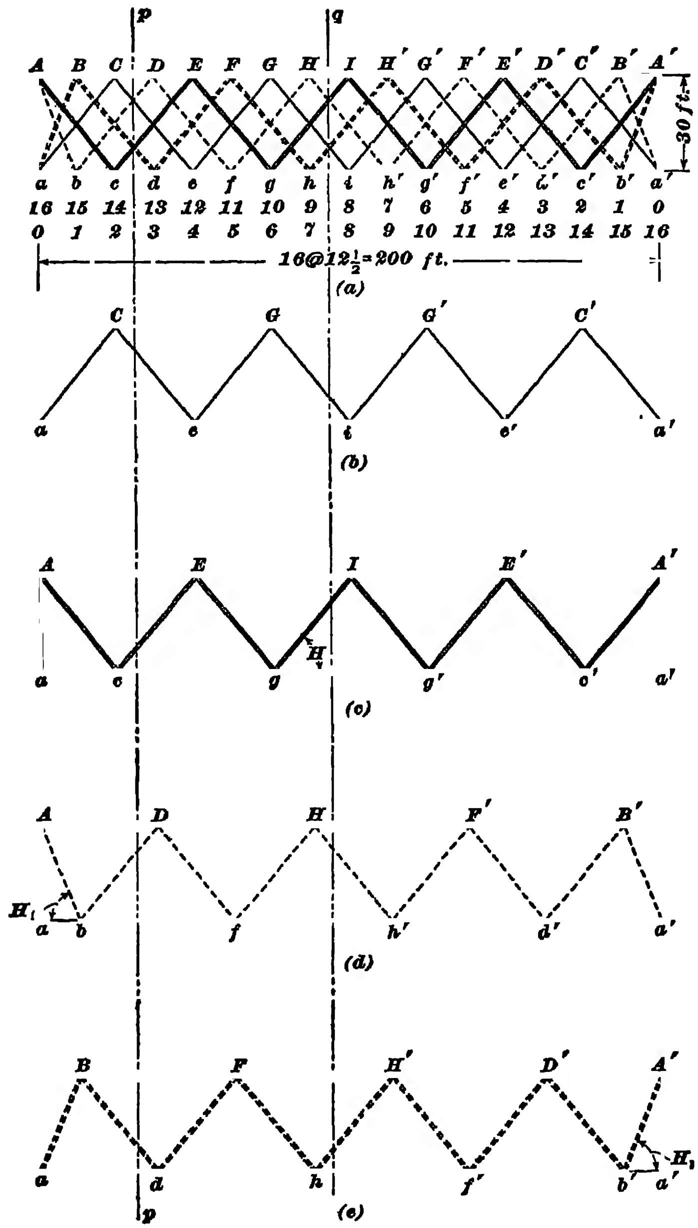
### EXAMPLE FOR PRACTICE

If the sixteen-panel through double Warren truss shown in Fig. 18 ($a$) has a span length of 192 feet and a height of 30 feet, and the dead load is 1,200 pounds per linear foot of bridge, all applied at the joints of the loaded chord, what are the dead-load stresses in the members $C\,E$, $e\,f$, $G\,I$, $F\,f$, $C\,D$, $e\,F$, and $H\,I$?

| | Member | Stress, in Pounds |
|---|---|---|
| Ans. | $C\,E$ | + 57,600 |
| | $e\,f$ | − 77,800 |
| | $G\,I$ | + 92,200 |
| | $F\,f$ | − 7,200 |
| | $C\,D$ | − 27,600 |
| | $e\,F$ | + 13,800 |
| | $H\,I$ | 0 |

## THE MULTIPLE-SYSTEM WARREN OR LATTICE TRUSS

**36. Description.**—When it is desirable to build a very deep truss of the Warren type, an economical inclination of diagonal and panel length may be used by adding two or three additional systems of web members to the simple type of truss, and subdividing the main panels into three or four



p q
A B C D E F G H I H' G' F' E' D' C' B' A'
30 ft.
a b c d e f g h i h' g' f' e' d' c' b' a'
16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
16@12½=200 ft.
(a)
C G G' C'
a e i e' a'
(b)
A E I E' A'
H
a c g g' c' a'
(c)
A D H F' B'
H₁
a b f h' d' a'
(d)
B F H' D' A'
H₁
a d h f' b' a'
p
(e)


Fig 19

equal panels. When there are four systems, as shown in Fig. 19 (*a*), the truss is called a **quadruple-system,** or **quadruple-intersection, Warren truss.** All trusses of this type are included under the general heading of "multiple-system" or "lattice trusses," and are usually built as riveted trusses. The web members of the different systems are riveted together at their intersections. Fig. 19 (*a*) represents a quadruple-intersection Warren truss with the four systems shown in single and double full and dotted lines. For convenience of reference, the system shown in single full lines in Fig. 19 (*b*) will be called the *primary system;* that shown in double full lines, Fig. 19 (*c*), the *secondary system;* that shown in single dotted lines, Fig. 19 (*d*), the *tertiary system;* and that shown in double dotted lines, Fig. 19 (*e*), the *quaternary system.*

**37. Analysis of Stresses.**—It is customary to assume that the only stresses in each system are those caused by the loads directly applied to it. Although this assumption is not strictly correct, it is probably as close as any assumption that can be made concerning the distribution of the stresses. The effect of connecting the web members to each other at every intersection is ignored in the calculation of stresses.

The stresses may be found in the same general way as the stresses in the double Warren truss, by dividing the truss into separate systems and calculating the stresses in the members of each system due to the loads that come on it. The stresses in those members (such as web members) that occur in only one system are the actual stresses; in those that occur in more than one system, they are component stresses, and the actual stresses are found by properly combining the component stresses in the different systems in which the members occur. The analytic method is best adapted to the determination of the stresses.

**38. Stresses in Primary System.**—Fig. 19 (*b*) shows the primary system, which is a single-intersection four-panel symmetrical Warren truss supported at the points *a* and *a'*.

The maximum and minimum stresses are found in the ordinary way; the stresses in the web members are the actual stresses in these members. The stresses in the chord members are component stresses.

**39. Stresses in Secondary System.**—Fig. 19 (*c*) shows the secondary system, which is a single-system symmetrical Warren truss supported at the points $A$ and $A'$ by the verticals $aA$ and $a'A'$. The maximum and minimum stresses are found in the ordinary way; the stresses in the members $aA$ and $a'A'$, and the stresses in the chord members are component stresses, as these members occur in more than one system; the stresses in the inclined web members are the actual stresses

**40. Stresses in Tertiary System.**—Fig. 19 (*d*) shows the tertiary system, which is an unsymmetrical single-system Warren truss in which the end diagonals slope differently from the others. The truss is supported at the point $A$ by the vertical $aA$, and at the point $a'$ by the abutment. The maximum and minimum stresses are found in the same way as for the single-system Warren truss. The stress in $aA$ is equal to the left reaction. The stress in any chord member is equal to the bending moment due to the loads on the system, at the joint opposite the member, divided by the height of the truss. The stress in any web member is equal to the shear in the panel in which the member is located, multiplied by the cosecant of the angle that the member makes with the horizontal. As this system is unsymmetrical, it is necessary to find the dead-load stress and the maximum and minimum live-load stresses in every member, instead of finding them for only those members that are situated on one side of the center, as heretofore.

The maximum live-load chord stresses will occur when the truss is fully loaded; the maximum live-load stress in any web member due to positive shear, when all joints to the right of the member are loaded; the maximum live-load web stresses due to negative shear, when all joints to the left of the member are loaded.

**41. Stresses in Quaternary System.**—Fig. 19 (*e*) shows the quaternary system, which is the same as the tertiary system turned end for end. The remarks made in connection with the tertiary system apply here. The maximum and minimum stresses in any member of the quaternary system are equal to the stresses in the corresponding member at the other end of the tertiary system. Thus, the stress in $D' b'$ is equal to the stress in $D b$.

**42. Stresses in End Members.**—In calculating the actual stress in the end diagonals, such as $a B$, $A b$, etc., it should be remembered that the angle $H_1$, Fig. 19 (*d*) and (*e*), for these members is different from the angle $H$ for the remaining diagonals, and that, therefore, the cosecant and cotangent will have different values for the two angles. If the stresses are found by the method of joints, it should be noted that $\cot H_1 = \frac{\cot H}{2}$, and that, then,

$$\text{hor. comp. in } a B = \text{vert. comp. in } a B \times \frac{\cot H}{2}$$
$$= \frac{\text{vert. comp. in } a B}{2} \times \cot H$$

This gives the value of the horizontal component in the end diagonal to be used in finding chord stresses by the method of joints, as explained in Art. **18**.

**43. Actual Stresses.**—The stresses found for the diagonals in the different systems are the actual stresses in the diagonals. The member $a A$ occurs in two systems, and the actual stress in it is equal to the sum of the stresses in $a A$, as found in the two systems. The chord member $A B$ occurs in two systems; $B C$, in three; $C D$, in four; etc.

**44. Determination of Stresses by Method of Sections.**—To illustrate the calculation of the stresses by the method of sections, let Fig. 19 (*a*) be a sixteen-panel through bridge having a span length of 200 feet and a height of 30 feet, let the dead load be 1,200 pounds per linear foot, one-third of which is assumed to be applied at the joints of the unloaded chord. Let it be required to calculate the

dead-load stresses in all the members cut by a plane at the section $pp$. This plane is extended so that it will cut all the separate systems represented in Fig 19 (*b*), (*c*), (*d*), and (*e*), and to each system the general method of sections is applied independently. When there is a large number of joints, as in this case, it is convenient to number them from each end of the truss, as shown in the lower part of Fig. 19 (*a*).

The dead panel load is equal to

$$\frac{1{,}200 \times 12.5}{2} = 7{,}500 \text{ pounds}$$

of which there is 5,000 at each of the joints $b, c, d$, etc., and 2,500 at each of the joints $B$, $C$, $D$, etc The half top-panel loads at $A$ and $A'$ will only affect the stresses in $aA$, and $a'A'$; they need not be considered in this example. The figure gives

$$\csc H = \frac{\sqrt{30^2 + 25^2}}{30} = \frac{39.05}{30} = 1.30$$

In the primary system, Fig. 19 (*b*), the left reaction is

$$\frac{5{,}000\,(4 + 8 + 12) + 2{,}500\,(2 + 6 + 10 + 14)}{16} = 12{,}500 \text{ pounds},$$

and the stresses in the various members are as follows

| MEMBER | STRESS, IN POUNDS |
|---|---|
| $Ce$ | $(12{,}500 - 2{,}500) \times 1.30 = -13{,}000$ |
| $CG$ | $\dfrac{12{,}500 \times 50 - 2{,}500 \times 25}{30} = +18{,}800$ |
| $ae$ | $\dfrac{12{,}500 \times 25}{30} = -10{,}400$ |

In the secondary system, Fig. 19 (*c*), the left reaction is

$$\frac{5{,}000\,(2 + 6 + 10 + 14) + 2{,}500\,(4 + 8 + 12)}{16} = 13{,}750 \text{ pounds}$$

and the stresses in the various members are as follows:

| MEMBER | STRESS, IN POUNDS |
|---|---|
| $cE$ | $(13{,}750 - 5{,}000) \times 1.30 = +11{,}400$ |
| $AE$ | $\dfrac{13{,}750 \times 25}{30} = +11{,}500$ |
| $cg$ | $\dfrac{13{,}750 \times 50 - 5{,}000 \times 25}{30} = -18{,}800$ |

In the tertiary system, Fig. 19 ($d$), the left reaction is

$$\frac{5{,}000\,(3+7+11+15)+2{,}500\,(1+5+9+13)}{16}$$
$$= 15{,}625 \text{ pounds,}$$

and the stresses in the various members are:

| Member | Stress, in Pounds |
|---|---|
| $bD$ | $(15{,}625-5{,}000)\times 1.30 = +13{,}800$ |
| $AD$ | $\dfrac{15{,}625\times 12.5}{30} = +\ \ 6{,}500$ |
| $bf$ | $\dfrac{15{,}625\times 37.5-5{,}000\times 25}{30} = -15{,}400$ |

In the quaternary system, Fig. 19 ($e$), the left reaction is

$$\frac{5{,}000\,(1+5+9+13)+2{,}500\,(3+7+11+15)}{16}$$
$$= 14{,}375 \text{ pounds,}$$

and the stresses are:

| Member | Stress, in Pounds |
|---|---|
| $Bd$ | $(14{,}375-2{,}500)\times 1.30 = -15{,}400$ |
| $BF$ | $\dfrac{14{,}375\times 37.5-2{,}500\times 25}{30} = +15{,}900$ |
| $ad$ | $\dfrac{14{,}375\times 12.5}{30} = -\ \ 6{,}000$ |

The actual stresses in the various members are as follows:

| Member | | Stress, in Pounds |
|---|---|---|
| $bD$ | | $+13{,}800$ |
| $cE$ | | $+11{,}400$ |
| $Ce$ | | $-13{,}000$ |
| $Bd$ | | $-15{,}400$ |
| $CD$ | sum of stresses in $AD$, $AE$, $BF$, $CG$, $= 6{,}500+11{,}500+15{,}900+18{,}800 =$ | $+52{,}700$ |
| $cd$ | sum of stresses in $ad$, $ae$, $bf$, $cg$, $= -6{,}000-10{,}400-15{,}400-18{,}800 =$ | $-50{,}600$ |

### EXAMPLES FOR PRACTICE

1. Using the same dead loads and dimensions as in Fig. 19, and a live load of 2,000 pounds per linear foot, determine the maximum and minimum stresses due to combined live and dead load in all the members cut by a plane at the section $q$

| | Member | Stress, in Pounds | |
|---|---|---|---|
| | | Maximum | Minimum |
| Ans. | $HI$ | + 268,800 | + 99,500 |
| | $Hh'$ | − 11,000 | + 5,300 |
| | $gI$ | + 9,750 | − 6,500 |
| | $Gi$ | − 15,400 | + 800 |
| | $H'h$ | − 11,000 | + 5,300 |
| | $hi$ | − 256,300 | − 97,400 |

2. With the same data as in example 1, find the maximum stress in $aA$ due to combined live and dead load. Assume that the half-dead panel load at $A$ is included in the secondary system

Ans. + 83,750 lb

3. With the same data as in example 1, find, by the method of joints, the dead-load stress in the member $FG$. Ans. + 90,100 lb

### THE PRATT TRUSS

**45. Description.**—The **Pratt truss**, Fig 20 ($a$), is a simple type of truss in which the web members are alternately vertical and inclined; there is a vertical web member at each panel point and an inclined member in each panel, connecting the top of one vertical with the bottom of the next. The Pratt truss is used in deck, through, and half-through bridges; is built pin-connected more frequently than riveted; and is especially adapted to span lengths of 100 to 250 feet. For the longer spans, multiple systems of web or subdivided panels are sometimes built, although such forms are going out of use.

**46. Diagonals.**—In all except the end panels, *the diagonals are designed to resist tension only*. It was shown in the analysis of the Warren truss that positive shear causes tension in diagonals that slope upwards to the left, and negative shear tension in diagonals that slope upwards to the right. Therefore, in order to have the diagonals in tension, they must slope upwards to the left in the panels where the

shear is positive, and upwards to the right in the panels where the shear is negative. In the panels near the center, in which the maximum combined shear is opposite to the minimum shear, two diagonals will be required, one sloping in each direction. In a panel where there are two diagonals, one of them, the main diagonal, will be in tension when the combined shear is a maximum; the other, called a **counter**, will be in tension when the combined shear is a minimum. When any loading causes tension in one of these diagonals,

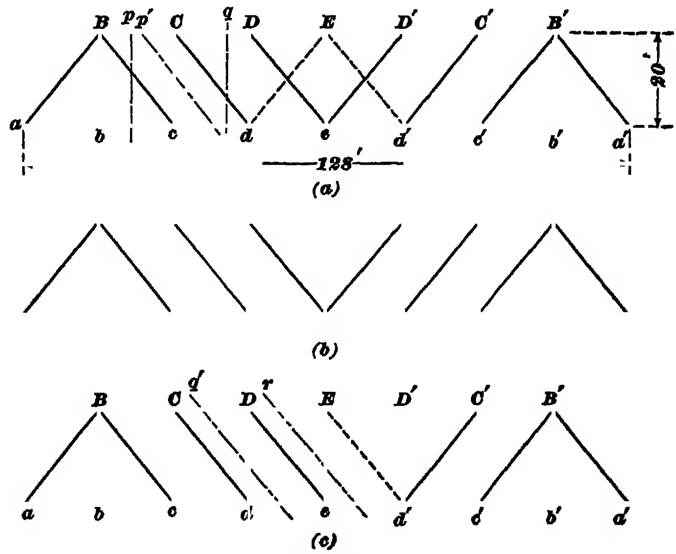



Fig. 20

that diagonal is said to be in action, and the stress in the other is assumed to be zero; the latter diagonal is said to be out of action and need not be considered as part of the truss for that loading. For example, when there is a full load on the truss shown in Fig. 20 (*a*), the shear in panel *de* is positive; there is tension in the main diagonal *De*, and the counter *dE* is out of action; the shear in panel *ed′* is negative, there is tension in the main diagonal *eD′*, and the counter *Ed′* is out of action. The members of the truss that are in action for full load are shown in Fig. 20 (*b*); as

the counters are out of action, they are omitted from the diagram.

**47. Method of Calculation.**—The stresses in the members of the simple Pratt truss can be found analytically or graphically. The work of calculation by either of the analytic methods is so simple that the graphic method will not be employed. The work of calculation can be best illustrated by the consideration of special cases.

## THE THROUGH PRATT TRUSS WITH AN EVEN NUMBER OF PANELS

**48. Description.**—In Fig 20 (*a*) is represented a through Pratt truss that has an even number of panels, the vertical member $Ee$ being the center vertical. The span is 128 feet, and the height 20 feet. The end posts $aB$ and $a'B'$ are compression members; the verticals $Bb$ and $B'b'$ are tension members, and are called the **hip verticals** or **end suspenders**; all other verticals are compression members. The members $dE$ and $Ed'$ are the counters.

It will be assumed that the dead load is 800 pounds, all of which is applied at the joints of the loaded chord, and that the live load is 2,000 pounds per linear foot of bridge.

**49. Panel Loads and Reactions.**—The dead panel load is

$$\frac{800}{2} \times 16 = 6{,}400 \text{ pounds,}$$

and each dead-load reaction is

$$\frac{6{,}400 \times 7}{2} = 22{,}400 \text{ pounds}$$

The live panel load is

$$\frac{2{,}000}{2} \times 16 = 16{,}000 \text{ pounds,}$$

and each live-load reaction for full load is

$$\frac{16{,}000 \times 7}{2} = 56{,}000 \text{ pounds}$$

**50. Chord Stresses.**—To calculate the stress in any chord member, the truss may be considered cut by a surface

that intersects three members, one of which is the member whose stress is desired. The stress in the member is then equal to the bending moment on the truss at the intersection of the two other members, divided by the height of the truss. For example, for the member $B\,C$, Fig. 20 ($a$), the truss may be cut by the plane $p$, or by the plane $p'$; in either case, the center of moments is at $c$, and the stress in $B\,C$ is equal to the bending moment at $c$ divided by the height.

For the member $c\,d$, the truss may be cut by either $p'$ or $q$; in either case, the center of moments is at $C$. As $C$ is vertically over $c$, the bending moments at these two points are the same; therefore, the stress in $B\,C$ is numerically equal to the stress in $c\,d$. This may also be proved by applying the equation $\Sigma X = \Sigma S \cos H = 0$ to all the forces to the left of section $p'$: as the only horizontal forces are the stresses in $B\,C$ and $c\,d$, they must be equal and opposite. In like manner, it may be shown that the stress in $C\,D$ is equal to the stress in $d\,e$.

In calculating the moments at the various points, the work may be simplified by taking the panel length as the unit of length, that is, by expressing the lever arms in panel lengths, and multiplying the result by the panel length in feet. Thus, the moment of $R_1$ about $d$ may be written $R_1 \times 3$, the distance from the line of action of $R_1$ to $d$ being 3 panels. Likewise, the moments of the loads at $b$ and $c$, with respect to $d$, are, respectively, $W \times 2$ and $W \times 1$. The resultant moment, referred to the panel length as the unit of length, is

$$R_1 \times 3 - W \times 2 - W \times 1$$

The moment (in foot-pounds or foot-tons, as the case may be) is obtained by multiplying this result by the panel length, 16; thus,

$$\text{moment at } d = (R_1 \times 3 - W \times 2 - W \times 1) \times 16$$

The dead-load chord stresses are as follows:

Stress in $a\,b$, $b\,c$ (center of moments at $B$),

$$\frac{(22{,}400 \times 1) \times 16}{20} = -17{,}900 \text{ pounds}$$

Stress in $c\,d$ (center of moments at $C$),

$$\frac{(22{,}400 \times 2 - 6{,}400 \times 1) \times 16}{20} = -30{,}700 \text{ pounds}$$

Stress in $de$ (center of moments at $D$),

$$\frac{[22{,}400 \times 3 - 6{,}400(2+1)] \times 16}{20} = -38{,}400 \text{ pounds}$$

Stress in $BC$ (stress in $cd$), + 30,700 pounds.

Stress in $CD$ (stress in $de$), + 38,400 pounds.

Stress in $DE$ (center of moments at $e$),

$$\frac{[22{,}400 \times 4 - 6{,}400(3+2+1)] \times 16}{20} = +41{,}000 \text{ pounds}$$

The dead-load chord stresses are shown in Fig. 21.

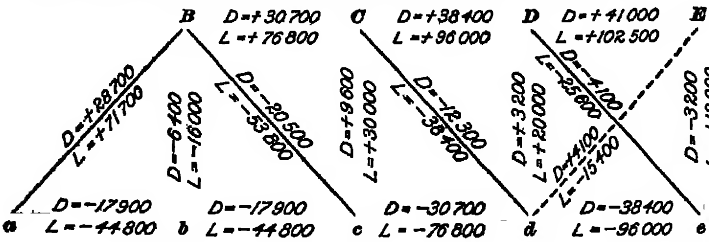


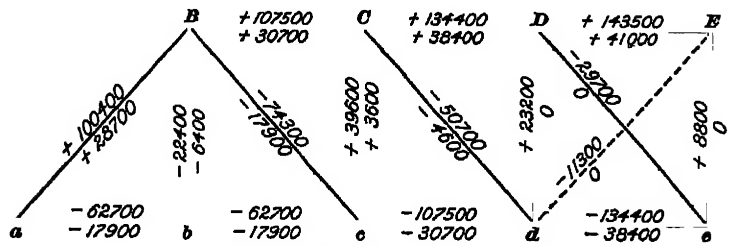

FIG 21

The live-load chord stresses may be found as above; but, as the dead load is all applied at the joints of the loaded chord, they are more conveniently determined by multiplying the dead-load stresses by the ratio of live to dead load, which is $\frac{2{,}000}{800}$, or $\frac{5}{2}$. The results are as follows:

| MEMBER | STRESS, IN POUNDS |
|---|---|
| $ab$, $bc$ | $17{,}900 \times \frac{5}{2} = - \quad 44{,}800$ |
| $cd$ | $30{,}700 \times \frac{5}{2} = - \quad 76{,}800$ |
| $de$ | $38{,}400 \times \frac{5}{2} = - \quad 96{,}000$ |
| $BC$ | $30{,}700 \times \frac{5}{2} = + \quad 76{,}800$ |
| $CD$ | $38{,}400 \times \frac{5}{2} = + \quad 96{,}000$ |
| $DE$ | $41{,}000 \times \frac{5}{2} = + 102{,}500$ |

The live-load chord stresses are shown in Fig. 21, and the combined chord stresses are shown in Fig. 22. The

maximum stresses are obtained by adding the dead- and live-load stresses; the minimum stresses are the dead-load stresses.




*Maximum and Minimum Stresses*

FIG 22

**51. Dead-Load Shears.**—As the chords are horizontal, the vertical component of the stress in any web member is equal to the shear on the plane of section that cuts such member and two chord members. Thus, the vertical component in $Cd$, Fig. 20 ($a$), is equal to the shear on section $q$. Since in this case there is no load at $C$, the shear on $p'$ is equal to the shear on $q$, and it follows that the stress in $Cc$ is equal to the vertical component of the stress in $Cd$. Likewise, the stress in $Dd$ is equal to the vertical component in $De$.

The dead-load shears are as follows:

| PANEL | SHEAR, IN POUNDS |
|---|---|
| *a b* | + 22,400 |
| *b c* | + 16,000 |
| *c d* | + 9,600 |
| *d e* | + 3,200 |
| *e d'* | − 3,200 |

and likewise for the remaining panels.

**52. Live-Load Shears.**—The approximate maximum positive live-load shear in any panel occurs when the truss is loaded on the *right* of the panel (see *Stresses in Bridge Trusses*, Part 1). The approximate values of the maximum positive live-load shears are as follows:

Panel $a\,b$ (loads at $b$, $c$, $d$, $e$, $d'$, $c'$, and $b'$),

$$\frac{16,000\,(1+2+3+4+5+6+7)}{8} = 56,000 \text{ pounds}$$

Panel *b c* (loads at *c*, *d*, *e*, *d'*, *c'*, and *b'*),

$$\frac{16{,}000\,(1+2+3+4+5+6)}{8} = 42{,}000 \text{ pounds}$$

Panel *c d* (loads at *d*, *e*, *d'*, *c'*, and *b'*),

$$\frac{16{,}000\,(1+2+3+4+5)}{8} = 30{,}000 \text{ pounds}$$

Panel *d e* (loads at *e*, *d'*, *c'*, and *b'*),

$$\frac{16{,}000\,(1+2+3+4)}{8} = 20{,}000 \text{ pounds}$$

Panel *e d'* (loads at *d'*, *c'*, and *b'*),

$$\frac{16{,}000\,(1+2+3)}{8} = 12{,}000 \text{ pounds}$$

Panel *d' c'* (loads at *c'* and *b'*),

$$\frac{16{,}000\,(1+2)}{8} = 6{,}000 \text{ pounds}$$

Panel *c' b'* (load at *b'*),

$$\frac{16{,}000 \times 1}{8} = 2{,}000 \text{ pounds}$$

In *b' a'*, there can be no positive shear.

The maximum negative shear in any panel is numerically the same as the maximum positive shear in the corresponding panel at the other end of the truss.

**53. Combined Shears.**—By combining dead-load with positive and negative live-load shears for the left half of the truss, the following results are obtained:

| Panel | Dead-Load Shear Pounds | Positive Live-Load Shear Pounds | Negative Live-Load Shear Pounds | Maximum Shear (Dead + Positive Live-Load) Pounds | Minimum Shear (Dead + Negative Live-Load) Pounds |
|---|---|---|---|---|---|
| *a b* | + 22,400 | + 56,000 | | + 78,400 | + 22,400 |
| *b c* | + 16,000 | + 42,000 | − 2,000 | + 58,000 | + 14,000 |
| *c d* | + 9,600 | + 30,000 | − 6,000 | + 39,600 | + 3,600 |
| *d e* | + 3,200 | + 20,000 | − 12,000 | + 23,200 | − 8,800 |

**54. Exact Live-Load Shears.**—For purposes of comparison, the exact maximum live-load shears may be

calculated from the formula given in *Stresses in Bridge Trusses*, Part 1; namely,

$$V'' = W'' \times \frac{m^2}{2(n-1)}$$

In the present case, $W''$, the panel load, is equal to 16,000 pounds, and $n$ is equal to 8 For the panel $a\,b$, $m = 7$; for the panel $b\,c$, $m = 6$; etc. The exact shears, the approximate shears, and the differences are as follows·

| Panel | Exact Shear Pounds | Approximate Shear Pounds | Difference Pounds |
|---|---|---|---|
| $a\,b$ | 56,000 | 56,000 | |
| $b\,c$ | 41,100 | 42,000 | 900 |
| $c\,d$ | 28,600 | 30,000 | 1,400 |
| $d\,e$ | 18,300 | 20,000 | 1,700 |
| $e\,d'$ | 10,300 | 12,000 | 1,700 |
| $d'\,c'$ | 4,600 | 6,000 | 1,400 |
| $c'\,b'$ | 1,100 | 2,000 | 900 |
| $b'\,a'$ | | | |

The approximate shears are greater than the exact in every panel except the two end panels. This is on the safe side, and, as the differences are not great, the approximate values may be used.

**55. Maximum and Minimum Stresses in the Diagonals.**—The maximum stresses in the diagonals are found from the maximum combined shears. The stress in any diagonal is equal to the combined shear in the panel in which the member is located multiplied by csc $H$, which in this case is

$$\frac{\sqrt{16^2 + 20^2}}{20} = 1.28$$

In all panels except $d\,e$, both the maximum and minimum combined shears are positive. In panel $d\,e$, they are of opposite kinds, when the truss is loaded on the right of this panel, the combined shear is positive; when loaded on

the left, it is negative. This reversal of shear requires the use of two diagonals in panel *de*. As explained in Art. **46**, the diagonal *De*, which slopes upwards toward the left, is the main diagonal and is in tension when the shear in panel *de* is positive; the diagonal *dE*, which slopes upwards toward the right, is the counter, and is in tension when the shear is negative. The maximum combined shear in the panel *de* is positive and equal to 23,200 pounds; then, the maximum tension in *De* is equal to $23{,}200 \times 1.28 = 29{,}700$ pounds. The minimum combined shear in panel *de* is negative and equal to 8,800 pounds; then the maximum tension in *dE* is equal to $8{,}800 \times 1.28 = 11{,}260$ pounds. The minimum stress in each diagonal in panel *de* is equal to zero, as it is assumed that when one is in action the other is out of action.

**56.** From the foregoing, it follows that the maximum tension in any main diagonal to the left of the center is equal to the maximum positive combined shear in the panel in which the diagonal is located, multiplied by csc *H*. When the minimum combined shear in any panel is positive, the minimum tension in the diagonal in that panel is equal to the minimum combined shear multiplied by csc *H*; when the minimum combined shear is negative, a counter is required, the maximum tension in which is equal to the minimum combined shear multiplied by csc *H*; in the latter case, the minimum tension in both the main diagonal and the counter are equal to zero.

**57. Maximum Stresses in the Verticals.**—The stress in the hip vertical *Bb* is found by considering the joint *b*, the maximum combined stress in *Bb* is tension, and equal to the sum of a dead and a live panel load, or 22,400 pounds. The stress in *Cc* is found by considering joint *C*. As the only vertical forces acting at this joint are the stresses in *Cc* and *Cd*, the maximum combined stress in *Cc* is compression and equal to the vertical component of the maximum combined stress in *Cd*, or 39,600 pounds. In like manner, the maximum combined stress in *Dd* is found to be 23,200 pounds compression, and in *Ee*, 8,800 pounds compression.

The stress in $Ee$ is a maximum when joints $d'$, $c'$, and $b'$ are loaded with live load. Under this condition of loading, the combined shear in panel $ed'$ is positive, counter $Ed'$ is in action, and main diagonal $eD'$ is out of action. The members of the truss that are in action for this loading are shown in Fig. 20 ($c$). The stress in $Ee$ is equal to the shear on section $r$, which is the maximum positive shear in panel $ed'$, or 12,000 − 3,200 = + 8,800 pounds compression.

When a portion of the dead load is assumed to be applied at the joints of the unloaded chord, the stress in any vertical to the left of the center, except the hip vertical, is equal to the vertical component of the stress in the diagonal in the panel to the right, plus the load at the joint of the unloaded chord.

**58. Minimum Stresses in the Verticals.**—The minimum stress in $Bb$ is tension, and equal to a dead panel load, or 6,400 pounds. The combined stress in $Cc$ is compression, and equal to the vertical component of the minimum combined stress in $Cd$, or 3,600 pounds. The minimum stress in $Dd$ is zero, and occurs when the stress in the diagonal $De$ is a minimum, that is, when the counter $dE$ is in action and the main diagonal $De$ is out of action. In like manner, the minimum stress in $Ee$ is zero, and occurs when the main diagonals $De$ and $eD'$ are both in action, the stresses in the counters $dE$ and $d'E$ being zero.

**59. Combined Stresses.**—The maximum and minimum stresses are shown in Fig. 22; the dead and live-load stresses, in Fig. 21. In the counter $dE$, Fig. 21, there is shown a compressive stress of 4,100 pounds due to dead load; and in $Ee$, a tensile stress of 3,200 pounds. These are the stresses that would occur in $dE$ and $Ee$ if the main diagonal $De$ in the panel $de$ were omitted when there is nc live load on the truss. No compression can actually occur in $dE$, and no tension in $Ee$, the values given being the amounts by which the live-load stresses in those members are reduced by the dead load when the main diagonal $De$ is out of action.

**60. Odd Number of Panels.**—The Pratt truss represented in Fig. 23 has an odd number of panels. In this truss, there is a center panel in which the dead-load shear is zero.

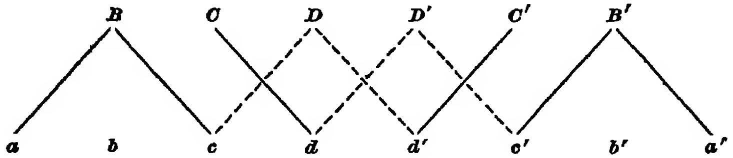



FIG 23

The dead-load stresses in the two diagonals in this panel are, therefore, equal to zero, and both may be considered as counters. Otherwise, this truss is the same as the one already analyzed.

**61. Chord Stresses for Partial Load.**—It is sometimes necessary to compute the stress in a chord member due to a live load over a portion of the span In the panels where there are no counters, the center of moments for any chord member will be the same for a partial as for a full load. In the panels where there are counters, the center of moments for the chord members depends on whether the counter or the main diagonal is in action for the specified loading. If the main diagonal is in action, the center of moments will be at the intersection of the main diagonal with the opposite chord; if the counter is in action, the center of moments will be at the intersection of the counter with the opposite chord.

EXAMPLE —Let it be required to calculate the stress in the chord member $e\,d'$, if the through Pratt truss shown in Fig 20 (*a*) carries a dead load of 6,400 pounds at each panel point, and a live load of 16,000 pounds at each of the panel points $e$, $d'$, $c'$, and $b'$.

SOLUTION —The dead-load reaction at the left end is

$$\frac{7 \times 6{,}400}{2} = 22{,}400 \text{ lb.}$$

The dead-load shear in the panel $e\,d'$ is

$$22{,}400 - 4 \times 6{,}400 = -3{,}200 \text{ lb.}$$

The live-load reaction at the left end is

$$\frac{16{,}000 \times (1 + 2 + 3 + 4)}{8} = 20{,}000 \text{ lb.}$$

The live-load shear in the panel $e\,d'$ is

$$20{,}000 - 16{,}000 = +4{,}000 \text{ lb.}$$

The combined shear in the panel $e\,d'$ is

$$-3{,}200 + 4{,}000 = +800 \text{ lb}$$

The combined shear is positive, and causes tension in the member sloping upwards to the left, which is the counter $d'E$. Therefore, the counter $d'E$ is in action for the specified loading, as shown in Fig 20 ($c$), and the center of moments for $e\,d'$ is at $E$, the intersection of $d'E$ and $E\,D'$. Then, the stress in $e\,d'$ is equal to the moment at $E$ divided by the height of the truss, or, the tension in $e\,d'$ is

$$\frac{[22{,}400 \times 4 - 6{,}400\,(1 + 2 + 3) + 20{,}000 \times 4] \times 16}{20} = 104{,}960 \text{ lb. Ans.}$$

### THE DECK PRATT TRUSS

**62. Description.**—The deck Pratt truss may be supported in any of the ways shown in Figs. 24, 25, and 26.

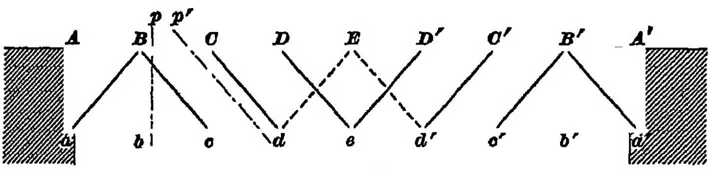



FIG 24

These three trusses are alike except for the end panels. In Fig 24, the truss is supported at the point $a$ in the same way

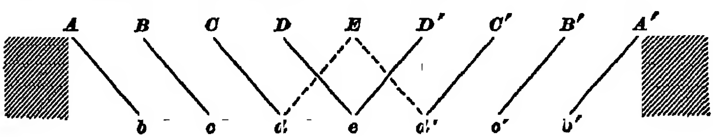



FIG 25

as the through Pratt truss; the loads in the end panel are supported by stringers $A\,B$, the ends of which either rest on

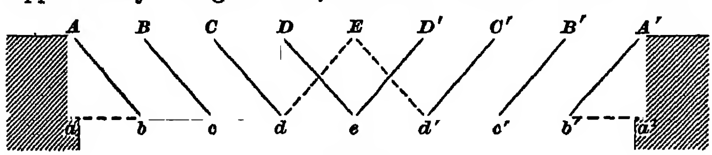



FIG 26

the masonry at $A$ or are supported by vertical members $a\,A$. In this truss the inclined member $a\,B$ is the end post, and is

a compression member. In Fig. 25, the truss is supported at the point $A$, and the end diagonal $A b$ is a tension member. In Fig. 26, the truss is supported at the point $a$; the vertical $a A$ is in compression, and is called the *vertical end post;* the web diagonal $A b$ is in tension; the stress in $a b$ due to dead and live loads is zero, this member being inserted for the sake of stiffness.

**63. Chord Stresses.**—Using the same loads and dimensions as for the through Pratt truss in Fig. 20 ($a$), the stress in any chord member of the deck Pratt truss will be the same as the corresponding member of the through Pratt truss. For example, the stress in $CD$, Fig. 20 ($a$), is equal to the moment at $d$ divided by the height of the truss; also, the stress in $CD$, Figs. 24, 25, and 26, is equal to the moment at $d$ divided by the height of the truss. In Figs. 25 and 26, the stress in $A B$ equals the stress in $b c$. For any other dimensions and loading, the chord stresses may be found in exactly the same way as for the through Pratt truss. The maximum chord stresses occur when there is a full live load; the minimum, when there is no live load.

**64. Stresses in Diagonal Members.**—The end diagonal in Fig. 24 is a compression member; the end diagonals in Figs. 25 and 26 are tension members The vertical component of the maximum stress in the end diagonal is equal to the maximum positive shear in the end panel; the vertical component of the minimum stress is equal to the dead-load shear. The stresses in all the other diagonals may be found in exactly the same way as for the through truss.

**65. Stresses in Vertical Members.**—The stresses in the verticals are different from those in the corresponding members of the through truss The vertical $B b$, Fig. 24, is the hip vertical; the stress in $B b$ is equal to zero, or to the dead load at $b$, if any. The maximum stress in $A a$, Fig. 26, is equal to the left reaction when there is a full live load; the minimum stress in $A a$ is equal to the left reaction when there is no live load. In calculating the

stress in $A\,a$, one-half of a panel load must be applied at $A$, as this is carried to the abutment by $A\,a$. The stress in any other vertical is, in general, equal to the shear on a plane of section cutting that vertical and two chord members. Thus, the stress in $Cc$, Fig. 24, is equal to the shear in the plane of section $p'$. Then, the maximum compression in any vertical on the left of the center (except the hip vertical and vertical end post) is equal to the maximum positive shear in a plane cutting that vertical and the two chord members between which the vertical lies. This occurs when the joint at the top of the member and all joints to the right are loaded.

The maximum positive combined shear in the panel to the left of the center vertical may be less than the sum of a dead and a live panel load, in which case the maximum compression in the vertical will be equal to the sum of a dead and a live panel load In this case, the shear in the panel to the right of the vertical will be negative, and the main diagonals in both center panels will be in action; as the stresses in the two counters that meet at the top of the center vertical will then be equal to zero, the vertical will simply support the load at its upper joint.

When the minimum combined shear in any panel on the left of the center is positive and *greater* than a dead panel load, the minimum stress in the vertical on the right of such panel is equal to the minimum combined shear on the plane cutting such vertical and the two chord members between which it lies. When the minimum combined shear is positive and *less* than a dead panel load, the shear in the panel to the right of the vertical is negative. The diagonal sloping upwards to the left will be in action in the panel on the left, the diagonal sloping upwards to the right, in the panel on the right, and the stress in the diagonal or diagonals that meet the vertical at the top will be zero Then, the only vertical forces acting at the top of the vertical member will be the dead panel load and the stress in the member; therefore, the minimum stress in such a vertical member will be equal to a dead panel load.

When the minimum shear in the panel to the left of any vertical is negative, the shear in that panel may have any value between the minimum shear, which is negative, and the maximum shear, which is positive. Therefore, under some conditions of loading, the shear in the panel to the left will be positive and less than a dead panel load, and then the stress in the vertical will, as shown, be a minimum and equal to a dead panel load In like manner, it may be shown that the minimum stress in any vertical between this member and the center will be equal to a dead panel load.

EXAMPLE —Let it be required to calculate the maximum and minimum stresses in the verticals of the deck Pratt truss shown in Fig 25, using the same dimensions and loads as for the through truss shown in Fig 20 (*a*) and described in Art 48.

SOLUTION —The maximum and minimum shears are the same as for the through truss, and are given in Art. 53. They are as follows:

| Panel | Maximum Shear Pounds | Minimum Shear Pounds |
|---|---|---|
| *AB* | + 78,400 | + 22,400 |
| *BC* | + 58,000 | + 14,000 |
| *CD* | + 39,600 | + 3,600 |
| *DE* | + 23,200 | − 8,800 |

The maximum stresses (which are the sum of the dead- and live-load stresses) are as follows:

| MEMBER | STRESS, IN POUNDS |
|---|---|
| *Bb* | + 22,400 + 56,000 = + 78,400 |
| *Cc* | + 16,000 + 42,000 = + 58,000 |
| *Dd* | + 9,600 + 30,000 = + 39,600 |
| *Ee* | + 3,200 + 20,000 = + 23,200 |

The maximum stress in *Ee* is equal to the maximum positive shear in panel *DE*, as this is greater than the sum of a full dead and a full live panel load (22,400 lb )

The minimum stresses are as follows·

| MEMBER | STRESS, IN POUNDS |
|---|---|
| *Bb* | + 22,400 |
| *Cc* | + 14,000 |
| *Dd* | + 6,400 |
| *Ee* | + 6,400 |

The minimum shear in panel $CD$ is positive and equal to 3,600 pounds, it occurs when joints $B$ and $C$ are loaded Under this condition of loading, the load at $D$ is a dead panel load, or 6,400 pounds. Then, the shear in panel $DE$ for the same loading is 3,600 − 6,400 = − 2,800 pounds, as this is negative, the counter $dE$ is in action and the stress in $De$ is zero Therefore, the stress in $Dd$ must be equal to the dead panel load at $D$, which is equal to 6,400 pounds In like manner, it may be shown that the minimum stress in $Ee$ is 6,400 pounds

### EXAMPLES FOR PRACTICE

1 Let Fig 27 be a ten-panel through Pratt truss having a span length of 180 feet and a height of 25 feet If the dead load is 1,200 pounds, one-third of which is applied at the joints of the unloaded chord, and the live load is 2,400 pounds, per linear foot of bridge,

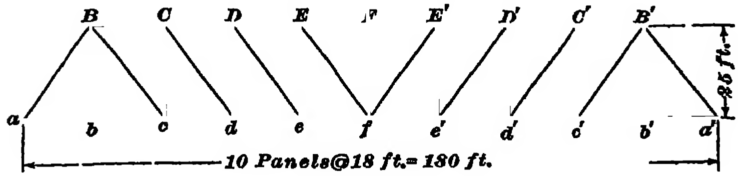



FIG 27

what are: (a) the maximum stresses in the counters and main diagonals in all the panels in which counters are required? (b) the maximum and minimum stresses due to combined live and dead load in the diagonals $aB$ and $Bc$? (c) the maximum and minimum stresses due to combined dead and live load in the verticals $Bb$, $Dd$, and $Ff$? (d) the maximum and minimum stresses due to combined dead and live load in the chord members $bc$, $BC$, and $DE$?

| | | Member | Stress, in Pounds: Maximum | Stress, in Pounds: Minimum |
|---|---|---|---|---|
| Ans. | (a) | $Ef$, $fE'$ | − 46,600 | 0 |
| | | $eF$, $Fe'$ | − 20,000 | 0 |
| | (b) | $aB$ | + 179,600 | + 59,900 |
| | | $Bc$ | − 142,400 | − 43,900 |
| | (c) | $Bb$ | − 28,800 | − 7,200 |
| | | $Dd$ | + 65,160 | + 6,840 |
| | | $Ff$ | + 19,800 | + 3,600 |
| | (d) | $bc$ | − 105,000 | − 35,000 |
| | | $BC$ | + 186,600 | + 62,200 |
| | | $DE$ | + 279,900 | + 93,300 |

2. Let Fig 28 be a nine-panel deck Pratt truss having a span length of 180 feet and a height of 26 feet If the dead load is 1,200 pounds, one-third of which is applied at the joints of the unloaded chord, and the live load is 2,200 pounds, per linear foot of bridge. (a) what are

the maximum combined stresses in the counters and main diagonals in all the panels in which counters are required? (*b*) what are the

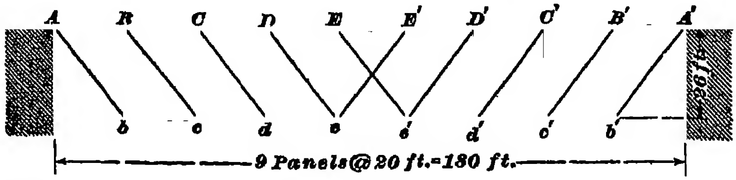



FIG 28

maximum and minimum combined stresses in the diagonals $Ab$ and $Cd$? (*c*) in the verticals $Bb$, $Dd$, and $Ee$? (*d*) in the chord members $bc$, $CD$, and $EE'$?

| | | Member | Stress, in Pounds Maximum | Stress, in Pounds Minimum |
|---|---|---|---|---|
| Ans. | (*a*) | $Ee'$, $eE'$ | − 30,800 | 0 |
| | | $De$, $e'D'$ | − 61,400 | 0 |
| | | $dE$, $d'E'$ | − 3,400 | 0 |
| | (*b*) | $Ab$ | − 171,600 | − 60,600 |
| | | $Cd$ | − 95,100 | − 21,000 |
| | (*c*) | $Bb$ | + 132,000 | + 44,000 |
| | | $Dd$ | + 71,300 | + 12,700 |
| | | $Ee$ | + 44,700 | + 8,000 |
| | (*d*) | $bc$ | − 104,600 | − 36,900 |
| | | $CD$ | + 235,400 | + 83,100 |
| | | $EE'$ | + 261,500 | + 92,300 |

## THE HOWE TRUSS

**66. Description.**—The **Howe truss**, shown in Fig. 29, was one of the earliest forms of simple bridge trusses in America. As originally constructed, it had short panels with

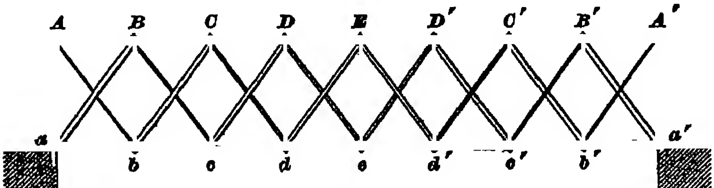



FIG 29

two diagonals in each panel, and vertical end posts; all parts of the truss were constructed of wood, except the intermediate verticals, which were iron rods This type of truss is still used to some extent in localities where timber is

plentiful. At present, however, all the members that receive no stress are omitted, and the lower chord is frequently constructed of steel. The diagonals are designed to resist compression only, therefore, they must slope downwards to the left in the panels in which the shear is positive; and downwards to the right in the panels in which the shear is negative. Two diagonals, one sloping in each direction, are required in each panel in which the sign of the maximum combined shear is opposite to that of the minimum combined shear. In the original trusses of this type, two diagonals were put in each panel, but the extra diagonals or counters in the panels near the ends were unnecessary. The verticals, except the vertical end post, were designed to resist tension only.

The modern forms of the Howe truss are shown in Fig. 30 as a through truss, and in Fig. 31 as a deck truss. The end posts, upper chord, and intermediate diagonals are constructed of wood; the verticals and bottom chord are of steel. The counters are shown in dotted lines, and are put in only where necessary.

**67. Calculation of Stresses.**—The stresses in the members are calculated in exactly the same way as the stresses in the members of the Pratt truss. The maximum stress in the center vertical $Dd$ of the through truss shown in Fig. 30 is equal to the maximum positive shear in panel $cd$

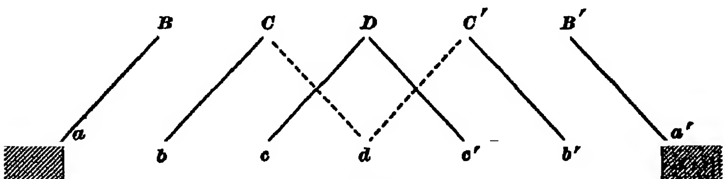



FIG 30

(when there is no dead load at $D$), or to a full dead and live panel load, whichever is the greater; the minimum stress is equal to a dead panel load. The maximum stress in the center vertical $dD$ of the deck truss shown in Fig. 31 is equal to the maximum positive shear in the panel $dc'$; the

minimum stress is equal to zero, and occurs when the main diagonals $Cd$ and $d\,C'$ are in action. (In reality, the stress

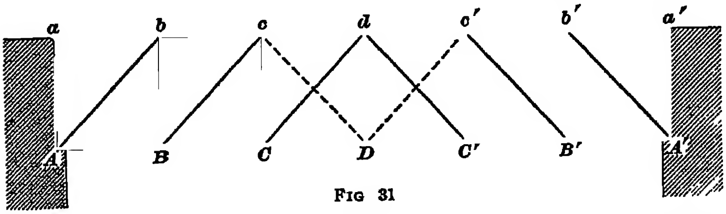



FIG 31

in $dD$, under this condition of loading, is equal to the dead load at $D$; but, if this is all assumed at the joints of the loaded chord, the stress in $dD$ will be zero.)

### EXAMPLE FOR PRACTICE

Let Fig 30 be a six-panel through Howe truss having a span length of 90 feet, consisting of six 15-foot panels, and a height of 12 feet If the dead load is 800 pounds, all applied at the loaded chord, and the live load is 2,000 pounds, per linear foot of bridge, find: (*a*) the maximum and minimum combined stresses in the chord members $BC$ and $bc$, (*b*) the maximum combined stresses in both the main diagonal and the counter in each panel to the left of the center in which a counter is required, (*c*) the maximum and minimum combined stresses in the verticals $Bb$ and $Dd$

| | | Member | Stress, in Pounds Maximum | Stress, in Pounds Minimum |
|---|---|---|---|---|
| Ans | (*a*) | $BC$ | + 65,600 | + 18,800 |
| | | $bc$ | − 105,000 | − 30,000 |
| | (*b*) | $cD$ | + 28,800 | 0 |
| | | $Cd$ | + 7,200 | 0 |
| | (*c*) | $Bb$ | − 52,500 | − 15,000 |
| | | $Dd$ | − 21,000 | − 6,000 |

## THE WHIPPLE TRUSS

**68. Description.**—If the simple type of Pratt truss is used for the longer spans to which the Pratt truss is adapted, it will be impossible to make use of an economical height of truss and inclination of diagonal without using very long panels, thereby making the floor system very heavy. For such spans, it is convenient to use a modified form of the Pratt truss, making use of multiple systems of web or

subdivided panels. The **Whipple truss,** sometimes called the **double-intersection Pratt truss,** is a modified form of the Pratt truss, in which there are two systems of web members, the diagonals running across two panels, as shown in Fig 32 (*a*) The two systems of web members are shown in full and dotted lines There is a large number of these trusses in use at the present time, but they are now

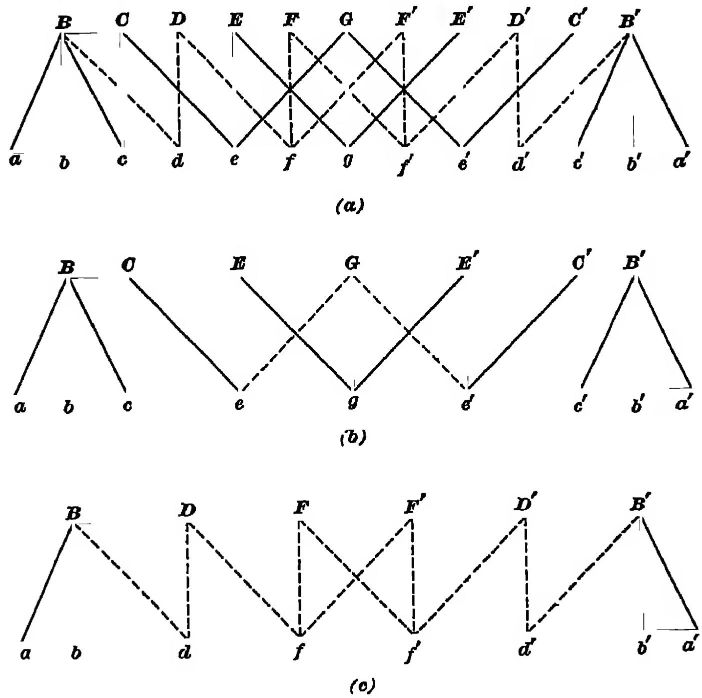



Fig 32

avoided by the best engineers, because the actual stresses in some members cannot be determined accurately.

**69. Method of Calculation.**—Applying the principles of the method of sections to the truss shown in Fig. 32 (*a*), it will be seen that, if the truss is cut by a plane in any panel except the end panel, the plane will cut at least four members in which the stresses are unknown, and it will be

impossible to determine those stresses by the principles of statics, unless some assumption is made regarding the distribution of the stresses among the members It is customary to assume that the truss is composed of two trusses lying in the same plane and having the chords and end posts in common. The stresses in each truss are supposed to be caused by the loads that are directly applied to it. The two systems in this case are shown separated in Fig. 32 (*b*) and (*c*) For convenience of reference, the system shown in full lines in Fig. 32 (*b*) will be called the *primary system*, and the system shown in dotted lines in Fig. 32 (*c*) will be called the *secondary system*.

The joints $B$, $b$, $B'$, and $b'$ are common to both systems, and one-half of each of the loads applied at those points may be assumed to be supported by each system. The stresses may be found in all the members of each system, for the loads that come on that system, in exactly the same way in which the stresses are found in the single-intersection Pratt truss. As the chords, end posts, and hip verticals are common to both systems, the stresses found in those members in each system are partial stresses, or component stresses, and must be added together to get the actual stresses. As each of the web members, except the end posts and hip verticals, occurs in but one system, the stresses found in them are the actual stresses. The assumption that the two web systems act independently is not strictly correct, but the resulting error is probably very small.

EXAMPLE —Let it be required to calculate the maximum stresses in the chord members $CD$, $DE$, $ef$, and $fg$, and in the web members $Bc$, $Ee$, $eG$, $Dd$, and $Df$, in the twelve-panel through Whipple truss shown in Fig. 32 (*a*), having a span length of 216 feet, and a height of 36 feet, when the dead load is 1,000 pounds (all applied at the joints of the loaded chord), and the live load is 1,600 pounds, per linear foot of bridge. The dead and the live panel load are 9,000 and 14,400 pounds, respectively, at the joints $b$ and $b'$, the panel loads for each system are to be taken at one-half these values

SOLUTION.—*Chord Stresses* —The truss may be separated into the primary system shown in Fig 32 (*b*) and the secondary system shown in Fig. 32 (*c*) To find the stresses in $CD$, $DE$, $ef$, and $fg$, it is

necessary to calculate the stresses in $CE$ and $eg$ of the primary system, and in $BD$, $DF$, $df$, and $ff'$ of the secondary system The total dead load on the primary system is $9{,}000 \times 6$, and each reaction is equal to

$$\frac{9{,}000 \times 6}{2} = 27{,}000 \text{ lb.}$$

The dead-load chord stresses are as follows

$$\text{Stress in } CE = \frac{(27{,}000 \times 4 - 4{,}500 \times 3 - 9{,}000 \times 2) \times 18}{36} = +38{,}250 \text{ lb}$$

The stress in $eg$ is equal to the stress in $CE = -38{,}250$ lb.

The total dead load on the secondary system is $9{,}000 \times 5$, and each reaction is equal to

$$\frac{9{,}000 \times 5}{2} = 22{,}500 \text{ lb}$$

The dead-load chord stresses are as follows·

| Member | | Stress, in Pounds |
|---|---|---|
| $BD$ | $\frac{(22{,}500 \times 3 - 4{,}500 \times 2) \times 18}{36}$ | $= +29{,}250$ |
| $DF$ | $\frac{(22{,}500 \times 5 - 4{,}500 \times 4 - 9{,}000 \times 2) \times 18}{36}$ | $= +38{,}250$ |
| $df$ | (stress in $BD$) | $-29{,}250$ |
| $ff'$ | (stress in $DF$) | $-38{,}250$ |

Then, the dead-load chord stresses in this truss are·

| Member | Stress, in Pounds |
|---|---|
| $CD$ | $38{,}250 + 29{,}250 = +67{,}500$ |
| $DE$ | $38{,}250 + 38{,}250 = +76{,}500$ |
| $ef$ | $38{,}250 + 29{,}250 = -67{,}500$ |
| $fg$ | $38{,}250 + 38{,}250 = -76{,}500$ |

As all the dead load is applied at the joints of the loaded chord, the live-load stresses will be equal to the dead-load stresses multiplied by $\frac{1{,}600}{1{,}000}$, and the maximum combined stresses will be equal to the dead-load stresses multiplied by $\frac{2{,}600}{1{,}000}$, or by 2 6

The maximum combined stresses are as follows:

| Member | Maximum Stress, in Pounds |
|---|---|
| $CD$ | $+67{,}500 \times 2\ 6 = +175{,}500$ |
| $DE$ | $+76{,}500 \times 2\ 6 = +198{,}900$ |
| $ef$ | $-67{,}500 \times 2\ 6 = -175{,}500$ |
| $fg$ | $-76{,}500 \times 2.6 = -198{,}900$ |

*Web Stresses*—The web members $Bc$, $Ee$, and $eG$ occur in the primary system The dead-load shears are as follows

shear in panel $bc = 27{,}000 - 4{,}500 = 22{,}500$ lb

shear in panel $eg = 27{,}000 - 4{,}500 - 9{,}000 \times 2 = 4{,}500$ lb

For the member $B\,c$,

$$\csc H = \frac{\sqrt{18^2 + 36^2}}{36} = 1\ 118;$$

for the member $e\,G$,

$$\csc H = \frac{\sqrt{36^2 + 36^2}}{36} = 1\ 414$$

The dead-load stress in $B\,c$ is $22{,}500 \times 1\ 118 = -25{,}200$ lb.

The dead-load stress in $E\,e$ is $+4{,}500$ lb

The dead-load stress in $e\,G$ (counter, assuming that the main diagonal $E\,g$ is left out) is $4{,}500 \times 1\ 414 = +6{,}400$ lb

The members $D\,d$ and $D\,f$ occur in the secondary system. The shear in the panel $d\,f$ is $22{,}500 - 4{,}500 - 9{,}000 = 9{,}000$ lb

The dead-load stress in $D\,d$ is $+9{,}000$ lb

The dead-load stress in $D\,f$ is $9{,}000 \times 1\ 414 = -12{,}700$ lb

The maximum live-load stresses for the primary system are as follows, the load at $b'$ being a half-panel load For the member $B\,c$, with the truss loaded to $c$ from the right end, the shear in panel $b\,c$ is

$$\frac{14{,}400\,(\frac{1}{2} + 2 + 4 + 6 + 8 + 10)}{12} = 36{,}600 \text{ lb}$$

The live-load stress in $B\,c$ is $36{,}600 \times 1\ 118 = -40{,}900$ lb.

For the member $E\,e$, with the truss loaded to $g$ from the right end, the shear in panel $e\,g$ is

$$\frac{14{,}400\,(\frac{1}{2} + 2 + 4 + 6)}{12} = 15{,}000 \text{ lb}$$

The live-load stress in $E\,e$ is $+15{,}000$ lb

For the counter $e\,G$, the member $e'\,G$ may be considered instead With the truss loaded to $e'$ from the right end, the shear in panel $g\,e'$ is

$$\frac{14{,}400\,(\frac{1}{2} + 2 + 4)}{12} = 7{,}800 \text{ lb.}$$

The live-load stress in $e'\,G$ is $7{,}800 \times 1\ 414 = -11{,}000$ lb

The maximum live-load stresses for the secondary system are as follows, the load at $b'$ being a half-panel load· For the members $D\,d$ and $D\,f$, with the truss loaded to $f$, from the right end, the shear in panel $d\,f$ is

$$\frac{14{,}400\,(\frac{1}{2} + 3 + 5 + 7)}{12} = 18{,}600 \text{ lb}$$

The live-load stress in $D\,d$ is $+18{,}600$ lb

The live-load stress in $D\,f$ is $18{,}600 \times 1\ 414 = -26{,}300$ lb.

The final maximum combined stresses are as follows

| Member | Stress, in Pounds |
|---|---|
| $B\,c$ | $-25{,}200 - 40{,}900 = -66{,}100$ |
| $E\,e$ | $4{,}500 + 15{,}000 = +19{,}500$ |
| $e\,G$ | $-11{,}000 + 6{,}400 = -4{,}600$ |
| $D\,d$ | $9{,}000 + 18{,}600 = +27{,}600$ |
| $D\,f$ | $-12{,}700 - 26{,}300 = -39{,}000$ |

### EXAMPLE FOR PRACTICE

Let Fig 32 (*a*) be a twelve-panel through Whipple truss having a span length of 180 feet and a height of 30 feet If the dead load is 1,200 pounds, all applied at the joints of the loaded chord, and the live load is 2,200 pounds, per linear foot of bridge, find. (*a*) the maximum and minimum combined stresses in the chord members $cd$, $de$, $EF$, and $FG$, (*b*) the maximum and minimum combined stresses in the web members $Bd$, $Eg$, $Ff'$, $Cc$, and $Ff$

| | | Member | Stress, in Pounds Maximum | Stress, in Pounds Minimum |
|---|---|---|---|---|
| Ans | (*a*) | $cd$ | − 102,000 | − 36,000 |
| | | $de$ | − 153,000 | − 54,000 |
| | | $EF$ | + 229,500 | + 81,000 |
| | | $FG$ | + 229,500 | + 81,000 |
| | (*b*) | $Bd$ | − 73,100 | − 24,500 |
| | | $Eg$ | − 30,700 | 0 |
| | | $Ff'$ | − 16,500 | 0 |
| | | $Cc$ | + 41,700 | + 10,100 |
| | | $Ff$ | + 11,700 | 0 |

## THE POST TRUSS

**70. Description.**—The **Post truss**, Fig. 33 (*a*), may be looked on as a modified Whipple truss with an odd number of panels in the bottom chord. This truss is seldom built at present on account of the uncertainty in the stresses. The compression web members are inclined so that their upper joints are one-half a panel nearer the center of the truss than their lower joints. The two center members, one on each side of the center panel of the lower chord, meet at the center of the upper chord. The main diagonals are tension members and slope across one and one-half panels. The diagonals shown in dotted lines are counters, and were formerly inserted in each panel, although those in the panels near the ends were not needed. At present, counters are used only where they are actually needed. There are in reality two systems of web members, but as they are connected at the center joint of the upper chord, and as the counters connect the compression web members of the two systems, it is impossible to calculate the stresses from the equations of equilibrium without making some

assumption as to their distribution. The common assumptions for this truss give only roughly approximate results. It is customary to divide the truss into two systems, as shown in full lines in Fig. 33 (*b*) and (*c*), and treat each system as an independent truss in much the same way as in the Whipple truss. It will be seen that each system is unsymmetrical, but that the two are alike end for end.

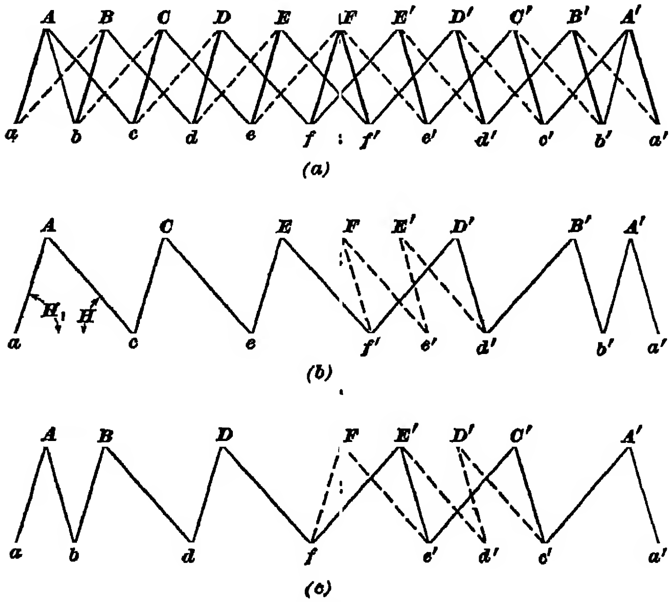



FIG 33

**71. Chord Stresses.**— It will be necessary to calculate the stresses in all the chord members of only one system. For example, the maximum stress in $AB$, Fig. 33 (*a*), is equal to the sum of the stresses in $AC$, Fig. 33 (*b*), and $AB$, Fig. 33 (*c*). But, since, when the whole truss is fully loaded, the stress in $A'B'$ is equal to that in $AB$, Fig. 33 (*c*), it is not necessary to calculate the latter stress separately. The stress in $AC$ is equal to the moment at $c$ divided by the height; the stress in $A'B'$ is equal to the moment at $b'$ divided by the height. In like manner, other chord stresses may be found.

**72. Stresses in Main Web Members.**—As the systems are alike end for end, it is only necessary to analyze each system for loads on the right of the center. The member $A\,a$, Fig. 33 (*a*), occurs in both systems; the stress in it is equal to the sum of the reactions at $a$ in the two systems, Figs 33 (*b*) and (*c*), multiplied by csc $H_1$; or, since the reaction at $a$ in (*c*) is equal to the reaction at $a'$ in (*b*), the stress in $a\,A$ is equal to the sum of the two reactions of either system multiplied by csc $H_1$. The same is true of the stress in $A'\,a'$. The maximum stresses in the other web members may be found from the maximum shears in the respective panels. For example, the vertical component of the maximum stresses in $A\,c$ and in $A'\,c'$ is equal to the maximum positive shear in panel $a\,c$, Fig. 33 (*b*); in $C\,c$ and $C\,e$, $C'\,c'$ and $C'\,e'$, to the maximum positive shear in panel $c\,e$, Fig. 33 (*b*); in $E\,e$ and $E\,f'$, $E'\,e'$ and $E'\,f$, to the maximum positive shear in panel $e\,f'$, Fig. 33 (*b*).

Also, the vertical component of the maximum stresses in $A\,b$ and $A'\,b'$ is equal to the maximum positive shear in panel $a\,b$, Fig. 33 (*c*); in $B\,b$ and $B\,d$, $B'\,b'$ and $B'\,d'$, to the maximum positive shear in panel $b\,d$, Fig. 33 (*c*); in $D\,d$ and $D\,f$, $D'\,d'$ and $D'\,f'$, to the maximum positive shear in panel $d\,f$, Fig 33 (*c*). The maximum positive shear in any panel obtains when all joints to the right are loaded with a live load.

**73. Stresses in Counters.**—The compression members $F\,f$ and $F\,f'$ may be considered as counters, as they are not in action for full load. In Fig 33 (*b*), when the combined shear in panel $f'\,d'$ is positive, the members $D'\,d'$ and $D'\,f'$ may be considered out of action, and the members $d'\,E'$, $E'\,e'$, $e'\,F$, and $F\,f'$ in action. In Fig. 33 (*c*), when the combined shear in panel $f\,e'$ is positive, the members $e'\,E'$ and $E'\,f$ may be considered out of action, and the members $e'\,F$ and $F\,f$ in action, when the combined shear in panel $e'\,c'$ is positive, $c'\,C'$ and $C'\,e'$ may be considered out of action, and $c'\,D'$, $D'\,d'$, $d'\,E'$, and $E'\,e'$ in action. When the live load extends to $e'$ from the right, the joints $d'$ and $b'$,

Fig. 33 ($b$), and $e'$ and $c'$, Fig. 33 ($c$), will be loaded. If the combined shear in panel $f e'$ is positive, the vertical component of the stresses in $F f$ and $F e'$ is equal to the shear in the panel $f e'$; if the combined shear in the panel $f' d'$ is also positive, the vertical component of the stress in $F f'$ and $E' d'$ is equal to the shear in panel $f' d'$, and the vertical component of the stress in $F e'$ is equal to the sum of the shears in panels $f' d'$ and $f e'$ The stresses in the other counters may be found in like manner.

It must be understood that for this truss the foregoing method of calculation is *roughly approximate* and is given here simply to afford a means of finding stresses in trusses of this nature already built rather than as a guide in designing. The Whipple truss answers the same purpose as the Post truss, and is preferable because the approximate method of calculation gives closer results for the former than for the latter.

### THE BALTIMORE TRUSS

**74. Description.**—The **Baltimore truss**, shown as a through bridge in Fig. 34, is in reality a Pratt truss with long panels, which are subdivided by the addition of short verticals and diagonals, such as $D d$ and $D c$, intersecting the main diagonals half way between the top and bottom chords This form of truss allows the use of an economical height and inclination of diagonal, without excessively long panels, and is used to a great extent at the present time for railroad and highway bridges. The short verticals ($D d$, $F f$, etc.), called the **subverticals**, are tension members; the short diagonals ($c D$, $e F$, etc.), called the **substruts**, are compression members; $e F$ also acts as a tension member as the lower half of the counter $e F G$ when that counter is in action. All the other members are similar to the corresponding members of the simple Pratt truss.

**75. Chord Stresses.**—The stress in any top-chord member may be found by cutting the truss by a plane that intersects the member whose stress is desired and the lower half of a main diagonal. For example, for the stress in $C E$,

Fig. 34 ($a$), the truss may be considered as cut by a plane at the section $q$; then the stress in $CE$ is equal to the moment of all the forces to the left of section $q$ about the point $e$

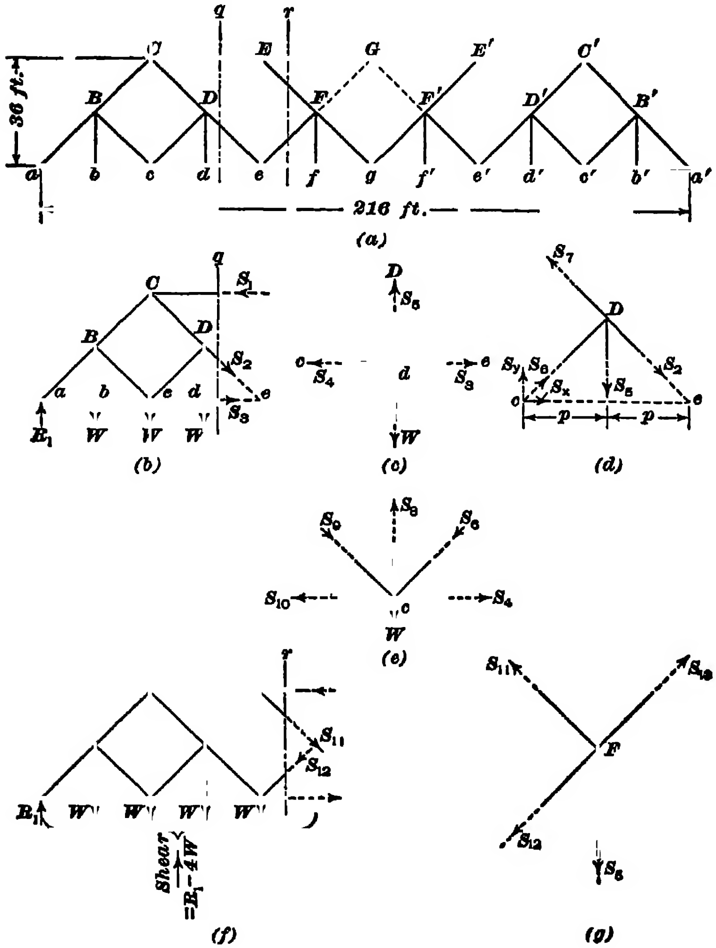



FIG. 34

divided by the height of the truss. In the present case, Fig. 34 ($b$),

$$S_1 = \frac{R_1 \times 4p - W(1 + 2 + 3)p}{h},$$

denoting the panel length by $p$, and the height of the truss by $h$.

The stresses in the bottom-chord members $ab$, $de$, $fg$, etc. may be found by cutting the truss by a plane that intersects the member whose stress is desired and the lower half of a main diagonal. For example, for the stress in $de$, Fig. 34 ($a$), the truss may be considered as cut by a plane at the section $q$; then the stress in $de$ is equal to the moment of all the forces to the left of section $q$ about the point $C$, divided by the height of the truss. In this case, loads at $d$ and $D$, if any, have positive or right-handed moments about the point $C$, similar to the moment of $R_1$, and this must be remembered in writing the equation of moments. *The moment of the forces on the left of section q about the point C is not the bending moment on the truss at the point C.* In the present case, Fig. 34 ($b$),

$$S_3 = \frac{R_1 \times 2p - W \times p + W \times p}{h}$$

The stresses in the bottom-chord members $bc$, $cd$, $ef$, etc. may be most easily found by considering the forces acting at the intermediate panel points. For example, for the stress in $cd$, Fig. 34 ($a$), the joint $d$ may be considered a free body, as shown in Fig. 34 ($c$). The only horizontal forces are $S_4$ and $S_3$; they must be equal and opposite; in other words, the stress in $cd$ is equal to the stress in $de$ In like manner, it may be shown that the stress in $bc$ is equal to the stress in $ab$, that in $ef$ equal to that in $fg$, etc.

The maximum chord stresses obtain when there is a full live load; the minimum stresses, when there is no live load.

**76. Stresses in the Subverticals.**—The stress in any subvertical may be found by considering the forces acting at an intermediate panel point. For example, for the stress in $Dd$, the joint $d$, Fig. 34 ($c$), may be considered. The only vertical forces are the stress in $Dd$ and the panel load $W$; they must be equal and opposite. Therefore, *the stress in any subvertical is tension: the maximum stress is equal to the sum*

*of a dead and a live panel load; the minimum stress is equal to a dead panel load.*

**77. Stresses in the Short Diagonals or Substruts.** The stresses in the short diagonals $Bc, Dc, Fe$, etc. may be found by considering the forces acting at the intermediate joints $B, D, F$, etc. For example, for the stress in $cD$, Fig. 34 ($a$), the joint $D$ may be treated as a free body, as shown in Fig. 34 ($d$). There are four forces ($S_3, S_8, S_6$, and $S_7$) acting on joint $D$, $S_6$ is required; $S_8$ is known; $S_3$ and $S_7$ are unknown, and are not required at the present time. $S_6$ may be resolved at the point $c$ in its line of action into $S_y$ and $S_x$, its vertical and horizontal components, respectively. If the point $e$ is taken as the center of moments, the moments of $S_3, S_7$, and $S_x$ will all be zero. Then,

$$\Sigma M = S_y \times 2p - S_8 \times p = 0;$$

whence

$$S_y = \frac{S_8}{2} = \frac{W}{2}$$

and

$$S_6 = S_y \times \csc H = \frac{W}{2} \csc H, \text{ compression}$$

In other words, the vertical component of the stress in a short diagonal is equal to one-half the panel load at the top and at the foot of the short vertical. Then, *the maximum compression in a short diagonal is equal to one-half the sum of a dead and a live panel load multiplied by csc H; the minimum compression (except in eF) is equal to one-half of a dead panel load multiplied by csc H.* The minimum stress in $eF$ will be discussed later in connection with counter $eFG$. In case part of the dead load is applied at the point $D$, this must be added to the panel load at $d$ in getting the stress in $cD$.

**78. Stresses in the Hip Verticals.**—The stress in the hip vertical $Cc$ may be found by considering the joint $c$ as a free body [Fig. 34 ($e$)]. The forces acting at $c$ are $W, S_4, S_6, S_5, S_9$, and $S_{10}$. The equation $\Sigma Y = \Sigma S \sin H = 0$ gives

$$\Sigma Y = S_5 - S_6 \sin H - S_9 \sin H - W = 0;$$

whence

$$S_5 = S_6 \sin H + S_9 \sin H + W, \text{ tension}$$

$S_9$ and $S_6$ are the stresses in the short diagonals $Bc$ and $cD$; therefore, $S_9 \sin H$ and $S_6 \sin H$ are each equal to $\frac{W}{2}$, and

$$S_8 = \frac{W}{2} + \frac{W}{2} + W = 2\,W$$

It is thus seen that *the maximum tension in the hip vertical is equal to twice the sum of a dead and a live panel load; and that the minimum tension is equal to twice a dead panel load.*

**79. Stresses in the Main Diagonals.**—The stress in the lower half of a main diagonal may be found by considering the portion of the structure to the left of a plane cutting that member and two chord members. For example, for the stress in $De$, Fig. 34 ($a$), the truss may be cut by a plane at the section $q$. The vertical component of the stress in the diagonal is equal to the shear on the section. The stress in the end post is compression; in the other diagonals, it is tension. The maximum stress in the lower half of a main diagonal to the left of the center is equal to the maximum positive shear multiplied by csc $H$, the minimum stress is equal to the minimum shear multiplied by csc $H$, when the minimum shear is positive. When the minimum shear is negative, a counter is required, and the minimum stress in the main diagonal is zero.

The stress in the upper half of a main diagonal may be found by considering the forces acting on the portion of the structure to the left of a plane of section that cuts the upper half of that diagonal, a short diagonal, and two chord members. For example, for the stress in $EF$, Fig. 34 ($a$), the portion of the truss to the left of section $r$ may be considered [see Fig. 34 ($f$)]. The equation $\Sigma Y = \Sigma S \sin H = 0$ gives

$$\Sigma Y = R_1 - 4\,W - S_{12} \sin H - S_{11} \sin H = 0;$$

whence

$$S_{11} \sin H = \text{shear on section } r - S_{12} \sin H$$

$S_{12}$ is the stress in the short diagonal $eF$, and is equal to one-half the panel load at $f$ multiplied by csc $H$. Then,

$$S_{12} \sin H = \frac{W}{2}$$

and

$$S_{11} = \left(\text{shear on section } r - \frac{W}{2}\right) \times \csc H$$

In other words, *the stress in the upper half of a main diagonal is equal to the algebraic sum of the shear in the panel in which the member is located and the vertical component of the stress in the short diagonal, multiplied by csc $H$.*

The maximum stress in $BC$ obtains when the truss is fully loaded, and is compression; the vertical component of the stress is equal to the shear in the panel $bc$ *plus* the vertical component of the stress in $Bc$. (The load at $b$ increases the vertical component of the stress in $BC$ by $\frac{11}{12}W - W + \frac{6}{12}W = \frac{5}{12}W$). The minimum stress obtains when there is no live load on the truss.

The maximum stress in $CD$ is tension, and obtains when all the joints from $d$ to $b'$ are loaded with a live load. (The load at $d$ increases the vertical component of the stress in $CD$ by $\frac{9}{12}W - \frac{6}{12}W = \frac{3}{12}W$.) The minimum stress obtains when joints $b$ and $c$ are loaded with live load.

The maximum stress in $EF$ is tension, and obtains when all the joints from $f$ to $b'$ are loaded with live load. (The load at $f$ increases the vertical component of the stress in $EF$ by $\frac{7}{12}W - \frac{6}{12}W = \frac{1}{12}W$.) The minimum stress obtains when all the joints from $b$ to $e$ are loaded with live load.

If the minimum combined shear in panel $ef$ is positive and greater than a dead panel load, the shear in panel $fg$ will also be positive, and the main diagonal $gFE$ will be in action. The stress in $EF$ is then equal to the shear in panel $ef$ minus one-half of the dead load at $f$, multiplied by csc $H$. If the minimum shear in the panel $ef$ is negative, or positive and less than the dead load at $f$, the shear in the panel $fg$ for the same loading will be negative, and the member $FG$ will be in action as a counter. Under this condition, the stress in the lower half of the main diagonal $gF$ will be zero, and the members that are in action at joint $F$ will be those shown in Fig. 34 ($g$). It will be seen that the stress in $EF$ is thus equal to one-half the dead load at $F$ multiplied by csc $H$; this is the minimum stress in $EF$, and is tension.

**80. Stresses in the Counters.**—The stress in $eF$ is a minimum (maximum tension) when the live load extends

from $b$ to $e$. If the combined shear in the panel $ef$ is then negative, the stress in $eF$ is tension and equal to the shear plus one-half the dead load at $f$, multiplied by csc $H$. If the combined shear in panel $ef$ is then positive and less than one-half the dead load at $f$, the stress in $eF$ is tension and equal to the difference between one-half the dead load at $f$ and the shear in the panel $ef$, multiplied by csc $H$. If the shear in panel $ef$ is positive and greater than one-half, but less than a full, panel dead load at $f$, the stress in $eF$ is compression and equal to the shear in the panel $ef$ minus one-half the dead load at $f$, multiplied by csc $H$.

The maximum tension in $FG$ is equal to the maximum negative shear in the panel $fg$ multiplied by csc $H$, and obtains when the live load extends from $b$ to $f$. The minimum stress is zero.

**81. Stresses in the Verticals.**—The maximum stress in $Gg$ is compression, and is equal to the vertical component of the maximum stress in $FG$ plus the dead load at $G$, if any; the minimum stress is zero, and obtains when the counters $FG$ and $F'G$ are out of action. The maximum stress in $Ee$ is compression, and is equal to the vertical component of the maximum stress in $EF$; the minimum stress is equal to the vertical component of the minimum stress in $EF$

**82. Modified Baltimore Truss.**—Fig. 35 shows a modified form of the Baltimore truss, in which the short

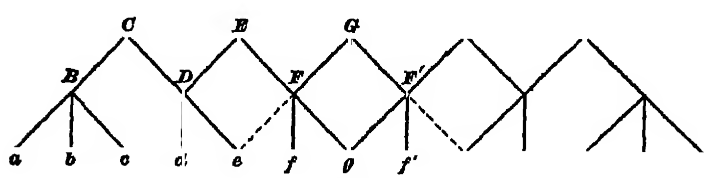



FIG 35

diagonals (except $Bc$) are attached to the upper-chord joints, and are tension members. The general method of analysis is the same as for the truss already treated.

**83. Deck Truss.**—When used in a deck bridge, the Baltimore truss may be supported as shown in Fig. 36.

In this form, the subverticals $bB, dD$, etc. are compression members, and the short diagonals $Bc, De$, etc. are tension members. The general method of analysis is the same as for the through truss.

**84. Distribution of Dead Load.** If the dead load is assumed to be divided between the loaded and unloaded chords, two-thirds of a dead panel load may be taken at the joints of the loaded chord ($b, c, d$, etc.), one-third at the joints of the unloaded chord ($C, E$, etc ), and one-third at the intermediate joints ($B, D$, etc.), although the latter are not on the unloaded chord. In the preceding discussion, it was assumed for convenience in explanation that all the dead load was applied at the joints of the loaded chord. The actual stresses in the verticals will be slightly different if the dead load is distributed among the several joints; the stresses in the other members will remain the same.

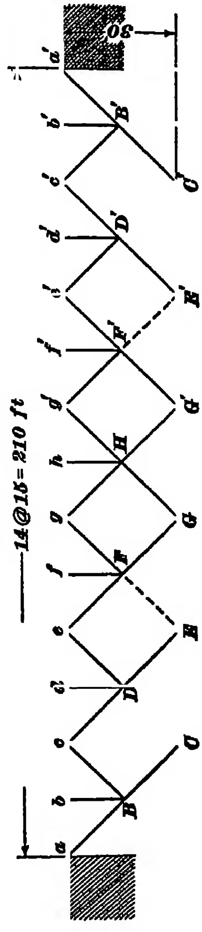



FIG. 36

EXAMPLE —Let Fig 36 be a fourteen-panel deck Baltimore truss having a span length of 210 feet and a height of 30 feet If the dead load is 1,200 pounds, two-thirds of which is applied at the loaded chord, and the live load is 2,400 pounds per linear foot, what are the maximum and minimum stresses in all the members?

SOLUTION —*Panel Loads and Reactions.* The dead panel load is

$$\frac{1,200}{2} \times 15 = 9,000 \text{ lb.}$$

of which 6,000 lb. is applied at each of the joints $b, c, d$, etc ; 3,000 lb. at $B, D, F$, etc , and 3,000 lb. at $C, E, G$, etc. The dead-load reaction is equal to

$$\frac{9,000 \times 13}{2} = 58,500 \text{ lb.}$$

The live panel load is equal to

$$\frac{2{,}400}{2} \times 15 = 18{,}000 \text{ lb.}$$

and the reactions for a fully loaded truss are each

$$\frac{18{,}000 \times 13}{2} = 117{,}000 \text{ lb.}$$

*Chord Stresses.*—As the loads of 6,000 lb. at $b$, $c$, $d$, etc. are vertically over the loads of 3,000 lb. at $B$, $C$, $D$, etc., the moment of 6,000 lb. at an upper joint, plus the moment of 3,000 lb. at a lower joint, about any point will be the same as the moment of 9,000 lb. about the same point. The minimum stresses in the chords are the dead-load stresses, they are as follows:

| Member | Dead-Load Stress, in Pounds |
|---|---|
| $ab$, $bc$ | $\frac{58{,}500 \times 15}{15} = +\ 58{,}500$ |
| $cd$, $de$ | $\frac{[58{,}500 \times 4 - 9{,}000\,(3+2)] \times 15}{30} = +\ 94{,}500$ |
| $ef$, $fg$ | $\frac{[58{,}500 \times 6 - 9{,}000\,(5+4+3+2)] \times 15}{30} = +112{,}500$ |
| $gh$ | $\frac{[58{,}500 \times 8 - 9{,}000\,(7+6+5+4+3+2)] \times 15}{30} = +112{,}500$ |
| $CE$ | $\frac{(58{,}500 \times 2 - 9{,}000 \times 1) \times 15}{30} = -\ 54{,}000$ |
| $EG$ | $\frac{[58{,}500 \times 4 - 9{,}000\,(3+2+1)] \times 15}{30} = -\ 90{,}000$ |
| $GG'$ | $\frac{[58{,}500 \times 6 - 9{,}000\,(5+4+3+2+1)] \times 15}{30} = -108{,}000$ |

As the total panel load is equal to 18,000 lb. live load plus 9,000 lb. dead load, or 27,000 lb., the maximum chord stresses are equal to the dead-load stresses multiplied by $\frac{27{,}000}{9{,}000}$, or 3. They are as follows:

| Member | Maximum Stress, in Pounds |
|---|---|
| $ab$, $bc$ | $+\ 58{,}500 \times 3 = +175{,}500$ |
| $cd$, $de$ | $+\ 94{,}500 \times 3 = +283{,}500$ |
| $ef$, $fg$ | $+112{,}500 \times 3 = +337{,}500$ |
| $gh$ | $+112{,}500 \times 3 = +337{,}500$ |
| $CE$ | $-\ 54{,}000 \times 3 = -162{,}000$ |
| $EG$ | $-\ 90{,}000 \times 3 = -270{,}000$ |
| $GG'$ | $-108{,}000 \times 3 = -324{,}000$ |

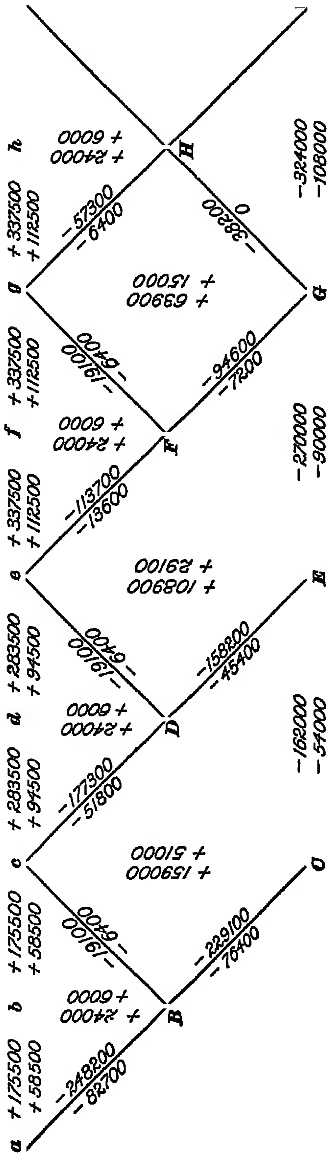


FIG 37

*Web Stresses*.—The dead-load shears in the various panels are as given in the following table

| Panel | Dead-Load Shear Pounds | Panel | Dead-Load Shear Pounds |
|---|---|---|---|
| *a b* | + 58,500 | *e f* | + 22,500 |
| *b c* | + 49,500 | *f g* | + 13,500 |
| *c d* | + 40,500 | *g h* | + 4,500 |
| *d e* | + 31,500 | | |

The maximum positive and negative live-load shears are as follows

| Panel | Positive Live-Load Shear Pounds | Negative Live-Load Shear Pounds |
|---|---|---|
| *a b* | + 117,000 | |
| *b c* | + 100,300 | − 1,300 |
| *c d* | + 84,900 | − 3,900 |
| *d e* | + 70,700 | − 7,700 |
| *e f* | + 57,900 | − 12,900 |
| *f g* | + 46,300 | − 19,300 |
| *g h* | + 36,000 | − 27,000 |

The maximum and minimum combined shears are as follows:

| Panel | Combined Shear Maximum | Combined Shear Minimum |
|---|---|---|
| *a b* | + 175,500 | + 58,500 |
| *b c* | + 149,800 | + 48,200 |
| *c d* | + 125,400 | + 36,600 |
| *d e* | + 102,200 | + 23,800 |
| *e f* | + 80,400 | + 9,600 |
| *f g* | + 59,800 | − 5,800 |
| *g h* | + 40,500 | − 22,500 |

From the figure, $\csc H = \frac{\sqrt{30^2 + 30^2}}{30} = 1.414$ The maximum stress in $bB$, $dD$, etc is 18,000 + 6,000 = 24,000 lb, compression The minimum stress in $bB$, $dD$, etc is 6,000 lb, compression. The maximum stress in $Bc$, $De$, and $Fg$ is

$$\left(\frac{18,000 + 6,000 + 3,000}{2}\right) \times 1.414 = 19,100 \text{ lb., tension}$$

The minimum stress in $Bc$, $De$, and $Fg$ is

$$\left(\frac{6,000 + 3,000}{2}\right) \times 1.414 = 6,400 \text{ lb, tension}$$

The maximum and minimum stresses (tension) in $aB$, $cD$, $eF$, and $gH$ are as folows:

| Member | Stress, in Pounds: Maximum | Minimum |
|---|---|---|
| $aB$ | 175,500 × 1.414 = 248,200 | 58,500 × 1.414 = 82,700 |
| $cD$ | 125,400 × 1.414 = 177,300 | 36,600 × 1.414 = 51,800 |
| $eF$ | 80,400 × 1.414 = 113,700 | 9,600 × 1.414 = 13,600 |
| $gH$ | 40,500 × 1.414 = 57,300 | 4,500 × 1.414 = 6,400 |

The maximum and minimum stresses (tension) in $BC$, $DE$, $FG$, and $GH$ are as follows:

| Member | Maximum Stress, in Pounds |
|---|---|
| $BC$ | (175,500 − 27,000 + 13,500) × 1.414 = 229,100 |
| $DE$ | (125,400 − 27,000 + 13,500) × 1.414 = 158,200 |
| $FG$ | (80,400 − 27,000 + 13,500) × 1.414 = 94,600 |
| $GH$ | (22,500 + 4,500) × 1.414 = 38,200 |

| Member | Minimum Stress, in Pounds |
|---|---|
| $BC$ | (58,500 − 9,000 + 4,500) × 1.414 = 76,400 |
| $DE$ | (36,600 − 9,000 + 4,500) × 1.414 = 45,400 |
| $FG$ | (9,600 − 9,000 + 4,500) × 1.414 = 7,200 |
| $GH$ | 0 |

The maximum and minimum stresses (compression) in $Cc$, $Ee$, and $Gg$ are as follows:

| Member | Stress, in Pounds: Maximum | Minimum |
|---|---|---|
| $Cc$ | 162,000 − 3,000 = 159,000 | 54,000 − 3,000 = 51,000 |
| $Ee$ | 111,900 − 3,000 = 108,900 | 32,100 − 3,000 = 29,100 |
| $Gg$ | 66,900 − 3,000 = 63,900 | 4,500 + 6,000 + 4,500 = 15,000 |

The maximum and minimum stresses are given in Fig. 37, the former above and the latter below the lines.

## EXAMPLE FOR PRACTICE

Let Fig. 34 (*a*) be a twelve-panel through Baltimore truss having a span length of 216 feet and a height of 36 feet. If the dead load is 1,200 pounds, of which two-thirds is applied at the loaded chord, and

the live load is 2,200 pounds, per linear foot of bridge, find the maximum and minimum combined stresses (*a*) in *C E*, *c d*, and *f g*; (*b*) in *D d* and *B c*; (*c*) in *B C*, *C D*, and *E F*, (*d*) in *e F*, *E e*, and *G g*

| | | Panel | Stress, in Pounds | |
|---|---|---|---|---|
| | | | Maximum | Minimum |
| Ans. | (*a*) | *C E* | + 244,800 | + 86,400 |
| | | *c d* | − 168,300 | − 59,400 |
| | | *f g* | − 260,100 | − 91,800 |
| | (*b*) | *D d* | − 27,000 | − 7,200 |
| | | *B c* | + 21,600 | + 7,600 |
| | (*c*) | *B C* | + 216,300 | + 76,400 |
| | | *C D* | − 136,800 | − 38,800 |
| | | *E F* | − 66,600 | − 7,600 |
| | (*d*) | *e F* | + 21,600 | − 8,100 |
| | | *E e* | + 50,800 | + 9,000 |
| | | *G g* | + 22,900 | + 3,600 |

## THE FINK AND THE BOLLMAN TRUSS

**85.** Fig. 38 shows the **Fink truss**, which has been used to some extent for bridge purposes in the past, and is at present employed in a modified form for roof trusses The analysis of stresses in it should present no difficulty; the method of joints is best adapted to this case. The stress in each short vertical is evidently equal to the panel load at its upper joint. The vertical component of the stress in a short

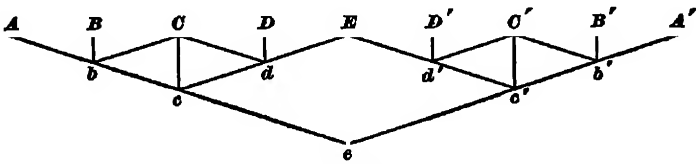
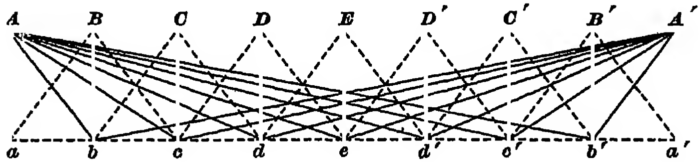


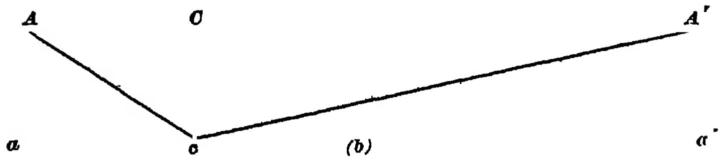

Fig 38

diagonal, such as *C d*, is equal to one-half the sum of the loads at the joints *D* and *d*. The stress in the vertical *C c* is equal to the sum of the vertical components of the stresses in *b C* and *C d*, and the load at *C*, etc. The maximum chord stresses obtain when there is a full load; under this condition the horizontal components in *b C*, *d E*, and *d′ C′* are equal, respectively, to those in *C d*, *E d′*, and *C′ b′*, and the compression in the top chord is constant from *A* to *A′* and equal to the horizontal component of the stress in *A b*.

**86.** The **Bollman truss,** shown in Fig. 39 (*a*), has been used to some extent for bridge purposes in the past, but is now practically obsolete The load that came on the lower joint of a vertical was carried directly to the ends of the top chord by the two diagonals, shown in full lines, that meet at the bottom of the vertical. The bottom chord and




(*a*)




(*b*)

Fig 39

the dotted diagonals are superfluous members, and were put in to stiffen the truss As shown in Fig 39 (*b*), the vertical components of the stresses in the main diagonals are equal, respectively, to the reactions due to the load that comes to the intersection of any pair. The stress in the top chord is compression, and is equal to the sum of the horizontal components of the stresses in all the main diagonals at one end of the truss.

# STRESSES IN BRIDGE TRUSSES

## (PART 3)

## CURVED- AND INCLINED-CHORD TRUSSES

### INTRODUCTORY AND GENERAL CONSIDERATIONS

**1. Description.**—All the trusses analyzed in *Stresses in Bridge Trusses*, Part 2, have parallel chords. As explained in Part 1, parallel-chord trusses are used for the shorter spans to which trusses are adapted, for the longer spans, curved- or inclined-chord trusses are more economical In a curved-chord truss, the inclination of the curved chord changes at every panel point; in an inclined-chord truss, the inclination is constant from the center toward the end In through bridges, the upper chord is curved or inclined, while in deck bridges, the lower, and in some bridges, both chords, are curved or inclined A length of one or two panels of the curved chord near the center of the truss is sometimes horizontal

**2. Stresses in Inclined Members.**—In calculating the stress in an inclined member, it is convenient to find first the vertical or horizontal component, and calculate the stress from it. The problem frequently arises, therefore, of calculating a stress from one of its components, or one component from the other, for this reason, the relations between a stress and its components, and between the components are restated. They are:

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$S = X \sec H$; $S = Y \csc H$; $X = Y \cot H$; $Y = X \tan H$

in which $S$ = stress;

$H$ = angle that line of action of stress makes with horizontal,

$X$ = horizontal component of stress,

$Y$ = vertical component of stress.

The method of sections and the method by the stress diagram are most useful in finding the stresses in inclined-chord trusses, and they will be used in what follows. The method of joints usually requires more work than either of the other two methods, and will not be used except in special cases.

**3. Illustrative Example.**—To illustrate the general method of calculating stresses, it will be well to consider a special case, such as the single-system curved-chord truss represented in Fig 1 (*a*). If it is desired to find the stress

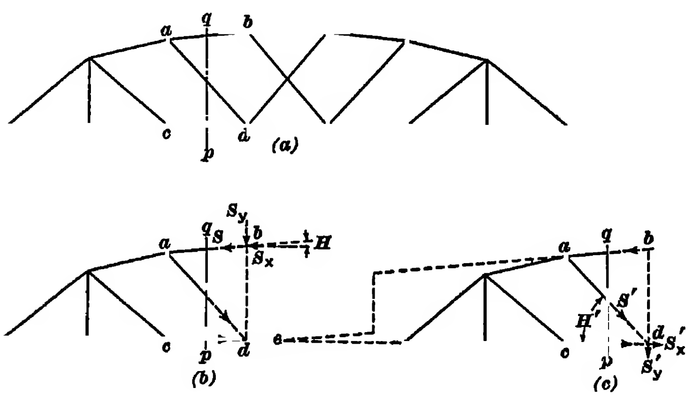



Fig 1

in the members $ab$, $ad$, and $cd$, the truss may be considered cut by the plane of section $pq$, and the part to the left of this section treated as a free body. Then the stress in the lower-chord member $cd$ is equal to the moment about $a$ of all the other forces acting on the part of the truss to the left of $pq$ divided by the height of the truss at the point $a$. The stress in the upper-chord member $ab$ is equal to the

bending moment at $d$ divided by the perpendicular distance from $d$ to $a\,b$, in this case, it is more convenient to consider the stress in $a\,b$ resolved into its vertical and horizontal components $S_y$ and $S_x$ at the point $b$, as represented in Fig 1 ($b$). The moment of $S_y$ about $d$ is equal to zero; then, $S_x$ is equal to the bending moment at the point $d$ divided by the height of the truss at the point $d$, and $S$ is equal to $S_x$ sec $H$.

The stress in the web member $a\,d$ may be found by the method of shears or by the method of moments. As the chord member $a\,b$ is inclined, the vertical component of the stress in $a\,d$ is not equal to the shear on section $p\,q$, but is equal to the algebraic sum of the shear and the vertical component of the stress in $a\,b$ due to the same loading. To find the stress in $a\,d$ by the method of moments, it is convenient to consider it resolved at the point $d$ into its vertical and horizontal components $S_y'$ and $S_x'$, respectively, as represented in Fig 1 ($c$). Then, if moments are taken about the point $e$, the intersection of $a\,b$ and $c\,d$, the moment of $S_x'$ about $e$ is equal to zero; $S_y'$ is equal to the moment about $e$ of all the other forces acting on the part of the truss to the left of section $p\,q$, divided by the distance from $d$ to $e$; and $S'$ is equal to $S_y'$ csc $H'$.

## THE CURVED-CHORD TRUSS

**4. Description.**—Fig. 2 ($a$) represents the ordinary type of curved-chord truss as used in a through bridge. The members are similar to the corresponding members of the Pratt truss, except that in this case the upper chord is curved. This truss is sometimes spoken of as the **curved-chord Pratt truss.** The method of determining the stresses will be illustrated by considering the eight-panel curved-chord truss represented in Fig. 2 ($a$). The dead load, all of which will be assumed to be applied at the joints of the loaded chord, is 800 pounds, and the live load 1,800 pounds per linear foot of bridge. The lengths of all the inclined members are shown in Fig. 2 ($b$).

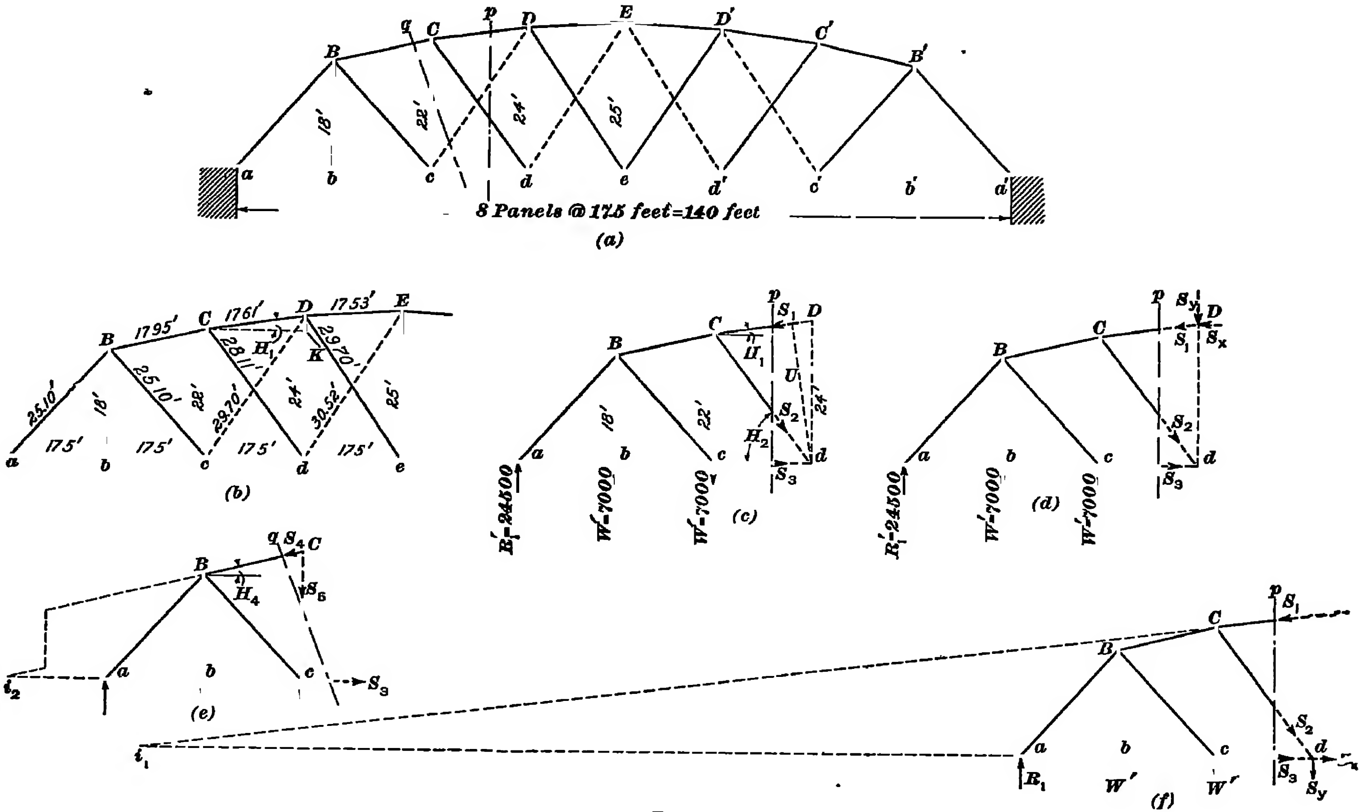



FIG 2

The dead panel load for one truss is

$$\frac{800 \times 17\ 5}{2} = 7{,}000 \text{ pounds,}$$

and each dead-load reaction is equal to

$$\frac{7{,}000 \times 7}{2} = 24{,}500 \text{ pounds}$$

The live panel load for one truss is

$$\frac{1{,}800 \times 17.5}{2} = 15{,}750 \text{ pounds,}$$

and each reaction for full live load is equal to

$$\frac{15{,}750 \times 7}{2} = 55{,}120 \text{ pounds}$$

### ANALYTIC METHOD

**5. Chord Stresses.**—The maximum chord stresses obtain when there is a full live load on the truss, the minimum, when there is no live load To find the stress in any chord member, such as *CD* or *cd*, Fig. 2 (*a*), the truss may be considered cut by a plane at the section *p*, and the left-hand part treated as a free body, as represented in Fig. 2 (*c*), the forces $S_1$, $S_2$, $S_3$ represent the stresses in *CD*, *Cd*, and *cd*, respectively. Then, $S_3$ is equal to the moment about the point *C* of all the other forces acting on this part of the truss, divided by the height at *c*. Therefore, for dead load alone,

$$S_3 = \frac{(24{,}500 \times 2 - 7{,}000 \times 1) \times 17\ 5}{22} = -33{,}400 \text{ pounds,}$$

using the minus sign to indicate tension.

The force $S_1$ may be considered replaced by its components $S_y$ and $S_x$ at *D*, Fig 2 (*d*). The moment of $S_y$ about *d* is equal to zero; $S_x$ is equal to the bending moment at *d* divided by the height at *d* Therefore, for dead load alone,

$$S_x = \frac{[24{,}500 \times 3 - 7{,}000\ (2 + 1)] \times 17\ 5}{24} = 38{,}300 \text{ pounds}$$

For *CD*, the angle $H_1 = DCK$, Fig. 2 (*b*), and

$$S_1 = S_x \sec DCK = S_x \times \frac{CD}{CK} = 38{,}300 \times \frac{17.61}{17.5}$$
$$= +\ 38{,}500 \text{ pounds}$$

In like manner, the stress in any chord member may be found.

If the truss is considered cut by an oblique plane that intersects two chord members and a vertical, and the part to the left of the section is considered as a free body, as represented in Fig. 2 (*e*), it will be seen that the only horizontal forces acting on this part are the stress in $cd$ and the horizontal component of the stress in $BC$. Therefore, these two forces are numerically equal. In like manner, it may be shown that the stress in $de$ is numerically equal to the horizontal component of the stress in $CD$

The dead-load chord stresses are the minimum chord stresses. They are as follows, expressed in pounds:

Stress in $ab$ and $bc$,

$$\frac{(24{,}500 \times 1) \times 17.5}{18} = -23{,}800$$

Stress in $cd$,

$$\frac{(24{,}500 \times 2 - 7{,}000 \times 1) \times 17.5}{22} = -33{,}400$$

Stress in $de$,

$$\frac{[24{,}500 \times 3 - 7{,}000 \times (2 + 1)] \times 17.5}{24} = -38{,}300$$

Stress in $BC$,

$$33{,}400 \times \frac{17.95}{17.5} = +34{,}300$$

Stress in $CD$,

$$38{,}300 \times \frac{17.61}{17.5} = +38{,}500$$

Stress in $DE$,

$$\frac{[24{,}500 \times 4 - 7{,}000 \times (3 + 2 + 1)] \times 17.5}{25} \times \frac{17.53}{17.5} = +39{,}300$$

The live-load chord stresses are found from the above by multiplying them by $\frac{1800}{800}$, or $\frac{9}{4}$. They are as follows:

| Member | Stress, in Pounds | Member | Stress, in Pounds |
|---|---|---|---|
| $ab$, $bc$ | − 53,600 | $BC$ | + 77,200 |
| $cd$ | − 75,200 | $CD$ | + 86,600 |
| $de$ | − 86,200 | $DE$ | + 88,400 |

The maximum chord stresses are found by adding the dead- and live-load chord stresses. They are as follows:

| Member | Maximum Stress, in Pounds | Member | Maximum Stress, in Pounds |
|---|---|---|---|
| $a b$, $b c$ | − 77,400 | $B C$ | + 111,500 |
| $c d$ | − 108,600 | $C D$ | + 125,100 |
| $d e$ | − 124,500 | $D E$ | + 127,700 |

**6. Web Stresses in General.**—The web stresses may be found by the method of shears or by the method of moments. Both methods will be explained; but, for the actual determination of stresses, the shorter method will be used in each case.

*Method of Shears.*—Applying the equation $\Sigma Y = \Sigma S \sin H = 0$ to the forces acting on the part of the truss represented in Fig. 2 (*c*), we have

$$\Sigma Y = R_1' - W' - W' - S_1 \sin H_1 - S_2 \sin H_2 = 0;$$

whence

$$S_2 \sin H_2 = R_1' - W' - W' - S_1 \sin H_1 \qquad (1)$$

Likewise, applying the same equation to the part shown in Fig 2 (*e*),

$$\Sigma Y = R_1' - W' - W' - S_4 \sin H_4 - S_3 = 0;$$

whence

$$S_3 = R_1' - W' - W' - S_4 \sin H_4 \qquad (2)$$

In equations (1) and (2), $R_1' - W' - W'$ is the shear on section $p$ and on section $q$; $S_1 \sin H_1$ and $S_4 \sin H_4$ are the vertical components of the stresses in $CD$ and $B C$, respectively; $S_2 \sin H_2$ is the vertical component of the stress in the diagonal $Cd$; and $S_3$ is the stress in the vertical $Cc$. Then, *the vertical component of the stress in any intermediate diagonal, and the stress in any intermediate vertical, of a curved-chord truss is equal to the algebraic sum of the shear on the plane of section that intersects the web member under consideration and two chord members, and the vertical component of the stress in the inclined chord member intersected by the plane* The chord stress referred to is that which obtains for the same loading that causes the desired web stress.

*Method of Moments*.—To find the stress in $Cd$ by the method of moments, the equation $\Sigma M = 0$ is applied to the forces acting on the part of the truss represented in Fig 2 ($c$), taking for the center of moments the point of intersection $i_1$ of the lines of action of $S_1$ and $S_3$. The work may be shortened by assuming that $S_2$ is replaced at the point $d$ by its vertical and horizontal components $S_y$ and $S_x$, respectively, as represented in Fig 2 ($f$). The moments of $S_1$, $S_3$, and $S_x$ about $i_1$ are each equal to zero, then, $S_y$ is equal to the moment about $i_1$ of all the external forces acting on the part shown, divided by the distance from $d$ to $i_1$. This distance is calculated as follows $CD$ slopes 2 feet vertical in one panel, or in 17 5 feet horizontal; then, since the height of the truss at $D$ is 24 feet, the point of intersection of the lines of action of $S_1$ and $S_3$ is $24 \div 2 = 12$ panel lengths to the left of $Dd$. Then, $i_1$ is $11 \times 17.5$ feet to the left of $c$, $10 \times 17.5$ feet to the left of $b$, and $9 \times 17.5$ feet to the left of $a$. Applying the equation $\Sigma M = 0$, we have

$$\Sigma M = (R_1' \times 9 - W' \times 10 - W' \times 11) \times 17.5 - S_y \times 12 \times 17.5 = 0,$$

whence

$$S_y = \frac{(R_1' \times 9 - W' \times 10 - W' \times 11) \times 17.5}{12 \times 17.5}$$

$$= \frac{R_1' \times 9 - W' \times (10 + 11)}{12}$$

and $S_2 = S_y \csc H_2$.

To find the stress in $Cc$ by the method of moments, the equation $\Sigma M = 0$ is applied to all the forces acting on the part of the truss represented in Fig 2 ($e$), the center of moments being taken at $i_2$, the intersection of $BC$ and $cd$ produced; $i_2$ is found to be $3.5 \times 17.5$ feet to the left of $a$. Applying the equation $\Sigma M = 0$, we have

$$\Sigma M = (R_1' \times 3.5 - W_1' \times 4.5 - W_1' \times 5.5) \times 17.5 - S_6 \times 5.5 \times 17.5 = 0,$$

whence $$S_6 = \frac{R_1' \times 3.5 - W_1'(4.5 + 5.5)}{5.5}$$

From the foregoing, the following general principle may be stated.

*The vertical component of the stress in an intermediate diagonal, or the stress in an intermediate vertical of a curved-chord truss is equal to the moment of the external forces on the left of the plane of section that cuts the web member and two chord members, about the point of intersection of the chord members cut by the plane, divided by the distance from this intersection to that of the web member under consideration with the horizontal chord.*

**7.** The method of moments is especially useful in the calculation of the maximum live-load stresses, as the left reaction is then usually the only external force to the left of the section. The method of shears is best suited to the calculation of the dead-load stresses, as the horizontal components of the chord stresses are usually known before the stresses in the web members are found, and the vertical components may be found from them

**8. Dead-Load Web Stresses.**—The dead-load web stresses will be found by the method of shears. The

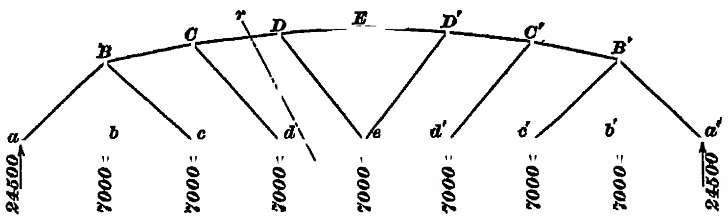



FIG 3

members in action, and the loads and reactions, are shown in Fig. 3. The shears are as follows:

| Panel | Shear, in Pounds | Panel | Shear, in Pounds |
|---|---|---|---|
| *a b* | + 24,500 | *c d* | + 10,500 |
| *b c* | + 17,500 | *d e* | + 3,500 |
| | | *e d'* | − 3,500 |

The vertical components of the chord stresses are found from the horizontal components given in Art. **5**. They are as follows:

| Member | Vertical Component of Stress, in Pounds |
|---|---|
| $BC$ | $33{,}400 \times \frac{4}{17.5} = 7{,}600$ |
| $CD$ | $38{,}300 \times \frac{2}{17.5} = 4{,}400$ |
| $DE$ | $39{,}200 \times \frac{1}{17.5} = 2{,}200$ |

Then, the vertical components of the stresses in the main diagonals are as follows:

| Member | Vertical Component, in Pounds |
|---|---|
| $aB$ | $24{,}500$ |
| $Bc$ | $17{,}500 - 7{,}600 = 9{,}900$ |
| $Cd$ | $10{,}500 - 4{,}400 = 6{,}100$ |
| $De$ | $3{,}500 - 2{,}200 = 1{,}300$ |

The actual stresses in the main diagonals are, therefore:

| Member | Stress, in Pounds |
|---|---|
| $aB$ | $24{,}500 \times \frac{25.10}{18} = +\ 34{,}200$ |
| $Bc$ | $9{,}900 \times \frac{25.10}{18} = -\ 13{,}800$ |
| $Cd$ | $6{,}100 \times \frac{28.11}{22} = -\ 7{,}800$ |
| $De$ | $1{,}300 \times \frac{29\ 70}{24} = -\ 1{,}600$ |

The stresses in the verticals are as follows:

| Member | Stress, in Pounds |
|---|---|
| $Bb$ | $W = -\ 7{,}000$ |
| $Cc$ | $10{,}500 - 7{,}600 = +\ 2{,}900$ |
| $Dd$ | $3{,}500 - 4{,}400 = -\ 900$ |
| $Ee$ | $2{,}200 + 2{,}200 = -\ 4{,}400$ |

The stress in $Dd$ is tension, because the vertical component of the stress in $CD$, which acts downwards on the

part of the truss to the left of section $r$, Fig. 3, is greater than the shear on section $r$; therefore, the stress in $Dd$ must act upwards from the joint $d$, and is tension. The stress in $Ee$ is equal to the sum of the vertical components of the stresses in $DE$ and $ED'$, both of which act upwards at the joint $E$; therefore, the stress in $Ee$ must act downwards from $E$, and is tension.

**9. Counters.**—When the counter $dE$ is in action, the members that are in action are shown in Fig. 4, the main diagonal $De$ is omitted from the diagram, as its stress is then zero. For purposes that will be explained later, it will be convenient here to find the dead-load stresses that would occur in the counter $dE$ and the verticals $Dd$ and $Ee$, if the main diagonal $De$ were omitted when there is no live load

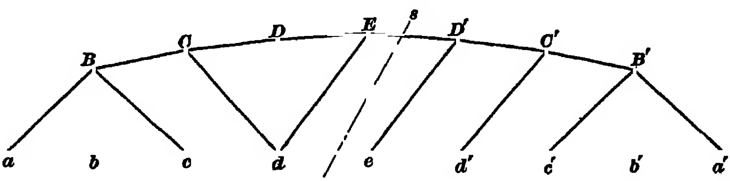


FIG 4

on the truss. The vertical component of the stress in $dE$ is then equal to the algebraic sum of the shear in panel $de$ and the vertical component of the stress in $DE$. The horizontal component of the stress in $DE$ under these conditions is equal to the moment at $d$ divided by the height at $d$; this is the same as the horizontal component of the stress in $CD$ found in Art. **5**, or 38,300 pounds. Then, the vertical component of the stress in $DE$, under these conditions, is equal to

$$38{,}300 \times \frac{1}{17.5} = 2{,}190 \text{ pounds}$$

The shear in panel $de$ is 3,500 pounds; therefore, the vertical component of the stress in $dE$ is $3{,}500 - 2{,}190 = 1{,}310$ pounds; and the stress in $dE$ is

$$1{,}310 \times \frac{30.52}{25} = +\,1{,}600 \text{ pounds}$$

The stress in $Dd$ is equal to the algebraic sum of the vertical components in $CD$ and $DE$; therefore, the stress in $Dd$ is

$$38{,}300 \times \frac{2}{17\ 5} - 38{,}300 \times \frac{1}{17\ 5} = -2{,}190 \text{ pounds}$$

The stress in $Ee$ is equal to the algebraic sum of the shear on section $s$ and the vertical component of the stress in $D'E$. The shear on the section $s$ is $+3{,}500$ pounds, the vertical component of the compression in $D'E$ is 2,200 pounds (see Art 8) Then, the stress in $Ee$ is $3{,}500 + 2{,}200 = -5{,}700$ pounds

When the counter $cD$ is in action, the members that are in action are shown in Fig 5; the main diagonals $Cd$ and $De$ are omitted, as the stresses in them are then zero. It will be convenient here to find the dead-load stresses that would

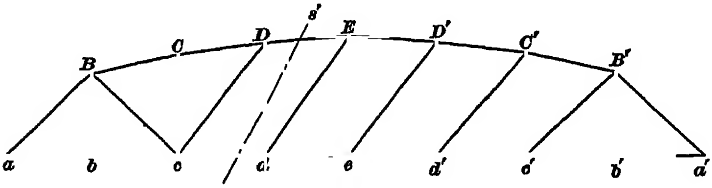



Fig 5

occur in the counter $cD$ and in the verticals $Cc$ and $Dd$, if the main diagonals were omitted when there is no live load on the truss. The vertical component of the stress in $cD$ is equal to the algebraic sum of the shear in panel $cd$ and the vertical component of the stress in $CD$. The center of moments for $CD$ is now $c$, and the horizontal component of the stress in $CD$ is equal to the horizontal component of the stress in $BC$ as found in Art. 5, or 33,400 pounds Then, the vertical component in $CD$ is equal to

$$33{,}400 \times \frac{2}{17\ 5} = 3{,}800 \text{ pounds}$$

The shear in the panel $cd$ is 10,500 pounds; therefore, the vertical component of the stress in $cD$ is $10{,}500 - 3{,}800 = 6{,}700$ pounds, and the stress in $cD$ is

$$6{,}700 \times \frac{29\ 70}{24} = +8{,}300 \text{ pounds}$$

The stress in $Cc$ is equal to the algebraic sum of the vertical components in $BC$ and $CD$, that is, the stress in $Cc$ is

$$33{,}400 \times \frac{4}{17\ 5} - 33{,}400 \times \frac{2}{17\ 5} = -3{,}800 \text{ pounds}$$

The stress in $Dd$ is equal to the algebraic sum of the shear on section $s'$ and the vertical component of the stress in $DE$ The vertical component of the stress in $DE$ is 2,190 pounds The shear on section $s'$ is 10,500 pounds, therefore, the stress in $Dd$ is $10{,}500 - 2{,}190 = -8{,}310$ pounds.

It must be understood that *the compressive stresses in the counters given above do not really exist for any loading; they are the amounts by which the live-load tensions in these members must be reduced in order to get the actual tensions in the counters.* If the live-load tension in any counter is less numerically than the dead-load compression, that means that the counter is not required.

**10. Live-Load Web Stresses.**—The positions of live load that cause maximum web stresses are in general the same as in the Pratt truss.

For example, the maximum stresses in $Cd$ and $Cc$, Fig 6,

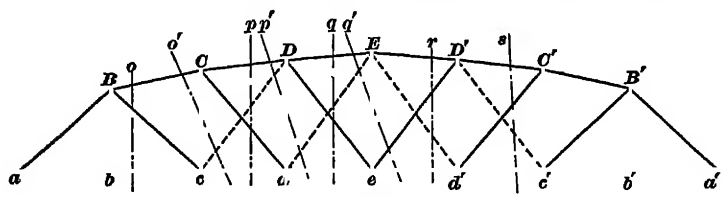



FIG 6

occur when all joints from $d$ to $b'$ are loaded, the maximum stresses in $De$ and $Dd$ occur when all joints from $e$ to $b'$ are loaded; etc.

The live-load web stresses will now be found. The maximum live-load stress in $aB$ is equal to the shear in panel $ab$ multiplied by csc $H$, or

$$\frac{15{,}750 \times 7}{2} \times \csc H = 55{,}120 \times \frac{25\ 10}{18} = +76{,}900 \text{ pounds.}$$

The maximum live-load stress in $Bb$ is a live panel load, or $-15{,}750$ pounds. The method of moments will be used

in calculating the remaining web stresses. The distances to the points of intersection of the various upper-chord members with the lower chord are as follows:

For $B\,C$, 3 5 panels to the left of $a$.

For $CD$, 9 panels to the left of $a$.

For $DE$, 21 panels to the left of $a$.

For $ED'$, 21 panels to the right of $a'$.

For $D'\,C'$, 9 panels to the right of $a'$.

For the member $Bc$, section $o$, Fig. 6, with loads from $c$ to $b'$, the center of moments is 3.5 panels to the left of $a$. Under this condition of loading,

$$R_1'' = \frac{15{,}750 \times (1 + 2 + 3 + 4 + 5 + 6)}{8} = 41{,}300 \text{ pounds}$$

and the stress in $Bc$ is

$$\frac{41{,}300 \times 3.5}{5\ 5} \times \frac{25.10}{18} = 26{,}300 \times \frac{25.10}{18} = -36{,}700 \text{ pounds}$$

For the member $Cc$, section $o'$, with loads from $d$ to $b'$, the center of moments is 3 5 panels to the left of $a$. Here,

$$R_1'' = \frac{15{,}750 \times (1 + 2 + 3 + 4 + 5)}{8} = 29{,}500 \text{ pounds}$$

and the stress in $Cc$ is

$$\frac{29{,}500 \times 3.5}{5\ 5} = +18{,}800 \text{ pounds}$$

For the member $Cd$, section $p$, with loads from $d$ to $b'$, the center of moments is nine panels to the left of $a$, and $R_1'' = 29{,}500$ pounds. Then, the stress in $Cd$ is

$$\frac{29{,}500 \times 9}{12} \times \frac{28.11}{22} = 22{,}100 \times \frac{28.11}{22} = -28{,}200 \text{ pounds}$$

For the member $Dd$, section $p'$, with loads from $e$ to $b'$, the center of moments is nine panels to the left of $a$, and $R_1'' = 19{,}700$ pounds. Then, the stress in $Dd$ is

$$\frac{19{,}700 \times 9}{12} = +14{,}800 \text{ pounds}$$

For the member $De$, section $q$, with loads from $e$ to $b'$, the center of moments is twenty-one panels to the left of $a$, and $R_1'' = 19{,}700$ pounds. Then, the stress in $De$ is

$$\frac{19{,}700 \times 21}{25} \times \frac{29\ 70}{24} = 16{,}500 \times \frac{29\ 70}{24} = -20{,}400 \text{ pounds}$$

For the member $Ee$, the stress is a maximum when the loads extend from $d'$ to $b'$ and the counter $Ed'$ is in action. Then (section $q'$), the center of moments is twenty-one panels to the left of $a$, $R_1'' = 11{,}800$ pounds; and, therefore, the stress in $Ee$ is

$$\frac{11{,}800 \times 21}{25} = +9{,}900 \text{ pounds}$$

For the counter $Ed'$, section $r$, with loads from $d'$ to $b'$, the center of moments is twenty-one panels to the right of $a'$, that is, twenty-nine panels to the right of $a$; $R_1'' = 11{,}800$ pounds, and the stress in $Ed'$ is

$$\frac{11{,}800 \times 29}{24} \times \frac{30.52}{25} = 14{,}300 \times \frac{30.52}{25} = -17{,}500 \text{ pounds}$$

For the counter $D'c'$, section $s$, with loads at $c'$ and $b'$, the center of moments is nine panels to the right of $a'$, that is, seventeen panels to the right of $a$; $R_1'' = 5{,}900$ pounds; and the stress in $D'c'$ is

$$\frac{5{,}900 \times 17}{11} \times \frac{29.70}{24} = -11{,}300 \text{ pounds}$$

**11. Maximum Web Stresses.**—Combining the maximum live-load stresses with the dead-load stresses, the following results are obtained:

| Member | Dead-Load Stress | Maximum Live-Load Stress | Combined Maximum Stress |
|---|---|---|---|
| $aB$ | + 34,200 | + 76,900 | + 111,100 |
| $Bb$ | − 7,000 | − 15,800 | − 22,800 |
| $Bc$ | − 13,800 | − 36,700 | − 50,500 |
| $Cc$ | + 2,900 | + 18,800 | + 21,700 |
| $Cd$ | − 7,800 | − 28,200 | − 36,000 |
| $Dd$ | − 900 | + 14,800 | + 13,900 |
| $De$ | − 1,600 | − 20,400 | − 22,000 |
| $Ee$ | − 5,700 | + 9,900 | + 4,200 |
| $dE$ or $Ed'$ | + 1,600 | − 17,500 | − 15,900 |
| $cD$ or $D'c'$ | + 8,300 | − 11,300 | − 3,000 |

As the resulting combined stresses in $dE$ and $cD$ are tension, both these counters are required The dead-load stress given for $Ee$ is the stress found on the supposition that the main diagonal in panel $de$ or $d'e$ is omitted from the truss. As the main diagonal is out of action for the loading causing the maximum live-load stress in $Ee$, the dead-load value given in the table is the one to use in obtaining the combined stress

**12. Minimum Web Stresses.**—The minimum stress in the end post $aB$ is equal to the dead-load stress of 34,200 pounds, compression For the minimum stress in $Bc$, the joint $b$ should be loaded. The center of moments for $Bc$, as before, is 3 5 panels to the left of $a$; the live-load reaction for the load at $b$ is

$$R_1'' = \frac{15{,}750 \times 7}{8} = 13{,}800 \text{ pounds}$$

and the live-load stress in $Bc$ is

$$\frac{13{,}800 \times 3.5 - 15{,}750 \times 4.5}{5\ 5} \times \frac{25.10}{18} = -4{,}100 \times \frac{25.1}{18}$$
$$= -5{,}700 \text{ pounds}$$

In writing this equation, the stress in $Bc$ was assumed as tension; the negative sign of the result indicates that the live load tends to cause compression in $Bc$. The dead-load stress in $Bc$ is 13,800 pounds, tension; therefore, the actual stress in $Bc$ for this loading is 13,800 − 5,700 = 8,100 pounds tension, which is the minimum stress in $Bc$. The minimum stress in each of the remaining main diagonals and in the counters is zero, when one diagonal in any panel is in action, the other is assumed to be idle.

**13. Determination of Panels in Which Counters Are Required.**—It should be noted that it is not always possible, in a curved- or inclined-chord truss, to determine directly from the shears the panels in which counters are required. In the analysis of the parallel-chord Pratt truss in *Stresses in Bridge Trusses*, Part 2, it was explained that counters are required only in those panels in which the minimum combined shears are of an opposite kind to the

maximum. If this rule is applied in the present case, it will be seen that the result is not correct. In the panel $cd$, for example, the maximum shear occurs when all the joints from $d$ to $b'$ are loaded with the live load, and is equal to the sum of 10,500 pounds dead, and 29,500 pounds live, or 40,000 pounds combined positive shear, the minimum shear occurs when the joints $b$ and $c$ are loaded, and is equal to 10,500 pounds dead, and − 5,900 pounds live, or 4,600 pounds combined positive shear. The minimum shear being of the same sign as the maximum, seems to indicate that no counter is required, although it was shown in Art. **11** that a counter is required in the panel $cd$. No mistake can be made, however, if the minimum stress in each main diagonal is calculated: if the minimum stress comes out tension, no counter is required; if it comes out compression, a counter is required. For example, if no counter were required in the panel $cd$, the minimum tension in the main diagonal $Cd$ would occur when the joints $b$ and $c$ are loaded with the live load, the left reaction being $\frac{15{,}750 \times (6+7)}{8} = 25{,}600$ pounds. The center of moments for $Cd$ is nine panels to the left of $a$; then the live-load stress in $Cd$ would be equal to

$$\frac{25{,}600 \times 9 - 15{,}750 \times 10 - 15{,}750 \times 11}{12} \times \frac{28.11}{22}$$
$$= -\ 10{,}700 \text{ pounds}$$

This equation was written assuming the stress in $Cd$ as tension; the negative sign of the result indicates that the live loads at $b$ and $c$ tend to cause compression in $Cd$. The dead-load stress in $Cd$ is 7,800 pounds tension. Then, the combined stress in $Cd$ would be equal to 10,700 − 7,800 = 2,900 pounds, compression; but, as no compression can exist in this member, the counter $cD$ is required.

**14. Verticals.**—The minimum stress in $Bb$ is equal to a dead panel load, which is 7,000 pounds, tension. The minimum stress in each of the other verticals $Cc$, $Dd$, and $Ee$ occurs when the stress in the diagonal that meets the vertical at its upper joint is a minimum, that is, when the

two diagonals that meet the vertical at its lower joint are in action. For this condition, the stress in the vertical is tension, and is equal to the algebraic sum of the vertical components of the stresses in the two chord members that meet the vertical at its upper joint. As the minimum stress is of an opposite kind to the maximum, it is necessary to find the maximum tension in the verticals

The maximum tension in $Cc$ occurs when the diagonals $Bc$ and $cD$ are in action The members of the truss that are then in action are shown in Fig. 5. The center of moments for both $BC$ and $CD$ is at $c$; therefore, the horizontal component in each of them is equal to the moment at $c$ divided by 22. The tension in $Cc$ is equal to the vertical component in $BC$, which acts upwards, minus the vertical component in $CD$, which acts downwards; then, the minimum live-load stress in $Cc$ is

$$\frac{\text{moment at } c}{22} \times \frac{4}{17.5} - \frac{\text{moment at } c}{22} \times \frac{2}{17.5}$$

$$= \frac{\text{moment at } c}{22} \times \frac{2}{17.5}, \text{ tension}$$

For the maximum live-load tension in $Cc$, the truss should be loaded as fully as possible without bringing the main diagonal $Cd$ into action. When there are live loads at $b$ and $c$, the total combined tension in the counter $cD$ is equal to 3,000 pounds (Art. 11) A live panel load at $d$ would tend to cause compression in $cD$, that is, decrease this tension by

$$\frac{\dfrac{15{,}750 \times 5}{8} \times 9}{11} \times \frac{29.70}{24} = 10{,}000 \text{ pounds}$$

As this is greater than the tension in $cD$, the live load cannot be applied at $d$, as then the main diagonal $Cd$ would be in action. The left reaction for loads at $b$ and $c$ is equal to 25,600 pounds. Then, the live-load moment at $c$ is

$$(25{,}600 \times 2 - 15{,}750 \times 1) \times 17.5$$

and the stress in $Cc$ is

$$\frac{(25{,}600 \times 2 - 15{,}750 \times 1) \times 17\ 5}{22} \times \frac{2}{17.5}$$

$$= 3{,}200 \text{ pounds, tension}$$

The dead-load stress in $Cc$ when the main diagonal $Cd$ is left out was found in Art. **9**, and is 3,800 pounds, tension Then, the maximum tension, or minimum stress, in $Cc$ is equal to $3{,}200 + 3{,}800 = 7{,}000$ pounds. The reason for finding the stresses discussed in Art. **9** is now apparent.

The maximum tension in $Dd$ occurs when the diagonals $Cd$ and $dE$ are in action. The members of the truss that are in action are shown in Fig. 4. The center of moments for $CD$ and $DE$ is at $d$, and the horizontal component of the stress in each of these members is equal to the moment at $d$ divided by 24. The tension in $Dd$ is equal to the vertical component of the stress in $CD$, which acts upwards, minus the vertical component of the stress in $DE$, which acts downwards; then, the stress in $Dd$ is

$$\frac{\text{moment at } d}{24} \times \frac{2}{17.5} - \frac{\text{moment at } d}{24} \times \frac{1}{17.5}$$
$$= \frac{\text{moment at } d}{24} \times \frac{1}{17.5}, \text{ tension}$$

It is evident that, for the maximum live-load tension in $Dd$, the truss should be loaded as fully as possible without bringing the main diagonal $De$ into action. When the live load is applied at $b$, $c$, and $d$, the tension in $dE$ is 15,900 pounds (Art. **11**), and it was shown in the preceding paragraph that under this loading the main diagonal $Cd$ is in action. A live panel load at $e$ decreases the tension in $dE$ by

$$\left(\frac{15{,}750 \times 4}{8} \times \frac{21}{24}\right) \times \frac{30.52}{25} = 8{,}400 \text{ pounds}$$

leaving $15{,}900 - 8{,}400 = 7{,}500$ pounds, tension, in $dE$. A live load at $d'$ further decreases this tension by

$$\left(\frac{15{,}750 \times 3}{8} \times \frac{21}{24}\right) \times \frac{30.52}{25} = 6{,}300 \text{ pounds}$$

leaving $7{,}500 - 6{,}300 = 1{,}200$ pounds, tension, in $dE$. A live load at $c'$ further decreases this tension by

$$\left(\frac{15{,}750 \times 2}{8} \times \frac{21}{24}\right) \times \frac{30.52}{25} = 4{,}200 \text{ pounds}$$

As this result is greater than the 1,200 pounds, tension, in $dE$, the main diagonal $De$ would be in action if the live

load extended to $c'$. The joints, therefore, from $b$ to $d'$ may be loaded, and still have the counter $dE$ in action. Then,

$$R_1'' = \frac{15{,}750 \times (3 + 4 + 5 + 6 + 7)}{8} = 49{,}200 \text{ pounds}$$

and the live-load stress in $Dd$ is equal to

$$\frac{[49{,}200 \times 3 - 15{,}750 \times (2 + 1)] \times 17.5}{24} \times \frac{1}{17.5}$$
$$= 4{,}200 \text{ pounds, tension}$$

The dead-load stress in $Dd$, when the main diagonal $De$ is omitted, was found in Art. **9** to be 2,200 pounds. Then, the total maximum tension (minimum stress) in $Dd$ is equal to $4{,}200 + 2{,}200 = 6{,}400$ pounds

The maximum tension in $Ee$ occurs when the main diagonals $De$ and $D'e$ are in action. The center of moments for both $DE$ and $ED'$ is at $e$, and the horizontal component of the stress in each of these members is equal to the moment at $e$ divided by 25 The tension in $Ee$ is equal to the sum of the vertical component of the stresses in these two members, as both act upwards at the joint $E$. The stress in $Ee$ is, therefore,

$$\frac{\text{moment at } e}{25} \times \frac{1}{17.5} + \frac{\text{moment at } e}{25} \times \frac{1}{17.5}$$
$$= \frac{\text{moment at } e}{25} \times \frac{2}{17.5}, \text{ tension}$$

The maximum moment at $e$ occurs when there is a full load on the truss. The live-load stress in $Ee$ is, then,

$$\frac{[55{,}120 \times 4 - 15{,}750 \times (3 + 2 + 1)] \times 17.5}{25} \times \frac{2}{17.5}$$
$$= 10{,}100 \text{ pounds, tension}$$

The dead-load stress in $Ee$, when the main diagonals $eD$ and $eD'$ are in action, is equal to 4,400 pounds, tension. Then, the total maximum tension in $Ee$ is $10{,}100 + 4{,}400 = 14{,}500$ pounds.

In this case, the tension of 14,500 pounds in $Ee$, found under the head of minimum stresses, is numerically greater than the compression of 4,200 pounds, found under the head of maximum stresses; so that the former is really the maximum. In all the verticals, except the hip vertical, the

minimum stress is of opposite kind to the maximum, and they must be designed to resist both kinds of stress. In a longer truss, or in one with different proportions, there would probably be more than the hip vertical in which the maximum and minimum stresses would be of the same kind. For example, if the counter *c D* were not required, the minimum compression in *Cc* would be equal to the algebraic sum of the minimum positive shear in the panel *c d* or on the plane of sections *q*, Fig. 2 (*a*), and the vertical component of the stress in *B C*.

The maximum and minimum stresses are given in Fig. 7,

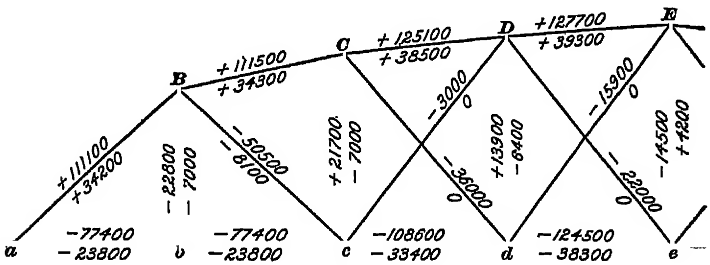



FIG 7

the former above and the latter below the lines representing the members.

**15. Distribution of Dead Load.**—In case it is desired to distribute the dead load between the two chords, one-third may be assumed to be applied at the joints of the unloaded chord. The stresses in the verticals only will be slightly different from what they are when all the dead load is assumed to be applied at the joints of the loaded chord

## GRAPHIC METHOD

**16. Dead-Load Stresses.**—The stress diagram for dead load is represented in Fig 8 (*b*); the truss, numbered according to Bow's notation, is shown in Fig. 8 (*a*). As the truss is symmetrical about the center, and therefore the stresses in the corresponding members on the two sides of

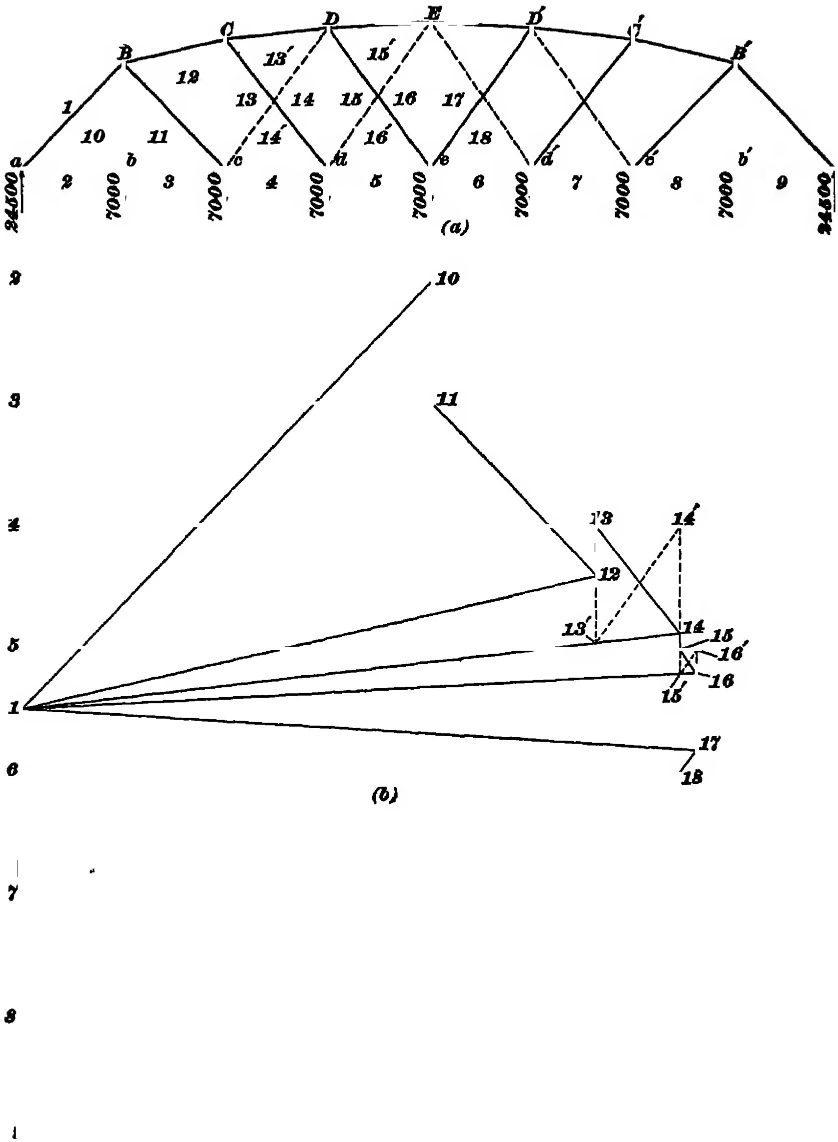



Fig. 8

the center are the same, the stress diagram has been drawn for the left end only, although the load line has been laid off for the whole truss. The left reaction *1–2* was first laid off, then the panel loads *2–3*, *3–4*, etc., and finally the right reaction *9–1*. The stress diagram was first drawn for the joint *a*, by drawing *1–10* parallel to *a B*, and *2–10* parallel to *a b*. The vectors *2–10* and *10–1* give the stresses in *a b* and *a B*, respectively. The diagram for the joint *b* was then drawn by drawing through *3* the line *3–11* parallel to *b c*, and through *10* the line *10–11* parallel to *B b*. Next, the diagram for joint *B*, then that for joint *c*, etc. were drawn in the manner explained in *Graphic Statics*.

The full lines in the stress diagram represent the dead-load stresses when the main diagonals are in action; the dotted lines represent the dead-load counter stresses. In drawing the vectors corresponding to the latter, counters were assumed in the panels in which they were most likely to be needed, and the counter stresses found by assuming that the main diagonals in these panels were left out. For example, if the main diagonal *D e* is left out, the diagram for joint *D* is *1–14–15′–1*; for joint *d*, *13–4–5–16′–15′–14–13*; and for joint *E*, *1–15′–16′–17–1*. If the main diagonal *C d* is left out, the diagram for joint *C* is *1–12–13′–1*; for joint *c*, *12–11–3–4–14′–13′–12*; for joint *D*, *1–13′–14′–15′–1*; and for joint *d*, *14′–4–5–16′–15′–14′*.

The characters and values of the stresses may be found from the stress diagram in the usual way; it is unnecessary to tabulate them here, as they are the same as found in the preceding articles.

**17. Live-Load Chord Stresses.**—As the maximum live-load stresses in the chords obtain when there is a full live load on the truss, they may be found by multiplying the dead-load stresses by the ratio of the live to the dead load, or a stress diagram similar to that shown in Fig. 8 (*b*) may be drawn, the dotted lines being omitted

**18. Live-Load Web Stresses.**—The maximum live-load stress in the end post may be found in the same way as

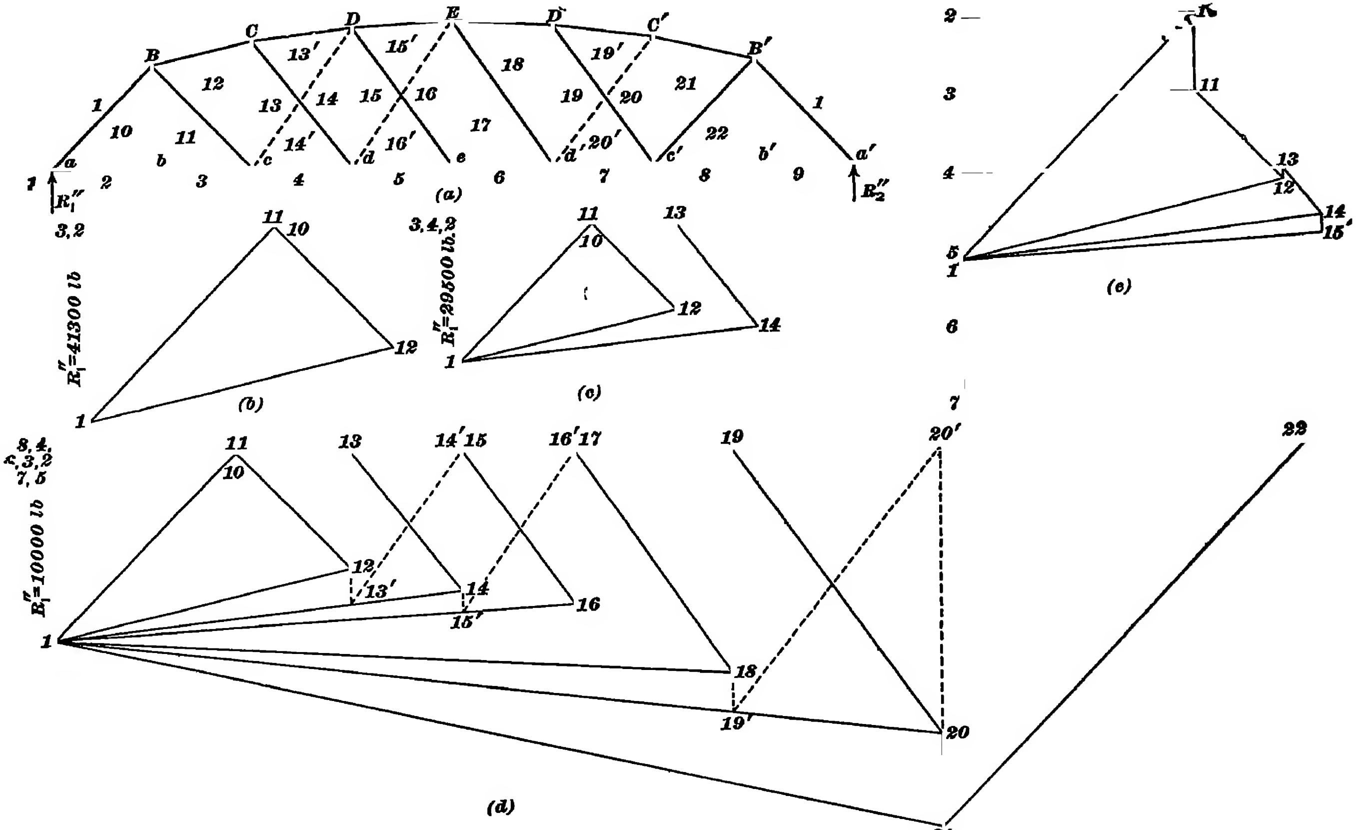


FIG 9

the live-load chord stresses, the maximum live-load stress in the hip vertical is tension and equal to a live panel load. In order to find the live-load stress in any other web member, it is necessary to consider the loading that causes the desired stress in the member, and the members of the truss that are in action for that loading. The maximum live-load stress in any main member on the left of the center obtains when the joints to the right of that member are loaded; the maximum stress in a counter on the left of the center obtains when the joints to the left of the member are loaded. As will be explained presently, it is more convenient to find the maximum stresses in the counters on the right of the center The only external force that acts on the left of a web member when the live-load stress in that member is greatest is then the left reaction, which can be found most easily by calculation. Each member must receive separate consideration.

The maximum stress in $Bc$ obtains when the truss is loaded from $b'$ to $c$, Fig. 9 ($a$); then, $R_1'' = 41{,}300$ pounds. It is unnecessary in this case to lay off the load line for all the external forces acting on the truss; the left reaction *1–2* may be laid off, Fig. 9 ($b$), and the stresses in the members that meet at $a$ found by drawing the stress diagram for the joint $a$, as represented in Fig. 9 ($b$); then the stress diagram may be drawn for the joints $b$ and $B$ until *11–12*, representing the stress in $Bc$, is found. The maximum stresses in $Cc$ and $Cd$ obtain when all the joints from $b'$ to $d$ are loaded; then, $R_1'' = 29{,}500$ pounds. This may be laid off, as shown at *1–2*, Fig. 9 ($c$), and the stress diagram drawn for the joints $a$, $b$, $B$, $c$, and $C$ until *12–13* and *13–14*, representing the stresses in $Cc$ and $Cd$, are found

As the vectors in Fig 9 ($c$), as far as point *12*, are parallel, respectively, to those in Fig. 9 ($b$), the two figures are similar; therefore,

$$\frac{(11\text{–}12)_b}{(11\text{–}12)_c} = \frac{(1\text{–}2)_b}{(1\text{–}2)_c} = \frac{41300}{29500};$$

whence $$(11\text{–}12)_b = \frac{41300}{29500} \times (11\text{–}12)_c$$

This principle is made use of in Fig. 9 (*d*); the force *1–2*, representing the left reaction, was laid off arbitrarily equal to 10,000 pounds, and the stress diagram drawn in the same way as in Fig. 9 (*b*) until *21–22*, representing the minimum live-load stress in $B'c'$, was found. In drawing the diagram, it was assumed that a counter would be required in the panel $ed'$, so the main diagonal $eD'$ was left out; in the panel $d'c'$, the stress diagram was drawn first on the assumption that the main diagonal $d'C'$ was left out, and then on the assumption that the counter $D'c'$ was left out. The vector *19–20* represents the stress in $D'c'$, and *19′–20′* represents the stress in $d'C'$; the vectors *13′–14′* and *15′–16′* were drawn in connection with the minimum stresses in the verticals $Cc$ and $Dd$, as will be explained later. It is evident that, as far as point *12*, Fig. 9 (*d*) is similar to Fig. 9 (*b*), and, as far as point *14*, is similar to Fig. 9 (*c*); whence

$$(11\text{–}12)_b = \frac{41800}{10000} \times (11\text{–}12)_d$$
$$(13\text{–}14)_c = \frac{28500}{10000} \times (13\text{–}14)_d$$

from which it follows that the stress in any member of the truss, when there are no loads on the left of the member, may be found from Fig. 9 (*d*), by multiplying the vector that represents the stress in the member by the ratio of the left reaction for the loading that causes the desired stress to 10,000.

The stresses in the various web members due to a left reaction of 10,000 pounds are first found by scaling the vectors in Fig. 9 (*d*). They are as follows:

| Member | Stress, in Pounds | Member | Stress, in Pounds |
|---|---|---|---|
| $Bc$ | − 8,900 | $Ee$ | + 8,400 |
| $Cc$ | + 6,400 | $Ed'$ | − 14,800 |
| $Cd$ | − 9,600 | $D'c'$ | − 19,200 |
| $Dd$ | + 7,500 | $d'C'$ | + 18,100 |
| $De$ | − 10,400 | $c'B'$ | + 28,500 |

The left reactions due to the loadings that cause the desired stresses are then found in the same way as in Art. **10**. They are as follows.

| Member | Reaction, in Pounds |
|---|---|
| $Bc$ | 41,300 |
| $Cc$, $Cd$ | 29,500 |
| $Dd$, $De$ | 19,700 |
| $Ee$, $Ed$ | 11,800 |
| $D'c'$ | 5,900 |
| $d'C'$ | $15{,}750 \times \dfrac{1+2}{8} = 5{,}900$ |
| $c'B'$ | $15{,}750 \times \frac{1}{8} = 2{,}000$ |

Then, the desired stresses in these members are as follows:

| Member | Maximum Live-Load Stress, in Pounds |
|---|---|
| $Bc$ | $\frac{41300}{10000} \times 8{,}900 = -36{,}800$ |
| $Cc$ | $\frac{29500}{10000} \times 6{,}400 = +18{,}900$ |
| $Cd$ | $\frac{29500}{10000} \times 9{,}600 = -28{,}300$ |
| $Dd$ | $\frac{19700}{10000} \times 7{,}500 = +14{,}800$ |
| $De$ | $\frac{19700}{10000} \times 10{,}400 = -20{,}500$ |
| $Ee$ | $\frac{11800}{10000} \times 8{,}400 = +9{,}900$ |
| $Ed'$ | $\frac{11800}{10000} \times 14{,}800 = -17{,}500$ |
| $D'c'$ | $\frac{5900}{10000} \times 19{,}200 = -11{,}300$ |

| Member | Minimum Live-Load Stress, in Pounds |
|---|---|
| $d'C'$ | $\frac{5900}{10000} \times 18{,}100 = +10{,}700$ |
| $c'B'$ | $\frac{2000}{10000} \times 28{,}500 = +5{,}700$ |

The combined stresses in these members may be found in the same way as in Art. **11**.

**19. Verticals.**—The minimum stresses in $Ee$ and $Dd$, and in $Cc$ when a counter is needed in panel $cd$, as in this case, require special consideration. The stress in each case is tension, and, as this is of opposite kind to the maximum stress, it is necessary to find the maximum live-load tension in these verticals, in order to find the minimum stresses. For this reason, it is necessary to go through the same operations as in the analytic method to determine the proper loadings.

The maximum tension in $Ee$ occurs when the counters $dE$ and $d'E$ are out of action, and the truss is fully loaded, it may be found from the vector *16–17* in the stress diagram for dead load, Fig. 8 ($b$), by multiplying the dead-load stress in $Ee$ by the ratio of the live to the dead load.

The maximum tension in $Cc$ occurs when the counter $cD$ is in action (if that counter is required), and the truss is loaded as fully as possible without bringing the main diagonal $Cd$ into action. The counter $cD$ is in action when the joints $b$ and $c$ are loaded, and the stress is evidently equal to the stress in $c'D'$ when the joints $b'$ and $c'$ are loaded. In Fig. 9 ($d$), the vector *19–20* represents the stress in $c'D'$ for a reaction of 10,000 pounds, and scales 19,200 pounds, tension. The left reaction for live loads at $b'$ and $c'$ is equal to

$$\frac{15{,}750 \times (1 + 2)}{8} = 5{,}900 \text{ pounds}$$

Then, the actual live-load stress is

$$\tfrac{5900}{10000} \times 19{,}200 = -11{,}300 \text{ pounds}$$

From Fig. 8 ($b$) it may be seen that the dead-load stress in $cD$ (vector *13′–14′*), when the main diagonal $Cd$ is out of action, is equal to 8,300 pounds, compression. Then, the combined stress in $cD$, when there are live loads at $b$ and $c$, is equal to 11,300 − 8,300 = 3,000 pounds, tension. To find whether or not a load at $d$ will bring the main diagonal $Cd$ into action, the vector *13′–14′*, Fig. 9 ($d$), representing the stress in $cD$, may be considered; this scales 10,200 pounds compression for a left reaction of 10,000 pounds. The left reaction due to a live panel load at $d$ is equal to

$$\frac{15{,}750 \times 5}{8} = 9{,}800 \text{ pounds}$$

Then, the actual stress in $cD$ due to the live load at $d$ is equal to

$$\tfrac{9800}{10000} \times 10{,}200 = +10{,}000 \text{ pounds}$$

This is greater than the tension in $cD$, which means that, with live loads at $b$, $c$, and $d$, the counter $cD$ will be out of action. Therefore, the maximum tension in $Cc$ obtains when the live load is applied at $b$ and $c$, and, for a reaction of

10,000 pounds, is given by the vector *20–21* (5,400 pounds), Fig. 9 (*d*). The actual stress in *Cc* due to the live load at *b* and *c* is then equal to

$$\tfrac{5900}{10000} \times 5{,}400 = -3{,}200 \text{ pounds}$$

The dead-load stress in *Cc*, when the main diagonal *Cd* is left out, is given in Fig. 8 (*b*) as 3,800 pounds, tension. Then, the combined tension in *Cc* is equal to 3,200 + 3,800 = 7,000 pounds, which is the minimum stress in *Cc*.

The maximum tension in *Dd* occurs when the diagonals *Cd* and *dE* are in action, and the truss is loaded as fully as possible without bringing the main diagonal *De* into action. The maximum live-load tension in *dE* occurs when the joints *b*, *c*, and *d* are loaded. It was shown in the preceding paragraphs that under this loading the main diagonal *dC* is in action. The stress in *dE* due to loads on the left is evidently the same as the stress in *d′E* due to loads on the right. In Fig. 9 (*d*), the stress in *d′E* is represented by the vector *17–18*, which is 14,800 pounds, tension; the left reaction for loads at *b′*, *c′*, and *d′* is equal to

$$\frac{15{,}750 \times (1 + 2 + 3)}{8} = 11{,}800 \text{ pounds}$$

and the actual live-load stress is

$$\tfrac{11800}{10000} \times 14{,}800 = -17{,}500 \text{ pounds}$$

From Fig. 8 (*b*), the dead-load stress in *dE* when the main diagonal *De* is left out is equal to 1,600 pounds, compression (vector *15′–16′*). Then, the combined stress in *dE* when there are live loads at *b*, *c*, and *d* is equal to 17,500 − 1,600 = 15,900 pounds, tension.

It remains to be seen how many joints to the right of *dE* can be loaded without bringing the main diagonal *De* into action. This can best be determined by considering the vector *15′–16′* representing the stress in *dE*, Fig. 9 (*d*). A left reaction of 10,000 pounds causes a compression in *dE* equal to 10,600 pounds, then, a reaction of 15,000 pounds will cause a compression in *dE* equal to 15,900 pounds, which is the numerical value of the combined tension just found. Therefore, if the left reaction due to live loads at *e*, *d′*, *c′*, etc. is less than 15,000 pounds, the counter *dE* will be in action; if

greater, the main diagonal $De$ will be in action. The left reaction for a live panel load at $e$ is equal to

$$\frac{15{,}750 \times 4}{8} = 7{,}900 \text{ pounds}$$

For loads at $e$ and $d'$, it is

$$\frac{15{,}750 \times (3 + 4)}{8} = 13{,}800 \text{ pounds}$$

For loads at $e$, $d'$, and $c'$, it is

$$\frac{15{,}750 \times (2 + 3 + 4)}{8} = 17{,}700 \text{ pounds}$$

From which it will be seen that live loads may be placed at the joints $e$ and $d'$, in addition to those at $b$, $c$, and $d$, without bringing the main diagonal $De$ into action.

The actual live-load tension in $Dd$ may be found by drawing a stress diagram, Fig. 9 ($e$), of the truss as far as joint $D$ for loads at $b$, $c$, $d$, $e$, and $d'$. The vector *14–15'*, equal to 4,200 pounds tension, is the stress in $Dd$, which may also be found, from Fig. 9 ($d$), by considering first the tension due to loads at $b$, $c$, and $d$, and then the tension due to loads at $e$ and $d'$. The former is evidently equal to the vector *18–19'* (2,100 pounds) multiplied by the ratio of the left reaction due to loads at $b'$, $c'$, and $d'$ to 10,000; the latter to vector *14–15'* (1,200 pounds), multiplied by the ratio of the left reaction due to loads at $e$ and $d'$ to 10,000. They are, respectively, equal to

$$\tfrac{11800}{10000} \times 2{,}100 = 2{,}500 \text{ pounds,}$$

and

$$\tfrac{13800}{10000} \times 1{,}200 = 1{,}700 \text{ pounds}$$

The dead-load tension in $Dd$ when the main diagonal $De$ is left out is equal to 2,200 pounds [vector *14–15'*, Fig. 8 ($b$)]. Then, the maximum combined tension in $Dd$ is equal to

$$2{,}500 + 1{,}700 + 2{,}200 = 4{,}200 + 2{,}200 = 6{,}400 \text{ pounds,}$$

which is the minimum stress in $Dd$.

**20. Comparison of Methods.**—It will be seen, from the application of the analytic and graphic methods to the preceding example, that the same course of reasoning is required in both, and the same amount of work required in

combining the stresses. The actual determination of the stresses is somewhat shorter by the graphic than by the analytic method, and the results are sufficiently accurate for all practical purposes.

### OTHER FORMS OF THE CURVED-CHORD TRUSS

**21. The Deck Truss.**—The method of calculating the stresses in the members of a deck truss with curved lower chord, such as that represented in Fig. 10, is, in general, the

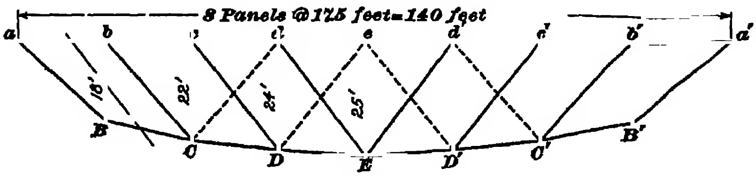



FIG. 10

same as for the through truss. The principal difference is due to the fact that the minimum stresses in the verticals, as well as the maximum, are compression. The maximum compression in any vertical on the left of the center is equal either to the algebraic sum of the maximum positive shear on the plane of section that intersects such member and two chord members, and the vertical component of the stress in the inclined-chord member for the same loading, or to the sum of a dead and a live panel load, whichever is the greater. It

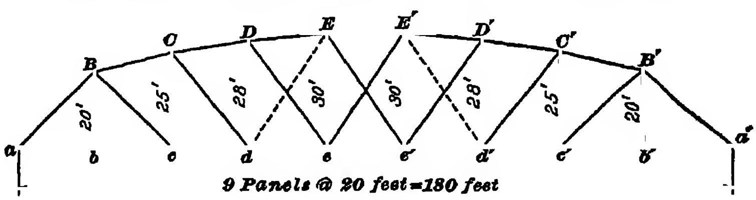



FIG 11

is evident that the maximum compression in a vertical cannot be less than the panel load supported at its upper joint. In a similar manner, the minimum compression is equal to the algebraic sum of the minimum positive shear on the section and the vertical component of the stress in the

inclined-chord member for the same loading, if the algebraic sum is greater than a dead panel load; if less, the minimum compression is equal to a dead panel load.

**22. Odd Number of Panels.**—When a curved-chord truss has an odd number of panels, as in Fig. 11, the chords in the center panel are made parallel, and the stresses in the members in that panel are found in the same way as for parallel-chord trusses.

### EXAMPLES FOR PRACTICE

1 Fig 10 represents a deck truss with dimensions as shown The dead panel load is 7,000, and the live panel load, 15,750 pounds. Assuming that all the dead load is applied at the joints of the loaded chord, find the maximum and minimum combined stresses in $bB$ and $cC$.

Ans.

| Member | Stress, in Pounds Maximum | Stress, in Pounds Minimum |
|---|---|---|
| $bB$ | + 61,900 | + 19,000 |
| $cC$ | + 47,500 | + 10,600 |

2 Fig 11 represents a curved-chord through truss with dimensions as shown The dead load is 1,500, and the live load, 2,800, pounds per linear foot of bridge, one-half being carried by each truss Determine the maximum stresses in the main diagonals and counters in the panels in which counters are required

Ans

| Member | Maximum Stress, in Pounds |
|---|---|
| $De$ | − 48,200 |
| $dE$ | − 25,600 |
| $Ee'$, $E'e$ | − 37,400 |

## THE INCLINED-CHORD TRUSS

**23. Description.**—Figs. 12 and 13 represent the type of inclined-chord truss most frequently used for bridge purposes. When the truss has an even number of panels, the inclined chord is usually made straight from the center of the truss to the first panel point from the end. When there is an odd number of panels, the center chord member is made horizontal, and the rest of the chord inclined. In some cases, when there is an even number of panels, the two chord members in the panels at the center are made

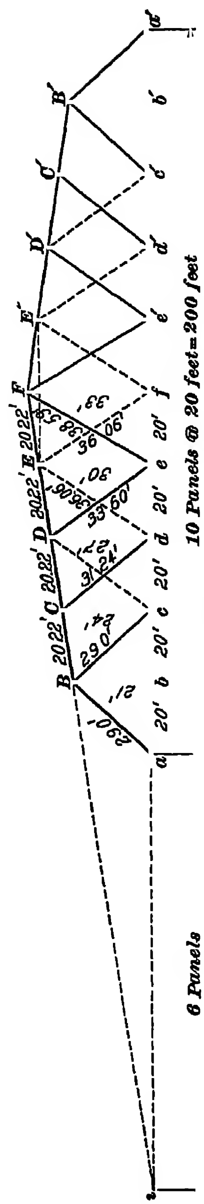


Fig 12

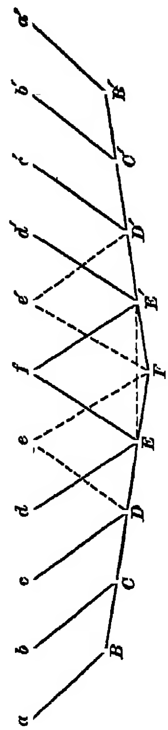


Fig 13

horizontal, as represented by the dotted lines $EE'$ in Figs. 12 and 13.

**24. Method of Calculation.**—The general method of analysis is the same as for the curved-chord truss. The actual calculation is somewhat simpler, because the inclined chord has the same inclination in the different panels; there is, therefore, but one intersection of the top and bottom chords to be considered in finding web stresses by the method of moments. In all the trusses analyzed in the preceding pages, the diagonals that slope upwards from the center toward the end have been called the main diagonals, and have been in tension for a full live load, or for no live load In the inclined-chord truss, it frequently happens that, under one of these conditions of loading, the vertical component of the stress in an inclined-chord member near the center is numerically greater than the shear in the panel, in which case the diagonal that slopes upwards from the center toward the end is out of action and there is tension in the other diagonal; this will be illustrated presently by an example. In such a truss, the diagonal that slopes downwards from

the center toward the end in that panel is called the main diagonal, and the other the counter. It is necessary to find which diagonal is in action in each panel before the chord stresses can be found; this can be determined very readily by assuming that the stress in either diagonal is tension and writing the expression for the value of the vertical component of the stress in that diagonal, taking moments about the intersection of the two chord members produced. If the vertical component comes out positive, the assumed diagonal is in action, if negative, the other diagonal is in action. As in the curved-chord truss, the panels in which counters are required cannot be determined directly from the shears. If the maximum and minimum combined stresses in any diagonal are found to be tension, no counter is required in that panel; if the maximum is found to be tension and the minimum compression, a counter is required. It is only necessary to calculate the minimum stresses in the diagonals from the end up to the first panel in which a counter is required; a counter is required in all panels from this up to the center and the minimum stresses in the diagonals in these panels are all zero. The verticals require special consideration.

**25. Through Truss.**—Fig. 12 represents a through truss. The stress in the hip vertical is evidently tension and is equal to the load at its lower joint; the maximum tension is equal to the sum of a dead and a live panel load; the minimum tension is equal to a dead panel load. The maximum stress in the other verticals is compression and is found, in the same way as for the curved-chord truss, by loading all joints to the right of the vertical under consideration. The minimum stress in the center vertical is tension; if the diagonals that meet at its lower joint are in action for full load, the maximum tension is equal to the sum of the vertical components of the stresses in the chord members that meet at its upper joint; if the diagonals that meet at its upper joint are in action for full load, the maximum tension is equal to the sum of the dead and the live panel load at its lower joint. The minimum stress in any

other vertical that has a counter adjacent, obtains when the two diagonals that meet at its lower joint are in action. Then the vertical components of the stresses in the chord members that meet at its upper joint are equal and opposite, and the stress in the vertical is equal to zero, or is compression and equal to the dead load at its upper joint, if any. The minimum stress in any vertical with no counter adjacent may be found by loading all joints to the left of the member, including the joint at its lower end.

**26. Deck Truss.**—Fig. 13 represents a deck truss. The maximum stress in any vertical is compression, and is found, in the same way as for the curved-chord truss, by loading all the joints to the right of the member, including the joint at its upper end; the maximum compression, however, cannot be less than the sum of a dead and a live panel load. The minimum stress is also compression, and is found by loading all joints to the left of the member; the minimum compression cannot be less than the dead panel load at the upper end of the member.

As the analysis of the inclined-chord truss differs in a slight degree from the curved, chord truss, the following example is given as an illustration.

### ILLUSTRATIVE EXAMPLE

**27. Data.**—Fig. 12 represents a ten-panel through inclined-chord truss with dimensions as shown. The dead load is 1,500 pounds per linear foot of bridge; one-third of it is assumed to be applied at the joints of the unloaded chord. The live load is 2,800 pounds per linear foot of bridge. The lengths, in feet, of all the members are shown in the figure. In what follows, the maximum and minimum stresses in all the members will be calculated by the analytic method.

**28. Panel Loads and Reactions.**—The dead panel load for one truss is

$$\frac{1,500 \times 20}{2} = 15,000 \text{ pounds}$$

of which 5,000 is applied at each of the upper-chord joints, and 10,000 at each of the lower-chord joints. Each dead-load reaction is equal to

$$\frac{15{,}000 \times 9}{2} = 67{,}500 \text{ pounds}$$

The live panel load for one truss is

$$\frac{2{,}800 \times 20}{2} = 28{,}000 \text{ pounds}$$

and each reaction for full load is equal to

$$\frac{28{,}000 \times 9}{2} = 126{,}000 \text{ pounds}$$

**29. Counters and Main Diagonals.**—To find which diagonals are counters, and which are main diagonals in the panels near the center, the expression for the vertical component of the stress in either diagonal for dead load alone will be written, assuming that the stress is tension, if the result is positive, the diagonal for which the vertical component was found is the main diagonal, and the other the counter, if negative, then the diagonal considered is the counter. The intersection of the chords is six panel lengths to the left of *a*.

*Panel ef.*—The member $Ef$ will be assumed in tension. Then, the vertical component in $Ef$ is

$$\frac{67{,}500 \times 6 - 15{,}000 \times (7 + 8 + 9 + 10)}{11} = -\frac{105000}{11}$$

As this comes out negative, $Ef$ is the counter and $eF$ the main diagonal.

*Panel de.*—The member $De$ will be assumed in tension. Then the vertical component in $De$ is

$$\frac{67{,}500 \times 6 - 15{,}000 \times (7 + 8 + 9)}{10} = +\frac{45000}{10}$$

As this comes out positive, $De$ is the main diagonal and $dE$ the counter, if one is required in this panel. It is evident that $Cd$ and $Bc$ are main diagonals.

**30. Dead-Load Chord Stresses.**—The dead-load chord stresses are found as follows, the horizontal components in the inclined members being found first:

| Member | Horizontal Component, in Pounds |
|---|---|
| $BC$ . . | $\dfrac{(67{,}500 \times 2 - 15{,}000 \times 1) \times 20}{24} = 100{,}000$ |
| $CD$ . . | $\dfrac{[67{,}500 \times 3 - 15{,}000 \times (2 + 1)] \times 20}{27} = 116{,}700$ |
| $DE, EF$ | $\dfrac{[67{,}500 \times 4 - 15{,}000 \times (3 + 2 + 1)] \times 20}{30} = 120{,}000$ |

From the figure, $\sec H = \dfrac{20.22}{20}$

Therefore, the dead-load chord stresses, computed from the horizontal components just found, are as follows:

| Member | Stress, in Pounds |
|---|---|
| $BC$ . | $100{,}000 \times \dfrac{20.22}{20} = +101{,}100$ |
| $CD$ . . . . | $116{,}700 \times \dfrac{20.22}{20} = +118{,}000$ |
| $DE, EF$ . . | $120{,}000 \times \dfrac{20.22}{20} = +121{,}300$ |
| $ab, bc$ . . | $\dfrac{67{,}500 \times 20}{21} = -\ 64{,}300$ |
| $cd$ | $-100{,}000$ = horizontal component in $BC$ |
| $de$ | $-116{,}700$ = horizontal component in $CD$ |
| $ef$ | $\dfrac{[67{,}500 \times 5 - 15{,}000 \times (4 + 3 + 2 + 1)] \times 20}{33} = -113{,}600$ |

**31. Live-Load Chord Stresses.**—The live-load chord stresses may be conveniently found from the dead-load stresses by multiplying the latter by the ratio of live to dead load, which in this case is $\frac{2800}{1500}$, or $\frac{28}{15}$.

**32. Maximum and Minimum Chord Stresses.**—The minimum stresses in the chords are equal to the dead-load stresses just given. The maximum stresses are equal to the sum of the dead- and live-load stresses, and will be found by multiplying the former by $\dfrac{2{,}800 + 1{,}500}{1{,}500} = \frac{43}{15}$. They are as follows:

| Member | Maximum Stress, in Pounds |
|---|---|
| $BC$ | $101{,}100 \times \frac{43}{15} = +289{,}800$ |
| $CD$ | $118{,}000 \times \frac{43}{15} = +338{,}300$ |
| $DE, EF$ | $121{,}300 \times \frac{43}{15} = +347{,}700$ |
| $ab, bc$ | $64{,}300 \times \frac{43}{15} = -184{,}300$ |
| $cd$ | $100{,}000 \times \frac{43}{15} = -286{,}700$ |
| $de$ | $116{,}700 \times \frac{43}{15} = -334{,}500$ |
| $ef$ | $113{,}600 \times \frac{43}{15} = -325{,}700$ |

**33. Dead-Load Web Stresses.**—The dead-load web stresses are found from the shears and the vertical components of the chord stresses. The shears are:

| Panel | Shear, in Pounds |
|---|---|
| $ab$ | 67,500 |
| $bc$ | 52,500 |
| $cd$ | 37,500 |
| $de$ | 22,500 |
| $ef$ | 7,500 |

The vertical components of the stresses in the chord members, found by multiplying the horizontal components by $\frac{3}{20}$, which is the tangent of the angle that the upper chord makes with the horizontal, are:

| Member | Vertical Components, in Pounds |
|---|---|
| $BC$ | $100{,}000 \times \frac{3}{20} = 15{,}000$ |
| $CD$ | $116{,}700 \times \frac{3}{20} = 17{,}500$ |
| $DE, EF$ | $120{,}000 \times \frac{3}{20} = 18{,}000$ |

Now,

$$\csc Bab = \frac{29.0}{21}$$

$$\csc Bcb = \frac{29.0}{21}$$

$$\csc Cdc = \frac{31.24}{24}$$

$$\csc Ded = \frac{33.60}{27}$$

$$\csc Fef = \frac{38.59}{33}$$

The dead-load stresses in the web members, when the main diagonals are in action, can now be computed They are

| Member | Stress, in Pounds |
|---|---|
| $aB$ | $67{,}500 \times \dfrac{29}{21} = +93{,}200$ |
| $Bb$ | $-10{,}000$ |
| $Bc$ | $(52{,}500 - 15{,}000) \times \dfrac{29}{21} = -51{,}800$ |
| $Cc$ | $42{,}500 - 15{,}000 = +27{,}500$ |
| $Cd$ | $(37{,}500 - 17{,}500) \times \dfrac{31\ 24}{24} = -26{,}000$ |
| $Dd$ | $27{,}500 - 17{,}500 = +10{,}000$ |
| $De$ | $(22{,}500 - 18{,}000) \times \dfrac{33\ 60}{27} = -\ 5{,}600$ |
| $Ee$ | $+\ 5{,}000$ |
| $eF$ | $(18{,}000 - 7{,}500) \times \dfrac{38\ 59}{33} = -12{,}300$ |
| $Ff$ | $-10{,}000$ |

It will be assumed that the counters $cD$, $dE$, and $E'f$ are required. Then, the dead-load stresses in these counters and the adjacent verticals, when the main diagonals $Cd$, $De$, and $e'F$ are left out, are as follows

| Member | Stress, in Pounds |
|---|---|
| $Ee$ | $17{,}500 - 18{,}000 = -\ 500$ |
| $Cc$ | $+\ 5{,}000$ |
| $Dd$ | $-15{,}000$ |
| $cD$ | $(37{,}500 - 15{,}000) \times \dfrac{33\ 60}{27} = +28{,}000$ |
| $dE$ | $(22{,}500 - 17{,}500) \times \dfrac{36\ 06}{30} = +\ 6{,}000$ |
| $E'f$ | $(17{,}000 - \ 7{,}500) \times \dfrac{36\ 06}{30} = +11{,}400$ |
| $Ff$ | $17{,}000 + 2{,}500 = -19{,}500$ |

**34. Live-Load Web Stresses.** — The maximum live-load web stresses are obtained most readily by first finding

the vertical components by the method of moments, as explained in Art. 3, and then multiplying each vertical component by csc $H$, if the member is inclined. The stresses, in pounds, are as follows:

Maximum live-load stress in $a\,B$,

$$126{,}000 \times \tfrac{29}{21} = +\,174{,}000$$

Maximum live-load stress in $B\,b$,

$$-\,28{,}000$$

Maximum live-load stress in $B\,c$,

$$\left[\frac{28{,}000 \times (1+2+3+4+5+6+7+8)}{10} \times \tfrac{6}{8}\right] \times \tfrac{29}{21}$$
$$= -\,104{,}400$$

Maximum live-load stress in $C\,c$,

$$\frac{28{,}000 \times (1+2+3+4+5+6+7)}{10} \times \tfrac{6}{8} = +\,58{,}800$$

Maximum live-load stress in $C\,d$,

$$\left[\frac{28{,}000 \times (1+2+3+4+5+6+7)}{10} \times \tfrac{6}{9}\right] \times \frac{31.24}{24}$$
$$= -\,68{,}000$$

Maximum live-load stress in $D\,d$,

$$\frac{28{,}000 \times (1+2+3+4+5+6)}{10} \times \tfrac{6}{9} = +\,39{,}200$$

Maximum live-load stress in $D\,e$,

$$\left[\frac{28{,}000 \times (1+2+3+4+5+6)}{10} \times \tfrac{6}{10}\right] \times \frac{33.60}{27}$$
$$= -\,43{,}900$$

Maximum live-load stress in $E\,e$,

$$\frac{28{,}000 \times (1+2+3+4+5)}{10} \times \tfrac{6}{10} = +\,25{,}200$$

Maximum live-load stress in $E\,f$,

$$\left[\frac{28{,}000 \times (1+2+3+4+5)}{10} \times \tfrac{6}{11}\right] \times \frac{36.06}{30} = -\,27{,}500$$

Maximum live-load stress in $Ff$,

$$\frac{28{,}000 \times (1 + 2 + 3 + 4)}{10} \times \tfrac{6}{11} = +\,15{,}300$$

Maximum live-load stress in $Fe'$ $(Fe)$,

$$\left[\frac{28{,}000 \times (1 + 2 + 3 + 4)}{10} \times \tfrac{16}{10}\right] \times \frac{38.59}{33} = -\,52{,}400$$

Maximum live-load stress in $E'e'$,

$$\frac{28{,}000 \times (1 + 2 + 3)}{10} \times \tfrac{16}{10} = +\,26{,}900$$

Maximum live-load stress in $E'd'$ $(Ed)$,

$$\left[\frac{28{,}000 \times (1 + 2 + 3)}{10} \times \tfrac{16}{9}\right] \times \frac{36.06}{30} = -\,35{,}900$$

Maximum live-load stress in $D'c'$ $(Dc)$,

$$\left[\frac{28{,}000 \times (1 + 2)}{10} \times \tfrac{16}{8}\right] \times \frac{33.60}{27} = -\,20{,}900$$

In case counters are not required in the panels $bc$, $cd$, and $de$, it is necessary to know the minimum live-load stresses in the members adjacent to those panels. For this reason, they will be calculated now. They are as follows:

| Members | Minimum Live-Load Stress, in Pounds |
|---|---|
| $Bb$ | $0$ |
| $Bc$ | $\left[\frac{28{,}000 \times 1}{10} \times \frac{16}{8}\right] \times \frac{29}{21} = +\ 7{,}700$ |
| $Cc$ | $\frac{28{,}000 \times (1 + 2)}{10} \times \frac{16}{8} = -\,16{,}800$ |
| $Cd$ | $\left[\frac{28{,}000 \times (1 + 2)}{10} \times \frac{16}{9}\right] \times \frac{31.24}{24} = +\,19{,}400$ |
| $Dd$ | $\frac{28{,}000 \times (1 + 2 + 3)}{10} \times \frac{16}{9} = -\,29{,}900$ |
| $De$ | $\left[\frac{28{,}000 \times (1 + 2 + 3)}{10} \times \frac{16}{10}\right] \times \frac{33.60}{27} = +\,33{,}400$ |

**35. Maximum and Minimum Combined Web Stresses.**—The maximum and minimum web stresses due to combined live and dead load are as follows:

| Member | Maximum Live-Load Stress | Minimum Live-Load Stress | Dead-Load Stress | Dead-Load Counter Stress | Maximum Combined Stress | Minimum Combined Stress |
|---|---|---|---|---|---|---|
| *aB* | + 174,000 | | + 93,200 | | + 267,200 | + 93,200 |
| *Bc* | − 104,400 | + 7,700 | − 51,800 | | − 156,200 | − 44,100 |
| *Cd* | − 68,000 | + 19,400 | − 26,000 | | − 94,000 | − 6,600 |
| *De* | − 43,900 | + 33,400 | − 5,600 | | − 49,500 | + 27,800 |
| *Ef* | − 27,500 | | | + 11,400 | − 16,100 | |
| *Fe* | − 52,400 | | − 12,300 | | − 64,700 | |
| *Ed* | − 35,900 | | | + 6,000 | − 29,900 | |
| *Dc* | − 20,900 | | | + 28,000 | + 7,100 | |
| *Bb* | − 28,000 | | − 10,000 | | − 38,000 | − 10,000 |
| *Cc* | + 58,800 | − 16,800 | + 27,500 | | + 86,300 | + 10,700 |
| *Dd* | + 39,200 | | + 10,000 | + 5,000 | + 49,200 | + 5,000 |
| *Ee* | + 25,200 | | + 5,000 | − 5,500 | + 19,700 | + 5,000 |
| *E′e′* | + 26,900 | | | + 500 | + 27,400 | |
| *Ff* | − 28,000 | + 15,300 | − 10,000 | − 19,500 | − 38,000 | − 4,200 |

It is found that the stress in *Ff* due to a full live load is numerically greater than that previously found for partial load.

Since a counter is required in panel *de*, there will be no live-load tension in the vertical *Dd*; for, when the joints to the left of *d* are loaded, the counter *dE* will be in action, and the stress in *Dd* is then the dead load of 5,000 pounds at *D*. The stress in *E′e′* for loads at *d′*, *c′*, and *b′* is found to be greater than that in *Ee* for loads from *f* to *b′*.

### EXAMPLES FOR PRACTICE

1 In Fig 14 is represented an eleven-panel inclined-chord truss

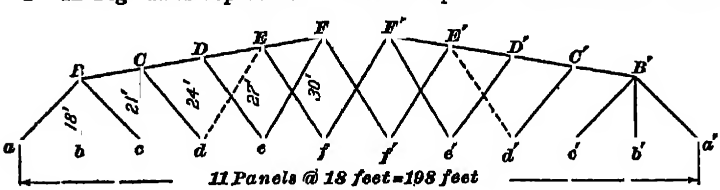



FIG 14

with dimensions as shown The dead load is 1,500, and the live load.

2,800 pounds per linear foot of bridge, one-half of the load being carried by one truss Assuming that one-third of the dead load is applied at the joints of the unloaded chord, find the maximum combined stresses in the diagonals $De$ and $dE$.

| Ans. | Member | Stress, in Pounds |
|---|---|---|
| | $De$ | − 52,000 |
| | $dE$ | − 24,900 |

2. Referring to the same truss as in the preceding example, find the maximum combined stress in the chord member $DE$

Ans + 366,200 lb.

3. For the truss referred to in the preceding example, determine the maximum and minimum combined stresses in $Ee$.

| Ans. | Stress, in Pounds | |
|---|---|---|
| | Maximum | Minimum |
| | + 23,700 | + 4,500 |

## THE PETIT TRUSS

**36. Description.**—When the method of subdivision used in the Baltimore truss (*Stresses in Bridge Trusses*, Part 2) is applied to the simple type of curved-chord truss, as shown in Figs. 15 and 16, the truss is called a **Petit truss**. This form of truss is well adapted to long spans, being very economical, and is at present the standard type of bridge truss in America for the longest spans in which simple trusses are

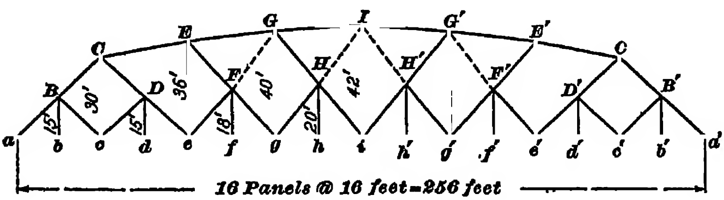



Fig 15

used. One of the longest simple-truss spans ever built is composed of Petit trusses 675 feet center to center of end supports, and 130 feet center to center of chords at the center of the truss. For very long spans, such as this, the modern tendency is toward the use of very long panels, those in the bridge mentioned being greater than 50 feet. Either substruts, as in Fig. 15, or subties, as in Fig. 16,

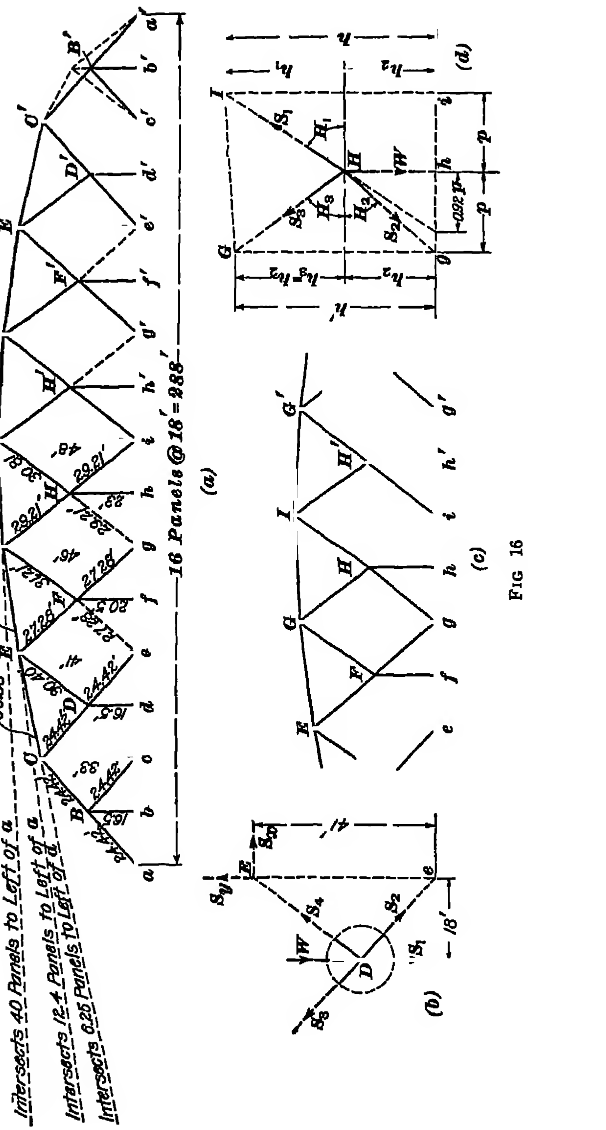

Intersects 40 Panels to Left of a
Intersects 12.4 Panels to Left of a
Intersects 6.25 Panels to Left of a
16 Panels @ 18' = 288'
(a)
(b)
(c)
(d)

Fig 16

may be used; the latter type is used when it is desired to make all the diagonals tension members. In both types, the short verticals divide the loaded chord (in this case the lower chord) and the main diagonal into two equal parts; the short diagonals are run from the intersection of the short vertical and main diagonal to the opposite chord, and therefore are not in straight lines with the respective counters.

**37. Method of Calculation.**—The method of analysis is precisely the same as for the Baltimore and the curved-chord truss, and the student should be thoroughly familiar with these trusses before attempting the analysis of the Petit truss. It will be impossible, however, for him to make a mistake if he applies the conditions of equilibrium correctly. In case of doubt regarding the stress in any member, the following steps should be taken in order:

1 *Consider the loading that produces the desired stress.*

2. *Determine which members are in action for that loading.*

3. *Consider the truss cut into two parts by a surface that intersects the member in which the stress is desired, and not more than two other members in which the stresses are unknown.*

4. *Consider the part of the truss on one side of the cutting surface, drawing a figure representing this part if necessary, replacing by external forces the stresses in the members cut.*

5. *Apply such equation or equations of equilibrium to the part considered as will give the desired stress in the most convenient way.*

To illustrate the method of calculation, the stresses will be found in the form of truss having subties, as represented in Fig. 16. This is a sixteen-panel through Petit truss with dimensions as shown The dead load will be taken as 2,400, and the live load as 3,000, pounds per linear foot of bridge, one-half being carried by one truss. One-third of the dead load will be assumed to be applied at the upper and intermediate joints. The lengths of all the inclined members are given in the figure.

**38. Panel Loads and Reactions.**—The dead panel load for each truss is

$$\frac{2,400 \times 18}{2} = 21,600 \text{ pounds}$$

of which 14,400 pounds is applied at each of the lower-chord joints, and 7,200 pounds at each of the upper and intermediate joints $B$, $C$, $D$, $E$, etc. Each dead-load reaction is equal to

$$\frac{21,600 \times 15}{2} = 162,000 \text{ pounds}$$

The live panel load for each truss is

$$\frac{3,000 \times 18}{2} = 27,000 \text{ pounds}$$

and each live-load reaction for full load is

$$\frac{27,000 \times 15}{2} = 202,500 \text{ pounds}$$

### ANALYTIC METHOD

**39. Chord Stresses.**—In the panels $ef$, $fg$, $gh$, and $hi$, the main diagonals $EFg$ and $GHi$ and the short diagonals $FG$ and $HI$ are in action when there is no live load, and when there is a full live load on the truss. The dead-load chord stresses, in pounds, are as follows:

Stress in $ab$ and $bc$,

$$\frac{162,000 \times 18}{16.5} = -176,700$$

Stress in $cd$ and $de$,

$$\frac{(162,000 \times 2 - 21,600 \times 1) \times 18}{33} = -164,900$$

Stress in $ef$ and $fg$,

$$\frac{[162,000 \times 4 - 21,600 \times (3 + 2 + 1)] \times 18}{41} = -227,600$$

Stress in $gh$ and $hi$,

$$\frac{[162,000 \times 6 - 21,600 \times (5 + 4 + 3 + 2 + 1)] \times 18}{46}$$
$$= -253,600$$

Stress in $CE$,

$$\frac{[162{,}000 \times 4 - 21{,}600 \times (3+2)] \times 18}{41} \times \frac{36.88}{36}$$

$$= 237{,}100 \times \frac{36.88}{36} = +242{,}900$$

Stress in $EG$,

$$\frac{[162{,}000 \times 6 - 21{,}600 \times (5+4+3+2)] \times 18}{46} \times \frac{36.35}{36}$$

$$= 262{,}000 \times \frac{36.35}{36} = +264{,}500$$

Stress in $GI$,

$$\frac{[162{,}000 \times 8 - 21{,}600 \times (7+6+5+4+3+2)] \times 18}{48}$$

$$\times \frac{36.06}{36} = 267{,}300 \times \frac{36.06}{36} = +267{,}700$$

The live-load chord stresses may be found by multiplying the foregoing stresses by $\frac{3000}{2400}$, or $\frac{5}{4}$.

**40. Maximum and Minimum Chord Stresses.** The dead-load stresses just given are the minimum stresses. The maximum combined chord stresses will be found by multiplying the minimum stresses by $\frac{3{,}000 + 2{,}400}{24} = \frac{9}{4}$. They are as follows:

| Member | Maximum Stress, in Pounds |
|---|---|
| $ab$, $bc$ | $176{,}700 \times \frac{9}{4} = -397{,}600$ |
| $cd$, $de$ | $164{,}900 \times \frac{9}{4} = -371{,}000$ |
| $ef$, $fg$ | $227{,}600 \times \frac{9}{4} = -512{,}100$ |
| $gh$, $hi$ | $253{,}600 \times \frac{9}{4} = -570{,}600$ |
| $CE$ | $242{,}900 \times \frac{9}{4} = +546{,}500$ |
| $EG$ | $264{,}500 \times \frac{9}{4} = +595{,}100$ |
| $GI$ | $267{,}700 \times \frac{9}{4} = +602{,}300$ |

**41. Dead-Load Web Stresses.**—The dead-load stress in each of the subverticals $Bb$, $Dd$, $Ff$, and $Hh$ is $-14{,}400$ pounds.

1. *Short Diagonals.*—As the short diagonal in any panel does not make the same angle with the horizontal as the main diagonal to which it connects, its stress will not be equal to the product of one-half the load at an intermediate

joint and the cosecant of the angle $H$, as in the case of the Baltimore truss. This stress is found most easily by considering one of the intermediate joints, such as $D$. The forces that act at joint $D$ are shown in Fig 16 ($b$): $S_4$, the stress in the short diagonal, may be replaced at $E$ in its line of action by its vertical and horizontal components $S_y$ and $S_x$, respectively. If the center of momen s is taken at $e$, the moments of $S_2$, $S_3$, and $S_y$ will all be zero; and the equation of moments is:

$$\Sigma M = (S_1 + W)\,18 - S_x \times 41 = 0;$$

whence
$$S_x = \frac{(S_1 + W) \times 18}{41}$$

Then,
$$S_4 = \frac{(S_1 + W) \times 18}{41} \times \frac{30.40}{18} = \frac{(S_1 + W) \times 30.40}{41}$$

and
$$S_y = \frac{(S_1 + W) \times 18}{41} \times \frac{24.5}{18} = \frac{(S_1 + W) \times 24.5}{41}$$

From these equations, the following principle may be stated·

*The dead-load stress in a short diagonal of the Petit truss with subties is equal to the sum of the stress in a short vertical and the dead load at an intermediate joint, multiplied by the ratio of the length of the short diagonal to the height of the truss at the point where the short diagonal joins the chord, also, the vertical component of the stress in a short diagonal is equal to the sum of the stress in a short vertical and the dead load at an intermediate joint multiplied by the ratio of the vertical projection of the short diagonal to the height of the truss at the point where the short diagonal joins the chord.*

The dead-load stresses in the short diagonals are as follows.

| MEMBER | DEAD-LOAD STRESS, IN POUNDS |
|---|---|
| $Bc$ | $(14{,}400 + 7{,}200) \times \frac{24\ 42}{33} = +\ 16{,}000$ |
| $DE$ | $(14{,}400 + 7{,}200) \times \frac{30.40}{41} = -\ 16{,}000$ |
| $FG$ | $(14{,}400 + 7{,}200) \times \frac{31\ 21}{46} = -\ 14{,}700$ |
| $HI$ | $(14{,}400 + 7{,}200) \times \frac{30\ 81}{48} = -\ 13{,}900$ |

2. *Main Diagonals*.—The dead-load stress in a main diagonal is found by multiplying the algebraic sum of the shear and the vertical components of the stresses in the other inclined members in the panel in which the member is located, by the cosecant of the angle that the main diagonal makes with the horizontal.

The vertical components of the stresses in the inclined chord members are·

| Member | Vertical Component, in Pounds |
|---|---|
| $CE$ | $237{,}100 \times \frac{8}{36} = 52{,}700$ |
| $EG$ | $262{,}000 \times \frac{5}{36} = 36{,}400$ |
| $GI$ | $267{,}300 \times \frac{2}{36} = 14{,}900$ |

The shears are as follows:

| Panel | Shear, in Pounds | Panel | Shear, in Pounds |
|---|---|---|---|
| $ab$ | 162,000 | $ef$ | 75,600 |
| $bc$ | 140,400 | $fg$ | 54,000 |
| $cd$ | 118,800 | $gh$ | 32,400 |
| $de$ | 97,200 | $hi$ | 10,800 |

The vertical components of the stresses in the short diagonals are as follows.

| Member | Vertical Component of Stress, in Pounds |
|---|---|
| $Bc$ | $21{,}600 \times \dfrac{16.5}{33} = 10{,}800$ |
| $DE$ | $21{,}600 \times \dfrac{24.5}{41} = 12{,}900$ |
| $FG$ | $21{,}600 \times \dfrac{25\ 5}{46} = 12{,}000$ |
| $HI$ | $21{,}600 \times \frac{25}{48} = 11{,}300$ |

The dead-load stresses, in pounds, in the main diagonals are as follows:

Dead-load stress in $aB$,

$$162{,}000 \times \frac{24.42}{16.5} = 162{,}000 \times 1.48 = +\,239{,}800$$

Dead-load stress in $BC$,

$$(140{,}400 + 10{,}800) \times \frac{24.42}{16.5} = 151{,}200 \times 1.48 = +223{,}800$$

Dead-load stress in $CD$,

$$(118{,}800 - 52{,}700) \times \frac{24.42}{16.5} = 66{,}100 \times 1.48 = -97{,}800$$

Dead-load stress in $De$,

$$(97{,}200 - 52{,}700 + 12{,}900) \times \frac{24.42}{16.5} = 57{,}400 \times 1.48$$
$$= -85{,}000$$

Dead-load stress in $EF$,

$$(75{,}600 - 36{,}400) \times \frac{27.28}{20.5} = 39{,}200 \times 1.33 = -52{,}100$$

Dead-load stress in $Fg$,

$$(54{,}000 - 36{,}400 + 12{,}000) \times \frac{27.28}{20.5} = 29{,}600 \times 1.33$$
$$= -39{,}400$$

Dead-load stress in $GH$,

$$(32{,}400 - 14{,}900) \times \frac{29.21}{23} = 17{,}500 \times 1.27 = -22{,}200$$

Dead-load stress in $Hi$,

$$(10{,}800 - 14{,}900 + 11{,}300) \times \frac{29.21}{23} = 7{,}200 \times 1.27 = -9{,}100$$

3 *Long Verticals.*—The stresses in the long verticals are found by considering the lower joint of each vertical as a free body. They are as follows:

| Member | Dead-Load Stress, in Pounds |
|---|---|
| $Cc$ | $10{,}800 + 14{,}400 = -25{,}200$ |
| $Ec$ | $57{,}400 - 14{,}400 = +43{,}000$ |
| $Gg$ | $29{,}600 - 14{,}400 = +15{,}200$ |
| $Ii$ | $7{,}200 - 14{,}400 + 7{,}200 = 0$ |

The fact that the stress in the middle vertical $Ii$ is equal to zero, when there is no live load on the truss, is not a general property of the Petit truss; it occurs in this case simply because the sum of the vertical components of the stresses in the two diagonals $Hi$ and $iH'$ that meet at the lower joint of the vertical, is equal to the dead-panel load at its lower joint

**42. Live-Load Web Stresses.**—1. The maximum live-load stresses in the hip vertical, short verticals, and short diagonals are as follows

| Member | | Maximum Live-Load Stress, in Pounds |
|---|---|---|
| $Bb$, $Dd$, $Ff$, $Hh$ | | $-27{,}000$ |
| $Cc$ | $27{,}000 + 13{,}500 =$ | $-40{,}500$ |
| $Bc$ | $27{,}000 \times \frac{24.42}{33} =$ | $+20{,}000$ |
| $DE$ | $27{,}000 \times \frac{30\ 40}{41} =$ | $-20{,}000$ |
| $FG$ | $27{,}000 \times \frac{31.21}{46} =$ | $-18{,}300$ |
| $HI$ | $27{,}000 \times \frac{30\ 81}{48} =$ | $-17{,}300$ |

2. *Main Diagonals* —The maximum live-load stresses in $aB$, $CD$, $EF$, and $GH$ occur when the live load extends from the right end up to the joints $b$, $d$, $f$, and $h$, respectively. The stress in $aB$ may be found directly from the reaction for full load; the stresses in the other members may be found by the method of moments. The upper-chord members $CE$, $EG$, and $GI$ produced intersect the lower chord produced at points distant 6 25, 12 4, and 40 panel lengths to the left of $a$, respectively.

In calculating the left reaction due to a partial live load on the truss, it is well to note that the sum of all whole numbers from 1 to $n$ is equal to $\frac{n(n+1)}{2}$; for example, if all the joints from $b'$ to $h$ are loaded, $n = 9$; and the sum $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = \frac{9 \times (9+1)}{2} = 45$ Then, in finding the left reaction for this loading, the expression may be written

$$\frac{27{,}000 \times \frac{9 \times (9+1)}{2}}{16}$$

instead of $\frac{27{,}000 \times (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9)}{16}$

The stresses in $aB$, $CD$, $EF$, and $GH$ are as follows

| Member | Maximum Live-Load Stress, in Pounds |
|---|---|
| $aB$ | $202{,}500 \times \frac{24.42}{16.5} = +299{,}700$ |
| $CD$ | $\dfrac{27{,}000 \times \frac{13 \times (13+1)}{2}}{16} \times \frac{6.25}{10.25} \times \frac{24.42}{16.5} = -138{,}600$ |
| $EF$ | $\dfrac{27{,}000 \times \frac{11 \times (11+1)}{2}}{16} \times \frac{12.4}{18.4} \times \frac{27.28}{20.5} = -\ 99{,}900$ |
| $GH$ | $\dfrac{27{,}000 \times \frac{9 \times (9+1)}{2}}{16} \times \frac{40}{48} \times \frac{29.21}{23} = -\ 80{,}400$ |

The maximum stresses in $BC$, $De$, $Fg$, and $Hi$ occur when the right end of the truss is loaded, the live load will extend at least as far as $c$, $e$, $g$, and $i$, respectively, whether or not $b$, $d$, $f$, and $h$ must be loaded will be determined by trial. When the correct loading for any member has been found, the stress may be calculated by multiplying csc $H$ by the vertical component of the stress found by applying the equation $\Sigma Y = \Sigma S \sin H = 0$ to all the forces that meet at the intermediate joint at the end of the member, or to the stresses in all the members cut by a vertical plane of section that intersects the member under consideration The former method is shorter for $De$, $Fg$, and $Hi$, the latter for $BC$.

For the member $BC$, a panel load will be tried at $b$. This load alone would cause a negative shear in panel $bc$ equal to 1,700 pounds, and compression in $Bc$, the vertical component of which would be 13,500 pounds Then, the load at $b$ would cause a stress in $BC$ equal to

$$(13{,}500 - 1{,}700) \times \csc H = +11{,}800 \csc H$$

As this is compression, the maximum compression will occur when the truss is fully loaded For this loading, the stress in $BC$ is

$$(175{,}500 + 13{,}500) \times \frac{24.42}{16.5} = +279{,}700 \text{ pounds}$$

For the member $De$, a panel load will be tried at $d$. This load alone would cause tension in $CD$ and $DE$, the vertical components of which are:

$$\frac{27{,}000 \times 13}{16} \times \frac{6.25}{10.25} = 13{,}400 \text{ in } CD$$

and
$$27{,}000 \times \frac{24.5}{41} = 16{,}100 \text{ in } DE$$

Applying the equation $\Sigma Y = \Sigma S \sin H = 0$ to the forces acting at the joint $D$, and multiplying the result by csc $H$ gives $-$ 2,500 csc $H$ for the stress in $De$ that would be caused by a load at $d$. As this is tension, the maximum tension in $De$ will occur when the truss is loaded from the right end up to $d$. For this loading, the vertical component in $CD$ is

$$27{,}000 \times \frac{\dfrac{13 \times (13+1)}{2}}{16} \times \frac{6\ 25}{10\ 25} = 93{,}600 \text{ pounds}$$

Applying the equation $\Sigma Y = \Sigma S \sin H = 0$ to the forces acting at the joint $D$, and multiplying the result by csc $H$, gives, for the maximum stress in $De$,

$$(93{,}600 + 16{,}100 - 27{,}000) \times \frac{24.42}{16.5} = -\ 122{,}400 \text{ pounds}$$

For the member $Fg$, a panel load will be tried at $f$. This load alone would cause tension in $EF$ and $FG$, the vertical components of which are:

$$\frac{27{,}000 \times 11}{16} \times \frac{12.4}{18.4} = 12{,}500 \text{ in } EF$$

and
$$27{,}000 \times \frac{25.5}{46} = 15{,}000 \text{ in } FG$$

Applying the equation $\Sigma Y = \Sigma S \sin H = 0$ to the forces acting at joint $F$, and multiplying the result by csc $H$, gives $-$ 500 csc $H$ for the stress in $Fg$ that would be caused by a load at $f$. As this is tension, the maximum tension in $Fg$ will occur when the truss is loaded from the

right end up to $f$. For this loading, the vertical component in $EF$ is

$$27{,}000 \times \frac{\frac{11 \times (11 + 1)}{2}}{16} \times \frac{12.4}{18.4} = 75{,}100 \text{ pounds}$$

and the stress in $Fg$ is

$$(75{,}100 + 15{,}000 - 27{,}000) \times \frac{27.28}{20.5} = -84{,}000 \text{ pounds}$$

For the member $Hi$, a panel load will be tried at $h$. This load alone would cause tension in $GH$ and $HI$, the vertical components of which are

$$\frac{27{,}000 \times 9}{16} \times \tfrac{40}{48} = 12{,}700 \text{ in } GH$$

and
$$27{,}000 \times \tfrac{25}{48} = 14{,}100 \text{ in } HI$$

Applying the equation $\Sigma Y = \Sigma S \sin H = 0$ to all the forces acting at the joint $H$, and multiplying the result by $\csc H$, gives $+ 200 \csc H$ for the stress in $Hi$ that would be caused by a load at $h$. As this is compression, the maximum tension in $Hi$ will occur when the truss is loaded from the right end up to $i$. For this loading, the vertical component in $GH$ is

$$27{,}000 \times \frac{\frac{8 \times (8 + 1)}{2}}{16} \times \tfrac{40}{48} = 50{,}600 \text{ pounds}$$

and the stress in $Hi$ is

$$50{,}600 \times \frac{29.21}{23} = -64{,}300 \text{ pounds}$$

3. *Main Verticals*.—The maximum stresses in the verticals $Ee$, $Gg$, and $Ii$ occur when the live load extends from the right end up to $f$, $h$, and $h'$, respectively. When the live load extends up to $f$, there is no load at $d$ and the live-load stress in $DE$ is zero. Then, if the truss is considered cut by a plane that intersects $CE$, $DE$, $Ee$, and $ef$, there will be but three unknown stresses, and the stress in $Ee$ may be found by taking moments about the intersection of $CE$ and $ef$. In like manner, the stress in $Gg$ may be found by taking moments about the intersection of $EG$ and $gh$, and that

in $Ii$ by taking moments about the intersection of $GI$ and $ih'$. The results are as follows:

| MEMBER | MAXIMUM LIVE-LOAD STRESS, IN POUNDS |
|---|---|
| $Ee$ | $\dfrac{27{,}000 \times \dfrac{11 \times (11+1)}{2}}{16} \times \dfrac{6.25}{10.25} = +67{,}900$ |
| $Gg$ | $\dfrac{27{,}000 \times \dfrac{9 \times (9+1)}{2}}{16} \times \dfrac{12.4}{18.4} = +51{,}200$ |
| $Ii$ | $\dfrac{27{,}000 \times \dfrac{7 \times (7+1)}{2}}{16} \times \frac{40}{48} = +39{,}400$ |

The dead-load stress in the vertical $Ii$ when there is no live load on the truss—that is, when both main diagonals $Hi$ and $iH'$ are in action—was found in 3, Art. **41**, to be zero. When the live load extends from $h'$ to $b'$, the position that causes the greatest live-load compression in $Ii$, the main diagonal $iH'$ is out of action, and it is necessary to find the dead-load counter stress in $Ii$. This stress can be most easily found by considering the joint $i$. The vertical dead-load forces that act at the joint $i$ are now the dead panel load of 14,400 pounds, the vertical component of the dead load in $Hi$, 7,200 pounds (2, Art. **41**), and the stress in $Ii$, the latter is 7,200 pounds, tension. Then, the combined stress in $Ii$ when the live load extends from $h'$ to $b'$ is 39,400 − 7,200 = 32,200 pounds, compression.

**43. Counters and Minimum Stresses in Main Diagonals.**—To find the panels in which counters are required, the minimum combined stress will be calculated in each main diagonal, starting at the left end of the truss. The minimum live-load stresses in $aB$ and $BC$ are each equal to zero; then, the minimum stresses are the dead-load stresses.

The minimum live-load stresses in $CD$ and $De$ occur when the joints $b$ and $c$ are loaded; it is evident that a load at $d$ will cause tension in $CD$, and it was shown in Art. **42** that

a load at $d$ causes tension in $De$; then, this joint and all to the right of it must be unloaded. As there is no load at $d$, the live-load stresses in $Dd$ and $DE$ are equal to zero, and the live-load stress in $CD$ is equal to the live-load stress in $De$. Considering the truss cut by a plane that intersects $CE$, $CD$, and $cd$, and applying the equation $\Sigma M = 0$ to the forces acting on the part of the truss to the right of this plane, taking moments about the intersection of $CE$ and $cd$, gives the minimum live-load stress in $CD$ and $De$ as follows:

$$\frac{27{,}000 \times (1+2)}{16} \times \frac{22.25}{10.25} \times \frac{24.42}{16.5} = +16{,}300 \text{ pounds}$$

The dead-load stresses in $CD$ and $De$ are equal, respectively, to $-97{,}800$ and $-85{,}000$ pounds (2, Art. **41**). Then, the minimum combined stresses in $CD$ and $De$ are as follows:

| Member | Minimum Combined Stress, in Pounds |
|---|---|
| $CD$ | $-97{,}800 + 16{,}300 = -81{,}500$ |
| $De$ | $-85{,}000 + 16{,}300 = -68{,}700$ |

As these are both tension, no counters are required in the panels $cd$ and $de$.

The minimum live-load stresses in $EF$ and $Fg$ occur when the joints $b$, $c$, $d$, and $e$ are loaded; it is evident that a load at $f$ will cause tension in $EF$, and it was shown in Art. **42** that a load at $f$ causes tension in $Fg$; then, this joint and all to the right of it must be unloaded. As there is no load at $f$, the live-load stresses in $fF$ and $FG$ are equal to zero, and the live-load stress in $EF$ is equal to the live-load stress in $Fg$. Considering the truss cut by a plane that intersects $EG$, $EF$, and $ef$, and applying the equation $\Sigma M = 0$ to the forces acting on the part of the truss to the right of this plane, taking moments about the intersection of $EG$ and $ef$, gives the minimum live-load stresses in $EF$ and $Fg$ as follows:

$$\frac{27{,}000 \times (1+2+3+4)}{16} \times \frac{28.4}{18.4} \times \frac{27\ 28}{20.5} = +34{,}700 \text{ pounds}$$

The dead-load stresses in $EF$ and $Fg$ are equal, respectively, to $-$ 52,100 and $-$ 39,400 pounds (2, Art. **41**). Then, the minimum combined stresses in $EF$ and $Fg$ are as follows:

| Member | Minimum Combined Stress, in Pounds |
|---|---|
| $EF$ | $-52{,}100 + 34{,}700 = -17{,}400$ |
| $Fg$ | $-39{,}400 + 34{,}700 = -\ \ 4{,}700$ |

As these are both tension, no counters are required in the panels $ef$ and $fg$.

The minimum combined stress, $-$ 4,700 pounds, in $Fg$, is so small, and the minimum combined stresses in the main diagonals decrease so rapidly as the center is approached, that it is evident that the minimum combined stress in $GH$ will come out compression. Then, $gH$ acts as the lower half of the counter $gHI$; $GH$ acts as a short diagonal or subtie; and $Hi$ is out of action, as represented in Fig. 16 ($c$). As the lower and upper halves, $gH$ and $HI$, of the counter $gHI$ are not in the same straight line, the stress in the subtie cannot be found by the principle explained in Art. **41**, but requires separate consideration. Applying the equations $\Sigma X = \Sigma S \cos H = 0$ and $\Sigma Y = \Sigma S \sin H = 0$ to all the forces acting at the joint $H$, as represented in Fig. 16 ($d$), letting $W$ represent the sum of the loads at $H$ and $h$, and assuming that the stresses in $gH$, $GH$, and $HI$ are tension, the following equations are obtained:

$$\Sigma Y = S_3 \sin H_3 - S_2 \sin H_2 + S_1 \sin H_1 - W = 0$$
$$\Sigma X = S_3 \cos H_3 + S_2 \cos H_2 - S_1 \cos H_1 \qquad = 0$$

Substituting for the trigonometric functions their algebraic values, and making the proper reductions, the following values of the vertical components of $S_2$ and $S_3$, in terms of the vertical components of $S_1$ and $W$, are obtained:

$$S_{2y} = S_2 \times \frac{h_2}{l_2} = S_{1y} \times \frac{\frac{h_1 + h_2}{2}}{h_1} - \frac{W}{2} = S_{1y} \times \frac{h}{2\,h_1} - \frac{W}{2} \quad (1)$$

$$S_{3y} = S_3 \times \frac{h_3}{l_3} = \frac{W}{2} - S_{1y} \times \frac{\frac{h_1 - h_2}{2}}{h_1} = \frac{W}{2} - S_{1y} \times \frac{h - h'}{2\,h_1} \quad (2)$$

The stress $S_1$ may be found most readily by the method of moments, taking as the center of moments the intersection of $GI$ and $hi$ produced, which is at a distance of 40 panel lengths to the left of $a$, and considering $S_1$ to be resolved into its vertical and horizontal components at the intersection of its line of action $HI$ with the lower chord, this intersection is at a distance equal to $\frac{23}{25}$, or 92, panel lengths to the left of $h$, or 46.08 panel lengths to the right of the center of moments. The dead-load counter stresses, in pounds, in $gH$, $GH$, and $HI$ are as follows:

Stress in $HI$,

$$\frac{-162{,}000 \times 40 + 21{,}600 \times (41 + 42 + 43 + 44 + 45 + 46 + 47)}{46.08} \times \frac{30.81}{25} = +4{,}600$$

Stress in $GH$,

$$(10{,}800 - 3{,}750 \times \tfrac{2}{50}) \times \frac{29.21}{23} = 13{,}500$$

Stresses in $gH$,

$$(3{,}750 \times \tfrac{48}{50} - 10{,}800) \times \frac{29.21}{23} = -9{,}100$$

In writing these equations, it was assumed that the stresses in $HI$, $GH$, and $gH$ were tension; the negative result in $H$ indicates that the dead-load counter stress in that member is compression.

The maximum live-load stress in $HI$, when it is a counter, occurs when the live load extends from $b$ to $h$, and is as follows:

$$\frac{27{,}000 \times \frac{7 \times (7 + 1)}{2}}{16} \times \frac{56}{46.08} \times \frac{30.81}{25} = -70{,}800 \text{ pounds}$$

For the maximum tension in $gH$, it is evident that the live load should extend from the left end at least as far as joint $g$; whether or not joint $h$ should be loaded will be determined by trial. A live panel load at $h$ alone would cause a tension in $HI$, the vertical component of which is equal to

$$27{,}000 \times \tfrac{7}{16} \times \frac{56}{46.08} = 14{,}400 \text{ pounds}$$

and a stress in $gH$, the vertical component of which is found by applying formula **1** and is equal to

$$14{,}400 \times \tfrac{48}{60} - 13{,}500 = 300 \text{ pounds}$$

As this comes out positive, the stress in $gH$ will be tension, and for the maximum tension in $gH$ the joints $b$ to $h$ should be loaded. For this loading, the live-load stress in $gH$ is equal to

$$(57{,}400 \times \tfrac{48}{60} - 13{,}500) \times \frac{29.21}{23} = 41{,}600 \times 1.27$$
$$= -52{,}800 \text{ pounds}$$

The maximum live-load stress in $GH$ as a main diagonal was found in Art. **42.** Now, if the truss is loaded from $b$ to $g$, the vertical component of the stress in $HI$ is equal to

$$\frac{27{,}000 \times \frac{6 \times (6+1)}{2}}{16} \times \frac{56}{46.08} = 43{,}100 \text{ pounds}$$

and the minimum live-load stress in $GH$ is equal to

$$(0 - 43{,}100 \times \tfrac{2}{60}) \times \frac{29.21}{23} = -2{,}200 \text{ pounds}$$

As this comes out negative, the minimum live-load stress in $GH$ is compression.

**44. Minimum Stresses in the Verticals.**—The minimum live-load stresses in the hip vertical $Cc$, and in the short verticals $Bb$, $Dd$, $Ff$, and $Hh$, are equal to zero The minimum live-load stresses in the main verticals $Ee$, $Gg$, and $Ii$ are tension; the loadings that cause the minimum stresses in these members are determined by trial.

*Vertical Ee*—It is first necessary to consider the loading that will probably cause the maximum tension in the vertical $Ee$ In Art **42,** it was found that the maximum compression occurs when all the joints from $b'$ to $f$ are loaded; therefore, for the maximum tension these joints should be unloaded. In finding the stress in $Ee$ due to loads at the left end, the truss may be considered cut by a plane that intersects $CE$, $DE$, $Ee$, and $ef$. Live panel loads at $b$ and $c$ cause a right reaction equal to $\frac{27{,}000 \times (1+2)}{16}$. Then,

taking moments, about the intersection of $CE$ and $ef$, of all the forces acting on the part of the truss to the right of the plane just mentioned (the stress in $DE$ being equal to zero), the stress in $Ee$ due to live loads at $b$ and $c$ only is

$$\frac{27{,}000 \times (1 + 2)}{16} \times \frac{22\ 25}{10\ 25} = -11{,}000 \text{ pounds}$$

Now, if any more joints to the left of $e$ are loaded, it will be seen that a panel load at $d$ tends to cause compression, and at $e$ tension, in $Ee$; the stress that would be caused by both of these together will be found, as it will be assumed impossible to have a full live load at $e$ and no load at $d$. It will be convenient to apply the equation $\Sigma Y = 0$ to all the forces acting on the part of the truss to the right of the plane of section just mentioned.

Thus, the right reaction, acting upwards, is

$$\frac{27{,}000 \times (3 + 4)}{16} = 11{,}800 \text{ pounds}$$

The vertical component in $CE$, acting upwards, is

$$\frac{27{,}000 \times (12 + 13)}{16} \times \frac{4 \times 18}{41} \times \tfrac{8}{36} = 16{,}500 \text{ pounds}$$

The vertical component in $DE$, acting downwards, is

$$27{,}000 \times \frac{24.5}{41} = 16{,}100 \text{ pounds}$$

Then, the live-load stress in $Ee$ due to loads at $d$ and $e$ is equal to

$$11{,}800 + 16{,}500 - 16{,}100 = 12{,}200 \text{ pounds, tension}$$

Adding this value to the tension due to loads at $b$ and $c$ gives 23,200 pounds for the maximum live-load tension in $Ee$. The dead-load stress in $Ee$ is equal to $+43{,}000$ pounds; then, the minimum combined stress is

$$43{,}000 - 23{,}200 = +19{,}800 \text{ pounds}$$

*Vertical Gg.*—It was shown in Art **42** that, for the maximum compression in $Gg$, the joints $h$ to $b'$ must be loaded. Then, for the maximum tension, these joints must be unloaded. There are two conditions when the joints at the left end of the truss are loaded that need to be considered; namely, when $GH$ is in action as a main diagonal,

d when $gH$ is in action as a counter. The vertical com-
nent of the dead-load tension in $GH$ as a main diagonal is
ual to 17,500 pounds. Live panel loads at $b, c, d$, etc.
crease the dead-load tension in $GH$ until the live-load
mpression in $GH$ is greater than the dead-load tension,
ien $gH$ comes into action as a counter. Denoting the
ht reaction for live loads at $b, c$, etc. by $R_2''$, and taking
ments about the intersection of $GI$ and $gh$, the vertical
nponent of the live-load compression in $GH$ is found to be
$\frac{''\times 56}{48}$. Placing this equal to 17,500, the vertical com-
nent of dead-load tension in $GH$, and substituting for $R_2''$

value $\dfrac{27{,}000 \times \dfrac{n(n+1)}{2}}{16}$, $n$ being the number of joints
ded at the left end, $n$ is found to be equal to 3.7. This
ans that, if three joints $b$, $c$, and $d$ are loaded, $GH$ will be
action as a main diagonal; if more than three are loaded,
$I$ will be in action as a counter. The live-load stress
$Gg$ due to loads at $b$, $c$, and $d$ may be found by taking
ments about the intersection of $EG$ and $gh$, the live-load
ess in $FG$ for this loading being equal to zero. Then, the
e-load stress in $Gg$ is

$$\frac{27{,}000 \times (1+2+3)}{16} \times \frac{28.4}{18\ 4} = 15{,}600 \text{ pounds, tension}$$

The dead-load stress in $Gg$ when $GH$ is in action as a
in diagonal is equal to 15,200 pounds, compression. Then,
minimum combined stress in $Gg$ for this loading is equal
15,600 − 15,200 = 400 pounds, tension.

When the joints $b$, $c$, $d$, and $e$ are loaded with live load,
$I$ is in action as a counter, and the live-load stress in $Gg$ is
und by considering the forces that act at joint $g$. They are
stresses in $Fg$, $Gg$, and $gH$. The vertical component of
stress in $Fg$ is found by taking the moment of the
ht reaction about the intersection of $EG$ and $fg$, and
equal to

$$\frac{000 \times (1+2+3+4)}{16} \times \frac{28.4}{18.4} = 26{,}000 \text{ pounds, compression}$$

The vertical component of the live-load stress in $gH$ is obtained from formula **1**, Art. **43**, first finding the stress in $HI$ for this loading. The vertical component of the stress in $gH$ is

$$\tfrac{48}{60} \times \frac{27{,}000 \times (1 + 2 + 3 + 4)}{16} \times \frac{56}{46.08}$$
$$= 19{,}700 \text{ pounds}$$

Then, the live-load stress in $Gg$ is 26,000 − 19,700 = 6,300 pounds, tension. If the live load is applied to any joints at the right of $e$ the tension in $Gg$ will decrease.

When the counter $gH$ is in action, the dead-load stress in $Gg$ may be found by considering the forces that act at the joint $g$. The vertical component of the dead-load stress in $Fg$ is equal to 29,600 pounds (Art. **41**), while that in $gH$ is equal to 7,200 pounds (Art **43**) The dead panel load at $g$ is equal to 14,400 pounds Then, the dead-load stress in $Gg$ is 29,600−7,200−14,400=8,000 pounds, compression The minimum combined stress in $Gg$ for this loading is 8,000 −6,300= +1,700 pounds As this is greater than the stress when the joints $b$, $c$, and $d$ are loaded, the minimum combined stress in $Gg$ is equal to 400 pounds, tension

*Vertical $Ii$.*—The maximum live-load tension in $Ii$ occurs when the truss is fully loaded. It may be found most easily by considering the forces that meet at the joint $I$. The vertical component of the stress in $GI$, or $G'I$, may be found from the dead-load stress given in Art. **41**, by multiplying the latter by the ratio of the live to the dead load, or $\frac{5}{4}$. The vertical component of the dead-load stress is equal to + 14,900 pounds, then, the vertical component of the live-load stress is equal to 14,900 × $\frac{5}{4}$ = + 18,600 pounds.

The vertical component of the live-load stress in $HI$ is equal to 27,000 × $\frac{24}{46}$ = 14,100 pounds. Then, the stress in $Ii$ is

$$2 \times 18{,}600 - 2 \times 14{,}100 = 9{,}000 \text{ pounds, tension}$$

As the dead-load stress in $Ii$ is equal to zero, the minimum combined stress is also equal to 9,000 pounds, tension.

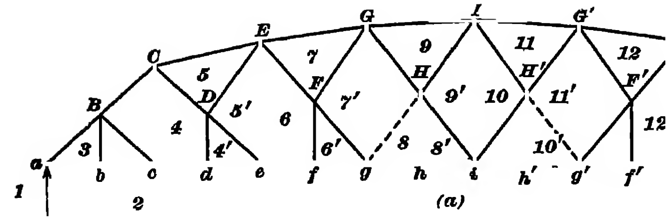
**45. Table of Stresses.**—The maximum and minimum combined stresses are given in the table on page 63.

| Member | Maximum Live-Load Stress | Minimum Live-Load Stress | Dead-Load Stress | Dead-Load Counter Stress | Maximum Combined Stress | Minimum Combined Stress |
|---|---|---|---|---|---|---|
| *Bb*, etc. | − 27,000 | | − 14,400 | | − 41,400 | − 14,400 |
| *Bc* | + 20,000 | | + 16,000 | | + 36,000 | + 16,000 |
| *DE* | − 20,000 | | − 16,000 | | − 36,000 | − 16,000 |
| *FG* | − 18,300 | | − 14,700 | | − 33,000 | − 14,700 |
| *HI* | − 70,800 | | | − 4,600 | − 75,400 | |
| *HI* | − 17,300 | | − 13,900 | | − 31,200 | − 13,900 |
| *aB* | + 299,700 | | + 239,800 | | + 539,500 | + 239,800 |
| *BC* | + 279,700 | | + 223,800 | | + 503,500 | + 223,800 |
| *CD* | − 138,600 | + 16,300 | − 97,800 | | − 236,400 | − 81,500 |
| *De* | − 122,400 | + 16,300 | − 85,000 | | − 207,400 | − 68,700 |
| *EF* | − 99,900 | + 34,700 | − 52,100 | | − 152,000 | − 17,400 |
| *Fg* | − 84,000 | + 34,700 | − 39,400 | | − 123,400 | − 4,700 |
| *GH* | − 80,400 | | − 22,200 | | − 102,600 | |
| *GH* | | + 2,200 | | − 13,500 | | − 11,300 |
| *Hi* | − 64,300 | | − 9,100 | | − 73,400 | |
| *gH* | − 52,800 | | | + 9,100 | − 43,700 | |
| *Ce* | − 40,500 | | − 25,200 | | − 65,700 | − 25,200 |
| *Ee* | + 67,900 | − 23,200 | + 43,000 | | + 110,900 | + 19,800 |
| *Gg* | + 51,200 | − 15,600 | + 15,200 | | + 66,400 | − 400 |
| *Ii* | + 39,400 | − 9,000 | | − 7,200 | + 32,200 | − 9,000 |
| *ab*, *bc* | $\frac{5}{4}$ × 176,700 | | − 176,700 | | − 397,600 | − 176,700 |
| *cd*, *de* | $\frac{5}{4}$ × 164,900 | | − 164,900 | | − 371,000 | − 164,900 |
| *ef*, *fg* | $\frac{5}{4}$ × 227,600 | | − 227,600 | | − 512,100 | − 227,600 |
| *gh*, *hi* | $\frac{5}{4}$ × 253,600 | | − 253,600 | | − 570,600 | − 253,600 |
| *CE* | $\frac{5}{4}$ × 242,900 | | + 242,900 | | + 546,500 | + 242,900 |
| *EG* | $\frac{5}{4}$ × 264,500 | | + 264,500 | | + 595,100 | + 264,500 |
| *GI* | $\frac{5}{4}$ × 267,700 | | + 267,700 | | + 602 300 | + 267,700 |

## GRAPHIC METHOD

**46. Dead-Load Stresses.**—The stress diagram for dead-load stresses is represented in Fig. 17 (*b*). In constructing the force polygon (the load line), the work was shortened somewhat by assuming that the dead load that is applied at *B*, *D*, *F*, etc. was applied at *b*, *d*, *f*, etc., respectively, together with the dead load already there. Then, the external forces acting on the truss are the reactions, equal to 162,000 pounds each, the partial loads at the upper-chord joints, 7,200 pounds each, at *C*, *E*, *G*, etc., the full panel loads of 21,600 pounds each, at *b*, *d*, *f*, *h*, etc., and the loads of 14,400 pounds each at *c*, *e*, *g*, *i*, etc., as represented in Fig. 17 (*a*). The result of assuming that the dead loads at *B*, *D*, *F*, etc. are carried at *b*, *d*, *f*, etc is that the stress diagram indicates a stress in each short vertical 7,200 pounds greater than the actual stress, and this must be kept in mind when these stresses are scaled from the stress diagram. The stresses in the other members are not affected by the assumption. The points *38* and *40* in the stress diagram coincide, indicating that the stress in *I i* (*38–40*) is equal to zero; this checks with the value of the dead-load stress in *I i*, found in Art. **41** by the analytic method

In finding the counter stresses in the panels *g h* and *h i*, the stress diagram was drawn for the panels *i h′* and *h′ g′* instead; *38–40′* is the vector representing the stress in *I i* when *H′ i* is out of action; *40′–44′* and *42′–43′* are the stresses in *H′ I* and *g′ H′*, respectively, as counters; *44′–43′* is the stress in *H′ G′* as a short diagonal, and *43′–45′* is the stress in *G′ g′*. In drawing the vector *43′–45′*, the diagram was drawn for joint *G′*. At this joint, *21–44′* and *44′–43′* were known; *43′–45′*, *45′–46*, and *46–20* were unknown. As there were three unknown forces, it was impossible to draw these three vectors directly. It was known, however, that the stress in *G′ E′* is equal to the stress in *G E*, which had already been found; then a line was drawn from *20* parallel to *20–46*, and the point *46* located so that *20–46* was equal to *23–33*, and, by drawing from *46* a line parallel to *45′–46*,


a
b
c
d
e
f
g
h
i
h′
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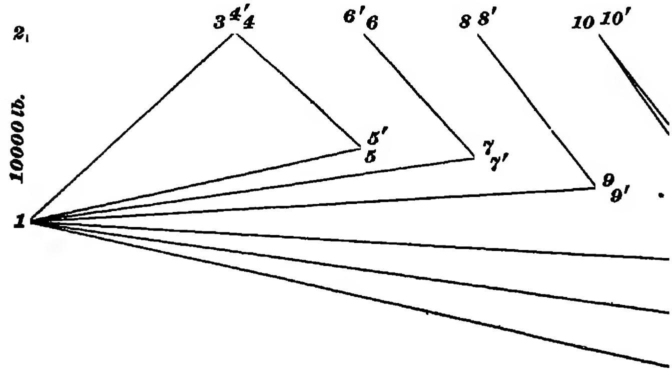
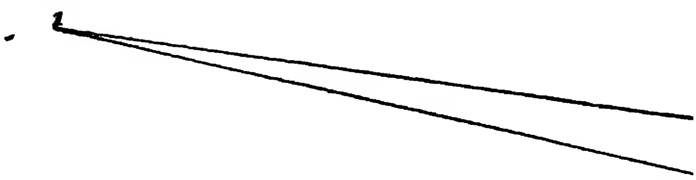
12

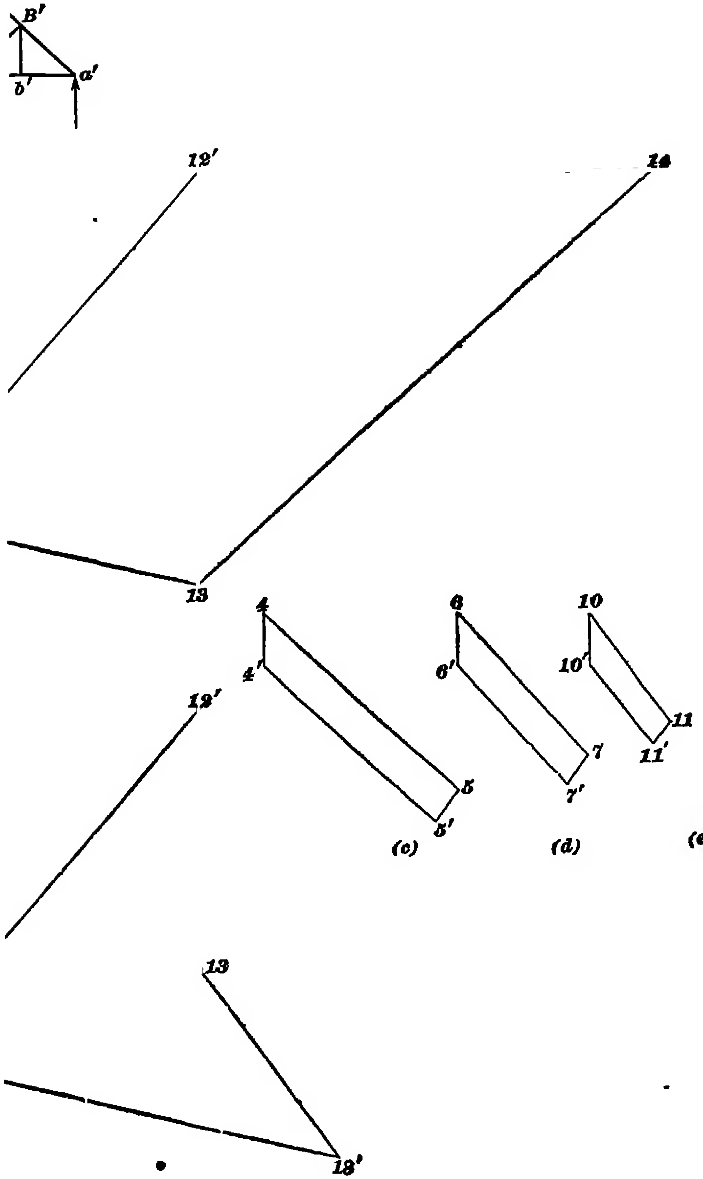
(a)


10000 lb.
1
2
3 4′ 4
5′
5
6′ 6
7
7′
8 8′
9
9′
10 10′

(b)


1

(f)


B'
b'
a'
12'
14
13
4
4'
6
6'
10
10'
11
11'
7
7'
5
5'
12'
(c)
(d)
(e)
13
13'

and from *43'* a line parallel to *43'–45'*, the point *45'* was found. It is unnecessary in the present case to scale the dead-load stresses, since they will be the same as found by the analytic method.

**47. Live-Load Chord Stresses.**—The chord stresses may be found by drawing a stress diagram for one-half the truss when there is a full live load, as represented in Fig. 18 (*b*), or by multiplying the dead-load chord stresses, scaled from Fig. 17 (*b*), by the ratio of the live to the dead load In the present case, the former method is preferable, as the stress diagram represented in Fig. 18 (*b*) gives also the desired stresses in some of the web members.

**48. Live-Load Web Stresses.**—The stresses in the end posts, hip vertical, short verticals, and short diagonals may be scaled directly from the stress diagram represented in Fig. 18 (*b*). The maximum and minimum live-load stresses in the main diagonals and remaining verticals may be found by drawing a stress diagram for the truss, from the left end up to the member $C'c'$, assuming that the only force acting on the left end of the truss is a reaction at $a$ equal to 10,000 pounds, as represented in Fig. 19 (*b*). In this diagram, the vectors representing the stresses in the short verticals and short diagonals do not appear; this is due to the fact that, if no loads act on the left end of the truss except the left reaction, there will be no panel loads at $b$, $d$, etc , and consequently the stresses in $Bb$ and $Bc$, $Dd$ and $DE$, etc will be equal to zero.

In finding the stresses in the main diagonals and verticals, it is best to find by the analytic method the loading that causes the desired stress. As these loadings have already been found in Art **42**, the work will not be repeated. When the live-load stress in one of the verticals $Ee$, $Gg$, and $Ii$, or in the upper half of one of the main diagonals $CD$, $EF$, and $GH$, or in $IH'$ as a counter, is a maximum, the only force that acts on the truss to the left of the member is the left reaction. Then, the maximum live-load stress in any one of these members may be found by multiplying the stress in

that member due to a left reaction of 10,000 pounds, as scaled from Fig. 19 (*b*), by the ratio of the left reaction caused by the loading that causes the maximum stress in the member under consideration to the assumed reaction of 10,000 pounds. The stresses in the members just referred to, caused by a left reaction of 10,000 pounds, are as follows

| MEMBER | STRESS, IN POUNDS | VECTOR |
|---|---|---|
| $Ee$ | + 6,100 | 5′– 6 |
| $Gg$ | + 6,700 | 7′– 8 |
| $Ii$ | + 8,300 | 9′–10 |
| $CD$ | – 9,000 | 4 – 5 |
| $EF$ | – 9,000 | 6 – 7 |
| $GH$ | – 10,600 | 8 – 9 |
| $IH'$ | – 15,000 | 10 –11 |

The left reactions caused by the loadings that cause maximum stresses in these members (as found by the analytic method in Art. 42) are as follows.

| MEMBER | LEFT REACTION, IN POUNDS | MEMBER | LEFT REACTION, IN POUNDS |
|---|---|---|---|
| $Ee$ | 111,400 | $EF$ | 111,400 |
| $Gg$ | 75,900 | $GH$ | 75,900 |
| $Ii$ | 47,300 | $IH'$ | 47,300 |
| $CD$ | 153,600 | | |

Then, the maximum live-load stresses in these members are as follows:

| MEMBER | MAXIMUM LIVE-LOAD STRESS, IN POUNDS |
|---|---|
| $Ee$ | $\frac{111400}{10000} \times 6{,}100 = +\ 68{,}000$ |
| $Gg$ | $\frac{75900}{10000} \times 6{,}700 = +\ 50{,}900$ |
| $Ii$ | $\frac{47300}{10000} \times 8{,}300 = +\ 39{,}300$ |
| $CD$ | $\frac{153600}{10000} \times 9{,}000 = -\ 138{,}200$ |
| $EF$ | $\frac{111400}{10000} \times 9{,}000 = -\ 100{,}300$ |
| $GH$ | $\frac{75900}{10000} \times 10{,}600 = -\ 80{,}500$ |
| $IH'$ | $\frac{47300}{10000} \times 15{,}000 = -\ 71{,}000$ |

When the stress in the lower half of the main diagonal $Hi$ is a maximum, there is no load to the left of $i$ except the left reaction, and the stress in $Hi$ is equal to the stress

in $GH$ that has just been found. When the stress in the lower half of one of the main diagonals $De$ and $Fg$, or in $H'g'$ as a counter, is a maximum, the joints $d$, $f$, and $h'$, respectively, at the left of these members are loaded; and, as the left reaction is then not the only force acting on the truss at the left of one of these members, the stress cannot be found by the stress diagram in Fig. 19 (*b*). These are the same loadings, however, that cause the maximum stresses that have just been found in the upper halves of the diagonals, and the stress in the lower half of any one of them may be found by constructing the polygon for the forces that meet at an intermediate joint. For example, the stress in $CD$, equal to − 138,200 pounds, is laid off to a convenient scale, as shown in Fig. 19 (*c*) by the line *5–4*, which is parallel to $CD$; *4–4′*, equal, by scale, to the stress in $Dd$ (a live panel load), is next drawn parallel to $Dd$; then the lines *4′–5′* and *5–5′* are drawn parallel, respectively, to $De$ and $DE$: the line *4′–5′* represents the stress in $De$. In like manner, *6′–7′*, Fig. 19 (*d*), represents the stress in $Fg$, and *10′–11′*, Fig. 19 (*e*), represents the stress in $H'g'$.

The vector *11–11′*, Fig 19 (*b*), that represents the stress in $H'G'$ when there are no loads to the left of $g'$ except the left reaction of 10,000 pounds, scales + 600 pounds. The left reaction for the loading that causes the minimum live-load stress in $H'G'$ (Art 43) is equal to 35,400 pounds. Then, the minimum live-load stress in $H'G'$ as a short diagonal is equal to

$$\tfrac{35400}{10000} \times 600 = +\,2{,}100 \text{ pounds}$$

When the stress in one of the main diagonals $g'E'$ or $e'C'$ is a minimum, there is no force acting on the truss to the left of the member under consideration except the left reaction, and the stresses in these members may be found from Fig 19 (*b*) in the usual way The stresses due to a left reaction of 10,000 pounds are as follows

| Member | Stress, in Pounds | Vector |
|---|---|---|
| $g'E'$ | + 20,500 | *12–12′* |
| $e'C'$ | + 32,000 | *13–14* |

The left reactions that obtain when the stresses in these members have their minimum values are as follows (see Art. **43**):

| Member | Reaction, in Pounds |
|---|---|
| $g' E'$ | 16,900 |
| $e' C'$ | 5,100 |

Then, the minimum live-load stresses in these members are as follows:

| Member | Stress, in Pounds |
|---|---|
| $g' E'$ | $\frac{16900}{10000} \times 20{,}500 = +34{,}600$ |
| $e' C'$ | $\frac{5100}{10000} \times 32{,}000 = +16{,}300$ |

The minimum stress in $Ii$ occurs when the truss is fully loaded, and may be found from the stress diagram represented in Fig. 18 (*b*). It was found in Art. **44** that the minimum stress in $G'g'$ ($Gg$) occurs when the joints $b'$, $c'$, $d'$, and $e'$ are loaded and $g' H'$ is in action as a counter. Then, the minimum stress in $G'g'$ may be found from Fig. 19 (*b*). The vector *12–11'* scales − 3,700 pounds. The left reaction is equal to 16,900 pounds; then, the minimum live-load stress in $Gg$ is equal to

$$\frac{16900}{10000} \times 3{,}700 = -6{,}300 \text{ pounds}$$

In Art. **44**, it was found that the minimum live-load stress in $E' e'$ occurs when the joints $b'$, $c'$, $d'$, and $e'$ are loaded. In this case, the stress in $E' e'$ cannot be found from Fig 19 (*b*), as the vector *12'–13* was drawn on the assumption that the stress in $E' D'$ is zero; this assumption is not true in the present case, as there is a load at $d'$. The method of finding the minimum stress in $E' e'$, when the loading is known, is illustrated in Fig. 19 (*f*). The stresses in $G' E'$ and $g' E'$ are found by multiplying the vectors *1–12* and *12–12'*, respectively, Fig. 19 (*b*), by $\frac{16900}{10000}$; the stress in $E' D'$ is the same as the stress in $DE$, which is given by the vector *5'–5*, Fig. 19 (*c*). Then, as three of the five forces that meet at $E'$ are known, the other two, one of which is the stress in $E' e'$, may be found by constructing the force polygon for the joint $E'$. The vector *12'–13*, Fig. 19 (*f*), equal to − 23,200 pounds, is the minimum live-load stress in $E' e'$.

### EXAMPLES FOR PRACTICE

1. The sixteen-panel through Petit truss with substruts represented in Fig 15, having dimensions as shown, supports one-half of a bridge The dead load is 2,200, and the live load, 3,200, pounds per linear foot of bridge. If one-third of the dead load is assumed to be applied at the upper and intermediate joints, what is the maximum combined stress in the member $GH$? Ans $-$ 65,100 lb.

2 For the same truss and loading referred to in example 1, what is the minimum combined stress (maximum tension) in $Ii$? Ans $-$ 60,000 lb

3 For the same truss and loading referred to in example 1, what is the maximum combined stress in $Ee$? Ans $+$ 82,200 lb.

4. For the same truss and loading referred to in example 1, what is the live-load stress in the member $gh$? Ans $-$ 317,400 lb.

## THE MULTIPLE-SYSTEM CURVED-CHORD TRUSS

**49. Description.**—The truss represented in Fig. 20 (*a*) is known as the **multiple-system curved-chord truss.** It is somewhat similar to the Whipple truss (*Stresses in Bridge Trusses*, Part 2) having two systems of web members, the difference between the two being that the upper chord of one is curved and that of the other is horizontal. The multiple-system curved-chord truss is not in common use at the present time, but, as there are some examples of it in existence, it is well to know how the stresses can be found.

**50. Method of Calculation.**—If the truss represented in Fig. 20 (*a*) is separated into two systems, as shown in single and double lines in Fig 20 (*b*) and (*c*), it will be seen that, on account of the angles of the curved chord, each system exerts an upward force on each upper-chord joint of the other system On this account, the systems do not act independently, and the stresses cannot be found on the assumption that the stresses in each system are caused by the loads that come on that system, unless some other assumption is made.

When there is a full live load or no live load on the truss, the stresses in all the members may be found with a

reasonable degree of accuracy without considering the truss separated into the two systems. If the members $GI$ and $G'I$, Fig 20 ($b$), were straight, the counters $gI$ and $g'I$ would be out of action when there is a full live load or no live load on the system. In like manner, if $FH$, $HH'$, and $H'F'$, Fig. 20 ($c$), were straight, the diagonals $fH$, $Hh'$, $hH'$, and

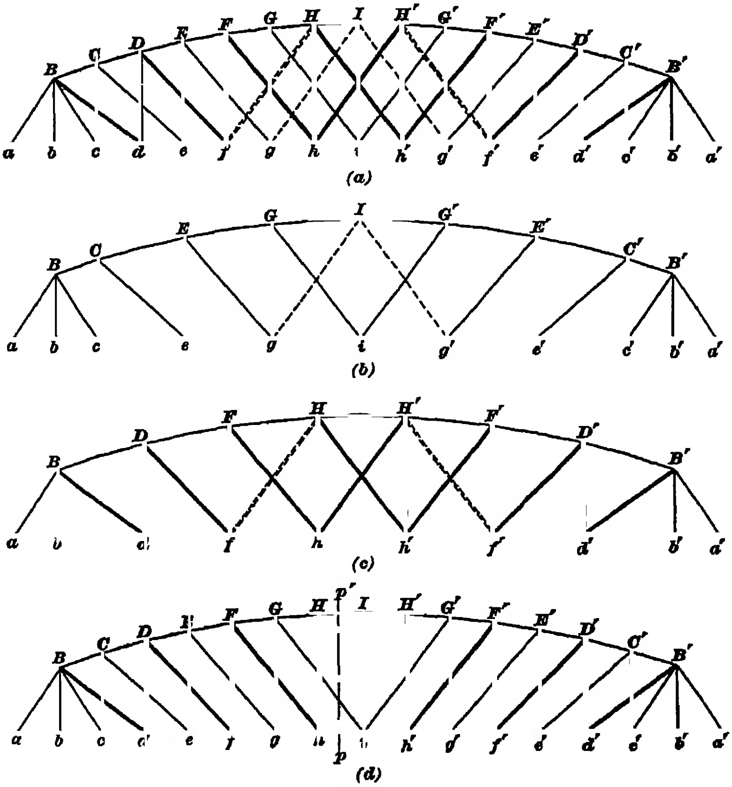



Fig 20

$H'f'$ would be out of action when there is a full live load or no live load on the system. In the present case, as the upper-chord members referred to are very nearly straight, these diagonals may be assumed to be out of action when there is a full live load or no live load on the truss. Then, the

members of the truss that will be in action for these loadings are shown in Fig. 20 (*d*). If the truss is considered cut by the plane of section $p'p$ that intersects $HI$, $Gi$, and $hi$, and the part to the left of the section is treated as a free body, it will be seen that the stress in $HI$ may be found by taking moments about point $i$; the stress in $Gi$, by applying the equation $\Sigma Y = \Sigma S \sin H = 0$ to all the forces acting on the part of the truss considered; and the stress in $hi$, by applying the equation $\Sigma X = 0$. The stresses in the remaining members may be found most readily by means of the stress diagram, only one-half of the truss being considered The stresses in $HI$, $Gi$, and $hi$, considered as external forces, and the panel loads and reaction on the part of the truss to the left of the section $pp'$, form a system in equilibrium, and may be laid off to scale in the force polygon. Then, starting at the right end with the joint $H$, the stress in $HI$ and the load at $H$, if any, being known, the stresses in $GH$ and $Hh$ may be found; at joint $h$, the stresses in $Hh$ and $hi$ and the load at $h$ being known, the stresses in $Fh$ and $gh$ may be found; at joint $G$, the stresses in $GH$ and $Gi$ and the load at $G$, if any, being known, the stresses in $FG$ and $Gg$ may be found; etc. By proceeding in this manner, the stresses in all the members, when there is a full live load or no live load on the truss, may be found.

The stresses in the members can also be found by the analytic method of joints; but, as the numerical work in connection with this method when applied to a curved-chord truss is very laborious, its use will not be considered.

**51.** The maximum live-load stress in almost every web member obtains when there is a partial load on the truss, and cannot be found accurately by the equations of equilibrium unless some assumption is made regarding the upper-chord members. As the upper-chord members of the separate systems, such as $EG$ and $GI$, Fig. 20 (*b*), are not straight, each system exerts an upward force at each upper-chord joint of the other systems; but, as the actual amount of this force at any joint is relatively small, it may be neglected, and the

stresses found on the assumption that the upper-chord members in the two systems are straight. The stresses in the web members of each system can then be found from the loads that come on that system, in the same way as in the simple type of curved-chord truss explained in Arts. 4 to 20. The graphic method by the stress diagram will give the results with the least work.

## OTHER TYPES OF CURVED- AND INCLINED-CHORD TRUSSES

**52. Inclined-Chord Truss.**—Fig 21 represents a type of inclined-chord truss in which the joints of the upper chord lie in a straight inclined line from the center to the end of the

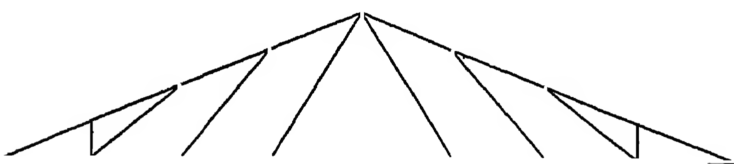


FIG 21

truss. This type of truss is not now used for bridge purposes, but is employed to a great extent for roofs. It possesses the advantage that the stresses in the web members never reverse; the diagonals are sometimes made tension members and sometimes compression; in the truss shown, they are tension members. When there is a full live load, or no live load on the truss, the stress in every diagonal is tension, when the live load covers a part of the span, the live-load stresses in the diagonals in the unloaded part of the truss are all zero. The method of calculation is precisely the same as for the inclined-chord truss explained in Arts. 23 and following. In finding stresses in the web members by the method of moments, it is well to note that the intersection of the chords, which is the center of moments, is the end joint of the truss, and that the reaction acts at this point. The moment of the reaction about this point is therefore zero, and it is unnecessary to compute the reaction.

**53. The Bowstring Truss.**—When the joints of the curved chord lie on a curve that passes through the end joints of the horizontal chord, the truss is called a **bowstring truss.** Two common types of this truss are represented in Figs. 22 and 23 as pony trusses. When a bowstring truss is used in

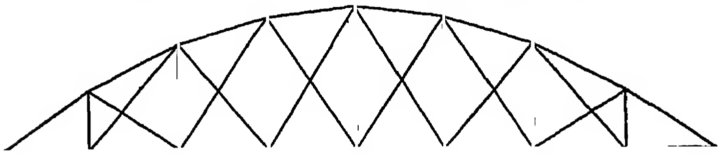


FIG 22

a deck bridge, the upper chord is horizontal and the lower chord curved. The curved chord is usually made a parabola. The height at the center is sometimes spoken of as the **rise.**

When there is a full live load or no live load on the truss, the stress in the horizontal chord and the horizontal component of the stress in the curved chord are equal and constant from end to end, and the stresses in the diagonals are all zero The diagonals in Fig. 22 are tension members, and two are required in every panel; one diagonal is in action when one end of the truss is loaded with live load, the other when the other end is loaded. In Fig 23, the web members are all inclined, and, as there is but one set, they will all get alternate stresses of tension and compression when the truss is partially loaded with live load

Bowstring trusses are not built at the present time to any great extent, but there are a large number in use. The theo-

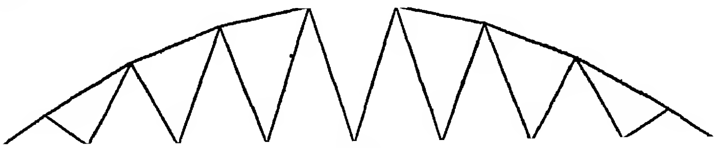


FIG 23

retical advantage in the use of this truss lies in the fact that the maximum chord stresses are almost constant, this is offset to a great extent by the number of counters required, or the reversal of stress in the web members, and the difficulty in getting an efficient system of lateral and sway bracing

**54. The Lenticular Truss.**—When both chords are curved, and the joints lie on curves that pass through the end joints of the truss, as represented in Fig. 24, the truss is called a **double bowstring, lenticular, or fish-belly truss.** The curves are usually parabolas; the lengths of the verticals vary as the ordinates to a parabola, and the action of the chords and diagonals is the same as in the bowstring truss represented in Fig. 22.

The floor may be supported along any of the lines *a b*, *c d*, or *e f*, Fig. 24 In the first case, the floorbeams are supported by short posts, in the second they are connected directly

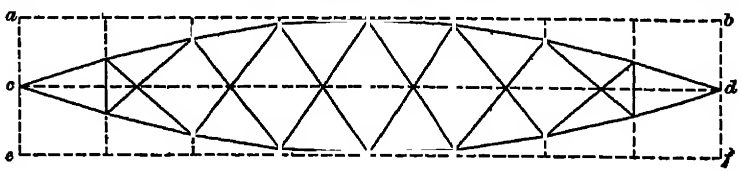



FIG 24

to the vertical web members, and in the third they are suspended from the lower chord by hangers.

The graphic method by the stress diagram is the shortest for this truss as well as for the bowstring truss. In case it is desired to determine the stresses in the lenticular truss by the analytic method, the work can be considerably shortened by drawing the truss to a large scale, and scaling the lever arms of the web members about the various intersections of the chord members.

# STRESSES IN BRIDGE TRUSSES

(PART 4)

## MOMENTS AND SHEARS DUE TO CONCENTRATED LOADS

### INTRODUCTION

**1.** The methods of finding the stresses due to uniformly distributed loads covering all or part of the span have been fully discussed. It is frequently necessary to determine the maximum stresses caused by a system of concentrated loads—such as the wheel loads of a locomotive and train of cars, street cars, road rollers, etc.—that moves over the span. The spacing of the wheels of such a system with respect to one another is fixed; the loads on the wheels are usually unequal, and the system is movable as a whole.

In designing trusses and plate girders, it is necessary to calculate the maximum moments and shears at several panel points, or sections along the span, in plate girders, it is also necessary to determine the section where the moment is greatest. This involves: (1) the determination of the position of the wheels that will cause the greatest moment or shear at any section; and (2) the actual calculation of the greatest moment or shear when the wheels occupy the required position. In practice, it is customary to consider the loads as moving from right to left, coming on the span at the right end and going off the span at the left end. This convention will be followed here.

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## MAXIMUM MOMENTS AND SHEARS IN SIMPLE BEAMS

### MAXIMUM MOMENT AT A GIVEN SECTION

**2. A Single Concentrated Moving Load.**—Let $AB$, Fig. 1 ($a$), be a simple beam over which a load $W$ is moving, and let it be required to find the position of the load for which the moment at a given section $C$ is a maximum. The distance $AC$ will be denoted by $a$. When $W$ is at any

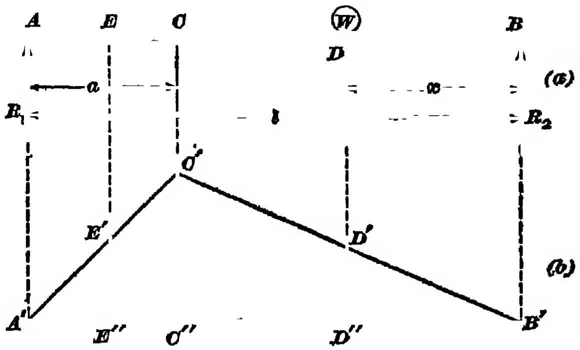



FIG. 1

point $D$ to the right of $C$, and at a distance $x$ from the right end, the reaction $R_1$ is $\frac{Wx}{l}$, and, therefore, the moment at $C$ is $R_1 a$, or $\frac{Wx}{l} \times a$; as $W$, $a$, and $l$ are constant, the moment at $C$ is greatest when $x$ is as great as possible. This occurs when $x$ is equal to $BC$, or $l - a$; that is, when $W$ is at $C$. When $W$ is at the left of $C$ at a distance $x_1$ from the left end, the moment at $C$ is equal to $\frac{Wx_1}{l} \times (l - a)$, which is a maximum when $x_1 = a$, that is, when $W$ is at $C$. It is thus seen that the maximum moment at $C$ is equal to $\frac{Wa}{l} \times (l - a)$, and occurs when the load is at $C$. (See also *Strength of Materials*, Part 1.)

**3. Influence Line for Moment.**—The variation in the bending moment at $C$ when $W$ moves along the beam may be represented graphically as in Fig. 1 ($b$). The horizontal line $A'B'$ having been drawn at a convenient distance from $AB$, the vertical $C''C'$ is drawn, representing the bending moment at $C$ when $W$ is at $C$. Since the moment at $C$ increases uniformly as the distance of $W$ from the right end increases, the line $B'C'$, drawn straight from $B'$ to $C'$, will represent the variation in moment as $W$ moves from $B$ to $C$; also, the line $C'A'$, drawn straight from $C'$ to $A'$, will represent the variation as $W$ moves from $C$ to $A$. For example, if $C''C'$ represents the moment at $C$ when $W$ is at $C$, then $D''D'$, vertically under $D$, represents the moment at $C$ when $W$ is at $D$, also, $E''E'$ represents the moment at $C$ when $W$ is at $E$

If the load $W$ is made equal to unity (as 1 pound or 1 ton), and the ordinate $C''C'$ is made equal to the maximum moment at $C$ for this load, the line $A'C'B'$ is called the **influence line** for the moment at $C$ Then, the moment at $C$ due to any load $W$ at $D$ is equal to $W \times D''D'$; with a load $W$ at $E$, the moment at $C$ is $W \times E''E'$, etc. If there are loads $W_1$ and $W_2$ at the same time at $E$ and $D$, respectively, the moment at $C$ is equal to $W_1 \times E''E' + W_2 \times D''D'$, and similarly for any number of loads. The influence line is of great value in determining the position of a system of loads for which the moment or shear at a given section of a beam or truss, or the stress in a member, is a maximum.

**4. Two Concentrated Loads.**—Let $AB$, Fig. 2 ($a$), be a simple beam over which the loads $W_1$ and $W_2$ are moving. These loads are assumed to be at a fixed distance $d$ apart. It is required to find the position of the loads for which the moment at a section $C$, distant $a$ from the left end of the beam, is a maximum. The influence line for the moment at the section $C$ is $A'C'B'$, Fig. 2 ($b$). Let $y_1'$ and $y_2'$ represent the ordinates under $W_1$ and $W_2$, respectively. As the loads approach $C$ from the right, both $y_1'$ and $y_2'$ increase until $W_1$ reaches $C$, when the moment is equal to

$W_1 y_1 + W_2 y_2$, Fig. 2, (*c*) and (*d*). As the loads move farther to the left, $y_1$ decreases and $y_2$ increases, and the moment at $C$ increases or decreases according as $W_1 y_1$ decreases less or more than $W_2 y_2$ increases. When $W_1$ is at a distance $b$ to the left of $C$, Fig. 2 (*c*), let $y_1''$ and $y_2''$ be the ordinates to the influence line corresponding to $W_1$ and $W_2$,

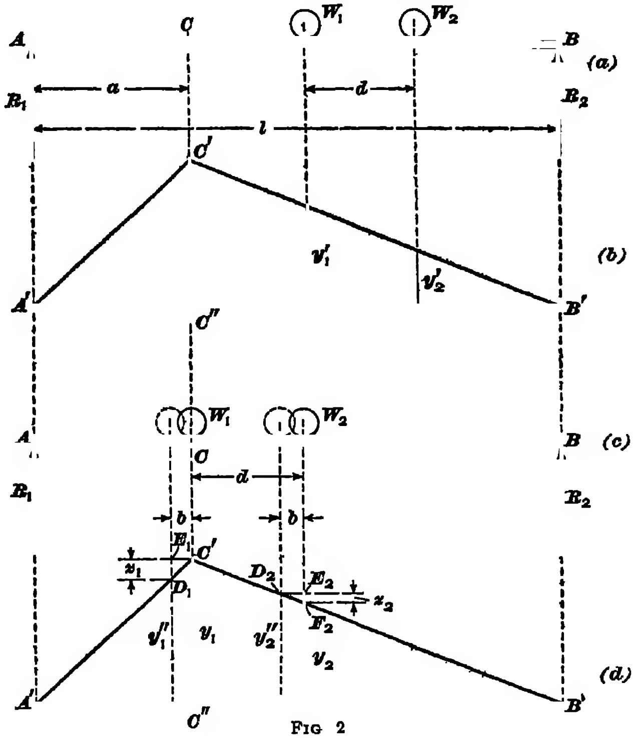



FIG 2

respectively, and let $z_1$ be the difference between $y_1$ and $y_1''$; and $z_2$, the difference between $y_2$ and $y_2''$. Then, the total change in moment is equal to $+W_2 z_2 - W_1 z_1$. The similar triangles $A' C' C''$ and $D_1 E_1 C'$, Fig 2 (*d*), give

$$D_1 E_1 : E_1 C' = C' C'' : A' C'';$$

that is,

$$z_1 : b = y_1 : A' C'';$$

whence

$$z_1 = b \times \frac{y_1}{A' C''}$$

Likewise, from the similar triangles $B' C' C''$ and $D_2 E_2 F_2$,

$$z_2 = b \times \frac{y_1}{B' C''}$$

Substituting these values for $z_1$ and $z_2$, the change in moment becomes

$$W_2 \left(b \times \frac{y_1}{B' C''}\right) - W_1 \left(b \times \frac{y_1}{A' C''}\right) = b y_1 \left(\frac{W_2}{B' C''} - \frac{W_1}{A' C''}\right)$$

The moment at $C$ will be increased if the change in moment is positive, and decreased if the change is negative; it is positive when $\frac{W_2}{B' C''}$ is greater than $\frac{W_1}{A' C''}$, and negative when $\frac{W_2}{B' C''}$ is less than $\frac{W_1}{A' C''}$. In other words, the moment at $C$ will be increased if the right-hand load divided by the right-hand portion of the span is greater than the left-hand load divided by the left-hand portion of the span, and decreased if the former quotient is less than the latter. Then, if the change in moment is negative, the maximum moment obtains when $W_1$ is at $C$; if positive, the moment will increase until $W_2$ reaches $C$, when it is a maximum; for, when $W_2$ passes $C$, both $y_1$ and $y_2$, and, therefore, the moment at $C$, decrease. Hence the following principle:

*The maximum bending moment caused at any given section of a beam by two concentrated moving loads obtains when one of the loads is at the section If the right-hand load divided by the length of the right-hand portion of the beam is greater than the left-hand load divided by the length of the left-hand portion of the beam, the moment is a maximum when the right-hand load is at the section; if the left-hand load divided by the left-hand portion of the beam is the greater, the moment is a maximum when the left-hand load is at the section.*

If the loads are equal, this principle will always bring the left-hand load at the section when the section is at the left of the center, and the right-hand load when the section is at the right of the center.

To find the value of the bending moment at $C$, the left reaction $R_1$ may first be found by taking moments about $B$.

Then, if $W_1$ is at $C$, the moment is equal to $R_1 \times a$; if $W_2$ is at $C$, the moment is equal to $R_1 \times a - W_1 \times d$.

EXAMPLE.—A beam 40 feet long supports a system of two concentrated loads 8 feet apart. The left-hand load is 10,000 pounds, and the right-hand load is 15,000 pounds. What are the maximum moments (*a*) at a section 10 feet from the left end? (*b*) at the center?

SOLUTION.—(*a*) The length of the left-hand portion of the beam is 10 ft., that of the right-hand portion is 30 ft. Then,

$$10{,}000 \div 10 = 1{,}000 \text{ and } 15{,}000 \div 30 = 500$$

As the first is the greater, the moment is a maximum when the left-hand load is at the section. Then,

$$R_1 = \frac{10{,}000 \times 30 + 15{,}000 \times 22}{40} = 15{,}750 \text{ lb.}$$

and the bending moment is equal to

$$15{,}750 \times 10 = 157{,}500 \text{ ft.-lb. Ans.}$$

(*b*) The left-hand and right-hand portions have each a length of 20 ft. Then,

$$10{,}000 \div 20 = 500 \text{ and } 15{,}000 \div 20 = 750$$

As the second is the greater, the moment is a maximum when the right-hand load is at the center. Then,

$$R_1 = \frac{10{,}000 \times 28 + 15{,}000 \times 20}{40} = 14{,}500 \text{ lb.}$$

and the bending moment is equal to

$$14{,}500 \times 20 - 10{,}000 \times 8 = 210{,}000 \text{ ft.-lb. Ans.}$$

**5. Any Number of Concentrated Loads.**—When the system consists of more than two loads, the method of treatment is practically the same as for two loads; the moment at a given section is a maximum when one of the loads is at the section. Let $AB$, Fig. 3 (*a*), be a simple beam over which a system of concentrated loads is moving, and let it be required to find the position of the loads when they cause the greatest moment at $C$, at a distance $a$ from the left end. Let $A' C' B'$, Fig. 3 (*b*), be the influence line for the moment at $C$, and let $y_1, y_2$, etc. be the ordinates corresponding to $W_1$, $W_2$, $W_3$, etc., respectively. As the loads move across the beam and $W_1$ approaches $C$, the bending moment at $C$ increases until $W_1$ reaches $C$, when it is equal to $W_1 y_1 + W_2 y_2 \ldots + W_n y_n$. When $W_1$ passes $C$, the moment due to it decreases, and that due to the remaining loads

increases; it can be shown, as in Art. 4, that the change in moment is equal to

$$b \times y_1\left(\frac{W_2 + W_3 + \dots W_n}{B'C''} - \frac{W_1}{A'C''}\right)$$

If $\frac{W_2 + W_3 + \dots W_n}{B'C''}$ is less than $\frac{W_1}{A'C''}$, the moment decreases after $W_1$ passes $C$, and is therefore greatest

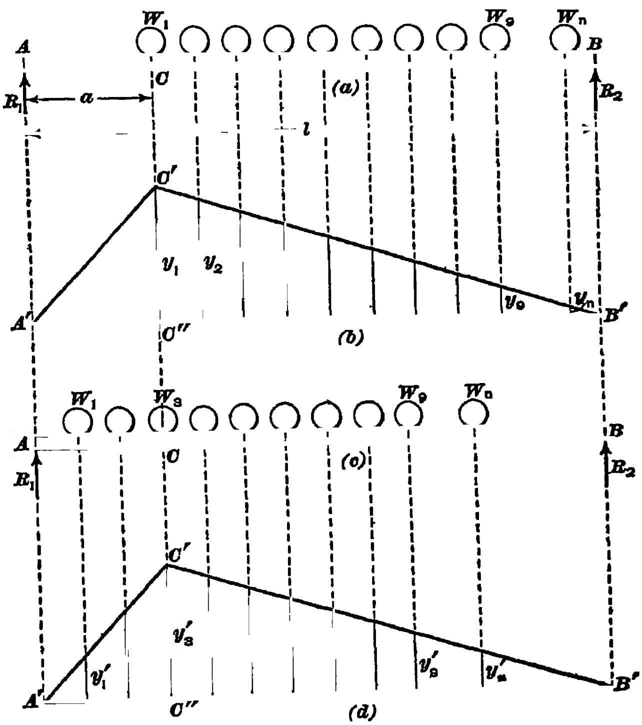



FIG 8

when $W_1$ is at $C$; if the former of these two fractions is greater than the latter, the moment increases until $W_2$ reaches $C$. When $W_2$ passes $C$, the change in moment is equal to

$$b \times y_1\left(\frac{W_3 + W_4 + \dots W_n}{B'C''} - \frac{W_1 + W_2}{A'C''}\right)$$

If $\frac{W_3 + W_4 + \ldots W_n}{B' C''}$ is less than $\frac{W_1 + W_2}{A' C''}$, the moment decreases after $W_2$ passes $C$, and is therefore greatest when $W_2$ is at $C$; if the former of these two fractions is greater than the latter, the moment increases until $W_3$ reaches $C$, etc. Suppose, in this case, that the moment increases after $W_1$ and $W_2$ pass $C$, and decreases after $W_3$ passes $C$; then, the moment is greatest when $W_3$ is at $C$ [see Fig. 3. ($c$) and ($d$)]. Then,

$$\frac{W_1 + W_2}{A' C''} \text{ is less than } \frac{W_3 + W_4 + \ldots W_n}{B' C''}$$

from the condition that the moment increases after $W_2$ passes $C$ until $W_3$ reaches $C$; and

$$\frac{W_1 + W_2 + W_3}{A' C''} \text{ is greater than } \frac{W_4 + W_5 + \ldots W_n}{B' C''}$$

from the condition that the moment decreases after $W_3$ passes $C$.

The quotient obtained by dividing the sum of all the loads on a part of a beam by the length of that part will be referred to as the **average intensity of load** on that part of the beam.

From the preceding discussion, the following general principle may be stated:

*The maximum bending moment caused at a given section of a beam by a system of moving concentrated loads obtains when such a load is at the section that, when it is counted with the loads on the right, the average intensity of load on the right is greater than the average intensity of load on the left, and when it is counted with the loads on the left, the average intensity of load on the left is greater than the average intensity of load on the right.*

When the loads are in this position, if part of the load at the section is counted with the loads on the left and the sum is called $W_l$, and the remainder counted with the loads on the right and the sum called $W_r$, it is possible to make $\frac{W_l}{a}$ equal to $\frac{W_r}{l-a}$. Putting $\frac{W_l}{a} = \frac{W_r}{l-a}$, and denoting the sum of all the loads on the span by $\Sigma W$, we have

$$W_r = \Sigma W - W_l,$$

and, therefore, $\dfrac{W_l}{a} = \dfrac{\Sigma W - W_l}{l - a};$

whence $W_l \times \dfrac{l - a}{a} = \Sigma W - W_l,$

or $W_l\left(\dfrac{l}{a} - 1\right) = \Sigma W - W_l,$

and $W_l \times \dfrac{l}{a} = \Sigma W,\ \dfrac{W_l}{a} = \dfrac{\Sigma W}{l},\ W_l = \dfrac{a}{l} \times \Sigma W$

The principle just given may therefore be stated more simply as follows:

*The maximum bending moment caused at a given section of a beam by a system of moving concentrated loads obtains when such a load is at the section that, if part of it is counted with the loads on the left, the average intensity of load on the left is equal to that on the whole beam.*

As $W_l$ is equal to the sum of all the loads to the left of $C$ plus a part of the load at $C$, the sum of all the loads to the left of $C$ must be less than $W_l$, or $\frac{a}{l} \times \Sigma W$, and this sum added to the load at $C$ must be greater than $W_l$, or $\frac{a}{l} \times \Sigma W$. If the system is such that all the loads are on the beam when the moment is a maximum, $W_l$ may be found directly from the formula $W_l = \frac{a}{l} \times \Sigma W$. If the system is such that some of the loads are off the beam when the moment is a maximum, the correct position may be found by trial by placing several of the heaviest loads at the section successively to see which fulfils the condition. It will frequently be found that several positions of the loads will satisfy the conditions for a maximum (on account of the fact that loads pass off at the left end and on at the right end as the loads move to the left to occupy the different positions); in this case, the actual value of the moment must be calculated for each position that satisfies the conditions, in order to determine which position causes the greatest moment. A load that comes exactly at $A$ or $B$ when one load is placed at $C$ should be considered as off the span.

**6. Value of the Moment.**—If $Wx$ represents the moment about $B$ of any load at a distance $x$ from the righ end, the moment of all the loads on the span about $B$ ma be denoted by $\Sigma\, Wx$, and the left reaction $R_1$ by $\frac{\Sigma\, Wx}{l}$. I $Wb$ represents the moment about $C$ of any load to the lef of $C$ at a distance $b$ from $C$, the moment of all the loads o the left end about $C$ may be written $\Sigma\, Wb$. Then, the bend ing moment at $C$ is $R_1 a - \Sigma\, Wb$; that is, $\frac{a}{l} \times \Sigma\, Wx - \Sigma\, Wb$

EXAMPLE 1—A beam 50 feet long supports the system of movin concentrated loads represented in Fig. 4. To find the maximur moment (*a*) at a point 10 feet from the left end; (*b*) at the cente

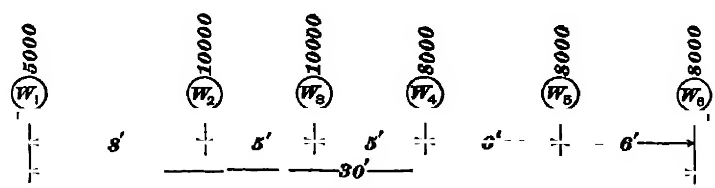



FIG 4

SOLUTION—(*a*) As the system of loads is much shorter than th beam, it may be assumed that all the loads will be on Ther $\Sigma\, W = 49{,}000$ lb, $W_l = \frac{a}{l} \times \Sigma\, W = \frac{10}{50} \times 49{,}000 = 9{,}800$ lb., $W$ $= 5{,}000$ lb, and $W_1 + W_2 = 15{,}000$ lb As one value is less and th other greater than 9,800 (Art 5), the moment is a maximum whe $W_2$ is at $C$ Then the distances of the various loads from the rig end are 48, 40, 35, 30, 24, and 18 ft, respectively, and

$$R_1 = (5{,}000 \times 48 + 10{,}000 \times 40 + 10{,}000 \times 35 + 8{,}000 \times 30 + 8{,}000 \times 2 + 8{,}000 \times 18) \div 50 = 31{,}320 \text{ lb}$$

The bending moment is equal to

$$31{,}320 \times 10 - 5{,}000 \times 8 = 273{,}200 \text{ ft-lb} \quad \text{Ans.}$$

(*b*) In this case, $W_l = \frac{25}{50} \times 49{,}000 = 24{,}500$ lb The sum of $W$ and $W_2$ is 15,000 lb, of $W_1$, $W_2$, and $W_3$, 25,000 lb As one value less and the other greater than 24,500, the moment is a maximu when $W_3$ is at $C$ Then, the distances of the loads from the rig end are 38, 30, 25, 20, 14, and 8 ft, respectively, and

$$R_1 = (5{,}000 \times 38 + 10{,}000 \times 30 + 10{,}000 \times 25 + 8{,}000 \times 20 + 8{,}000 \times + 8{,}000 \times 8) \div 50 = 21{,}520 \text{ lb.}$$

The bending moment is equal to

$$21{,}520 \times 25 - (5{,}000 \times 13 + 10{,}000 \times 5) = 423{,}000 \text{ ft.-lb.} \quad \text{Ans.}$$

EXAMPLE 2.—With the same system of loads as in the preceding example, what is the greatest moment at the center of a beam 25 feet long?

SOLUTION —In this case, the system of loads is longer than the beam, and some loads will be off, the position that causes the greatest moment will be determined largely by trial

When $W_1$ is at the center, $W_1$ and $W_2$ are on the beam Then,

$$W_l = \frac{a}{l} \times \Sigma W = \frac{12\ 5}{25} \times 15{,}000 = 7{,}500 \text{ lb}$$

As this is larger than $W_1$, the moment is not a maximum when $W_1$ is at the center

When $W_2$ is at the center, $W_1$, $W_2$, $W_3$, and $W_4$ are on the beam. Then,

$$W_l = \frac{12\ 5}{25} \times 33{,}000 = 16{,}500 \text{ lb}$$

As this is larger than $W_1 + W_2$ (15,000 lb ), the moment is not a maximum when $W_2$ is at the center.

When $W_3$ is at the center, $W_2$, $W_3$, $W_4$, and $W_5$ are on the beam. Then,

$$W_l = \frac{12\ 5}{25} \times 36{,}000 = 18{,}000 \text{ lb.}$$

As this is larger than $W_2$ (10,000 lb ), and smaller than $W_2 + W_3$ (20,000 lb ), the moment is a maximum when $W_3$ is at the center

When $W_4$ is at the center, $W_2$, $W_3$, $W_4$, $W_5$, and $W_6$ are on the beam Then,

$$W_l = \frac{12\ 5}{25} \times 44{,}000 = 22{,}000 \text{ lb.}$$

As this is larger than $W_2 + W_3$ (20,000 lb ), and smaller than $W_2 + W_3 + W_4$ (28,000 lb ), the moment is also a maximum when $W_4$ is at the center

When $W_5$ is at the center, $W_3$, $W_4$, $W_5$, and $W_6$ are on the beam Then,

$$W_l = \frac{12\ 5}{25} \times 34{,}000 = 17{,}000 \text{ lb}$$

As this is less than $W_3 + W_4$ (18,000 lb ), the moment is not a maximum when $W_5$ is at the center

There are then two positions that satisfy the conditions for maximum moment, namely, when $W_3$ is at the center, and when $W_4$ is at the center It is necessary to calculate both values

When $W_3$ is at the center,

$$R_1 = \frac{10{,}000 \times 17\ 5 + 10{,}000 \times 12\ 5 + 8{,}000 \times 7\ 5 + 8{,}000 \times 1\ 5}{25}$$

$$= 14{,}880 \text{ lb}$$

The bending moment is

$$14{,}880 \times 12\ 5 - 10{,}000 \times 5 = 136{,}000 \text{ ft.-lb.}$$

When $W_4$ is at the center,

$$R_1 = (10{,}000 \times 22\ 5 + 10{,}000 \times 17.5 + 8{,}000 \times 12.5 + 8{,}000 \times 6.5 + 8{,}000 \times 0\ 5) \div 25 = 22{,}240 \text{ lb}$$

The bending moment is

$$22{,}240 \times 12\ 5 - (10{,}000 \times 10 + 10{,}000 \times 5) = 128{,}000 \text{ ft -lb}$$

Therefore, the moment at the center is greatest when $W_3$ is at the center, and is equal to 136,000 ft -lb Ans

## EXAMPLES FOR PRACTICE

1. A beam 50 feet long supports a moving load consisting of two concentrations 10 feet apart The left-hand load is 12,000 pounds, and the right-hand load is 18,000 pounds What is the maximum moment: (*a*) at a section 10 feet from the left end? (*b*) at the center? (*c*) at a section 40 feet from the left end?

Ans $\begin{cases} (a) & 204{,}000 \text{ ft.-lb.} \\ (b) & 315{,}000 \text{ ft -lb.} \\ (c) & 216{,}000 \text{ ft -lb.} \end{cases}$

2 A beam 50 feet long supports a moving load consisting of two concentrations of 10,000 pounds each at a distance of 8 feet apart. What is the maximum moment at a section 20 feet from the left end?

Ans. 208,000 ft.-lb.

3. A beam 35 feet long supports a moving load consisting of the system shown in Fig 4 What is the greatest moment (*a*) at a section 10 feet from the left end? (*b*) at the center?

Ans $\begin{cases} (a) & 203{,}100 \text{ ft -lb.} \\ (b) & 239{,}250 \text{ ft.-lb.} \end{cases}$

4. A beam 30 feet long supports a moving load consisting of the

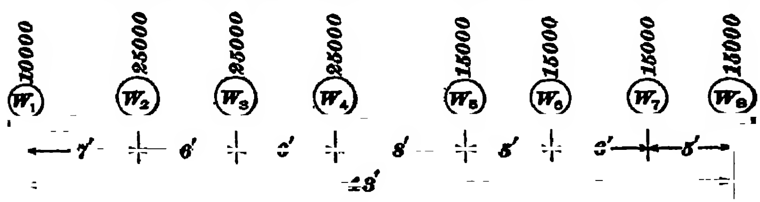



FIG 5

system shown in Fig 5 What is the greatest moment. (*a*) at a section 25 feet from the left end? (*b*) at the center?

Ans $\begin{cases} (a) & 247{,}500 \text{ ft.-lb.} \\ (b) & 430{,}000 \text{ ft.-lb.} \end{cases}$

## SECTION OF MAXIMUM MOMENT IN A BEAM

**7. Any Number of Loads.**—As a system of concentrated loads moves across a beam, there is a section where, under a certain position of the loads, the moment is greater than occurs at any other section under any position of the loads. The method of determining the location of this section will now be considered.

Let $AB$, Fig. 6, be a simple beam over which a system of concentrated loads is moving, and let it be required to find the section where the moment is a maximum as the loads move across the beam. The greatest bending moment will occur under one of the loads, and it is necessary to determine the position of the system for which the bending moment under any one of the given loads is a maximum.

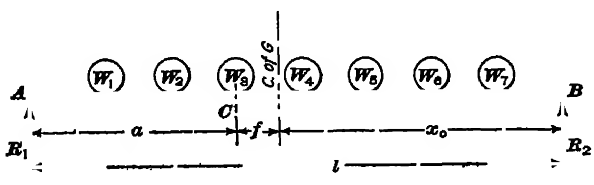



Fig 6

Let $C$ be the position of $W_3$ when the bending moment under $W_3$ is a maximum, and let $AC = a$. The left reaction $R_1$ is $\frac{\Sigma Wx}{l}$, denoting by $x$ the distance of any load from the right-hand support. If $\Sigma W$ represents the sum of all the loads on the beam, and $x_0$ the distance from the right end to the center of gravity of these loads, then

$$\Sigma Wx = x_0 \Sigma W,$$

and, therefore, $R_1 = \frac{x_0 \Sigma W}{l}$. The sum of the moments of the loads on the left end about $C$ is $\Sigma Wb$, denoting by $b$ the distance of any of those loads from $C$. The bending moment at $C$ is equal to $R_1 \times a - \Sigma Wb$, or $\frac{a x_0 \Sigma W}{l} - \Sigma Wb$. This expression is a maximum when $a x_0$ is a maximum, that is, when $x_0 = a$, or when $C$ is the same distance

from $A$ as the center of gravity of the loads is from $B$; in other words, when $C$ and the center of gravity of the loads are equidistant from the center of the beam.* Whence the principle:

*The maximum moment under a load occurs when that load and the center of gravity of all the loads on the beam are equidistant from the center.*

This principle, in connection with the principle that the loads on the left of the section of maximum moment must satisfy the condition $W_l = \frac{a}{l} \times \Sigma W$ (Art. 5), serves to determine the greatest bending moment in the beam.

In applying the preceding principles, the required position of the loads must be determined largely by trial. If the system of loads is longer than the beam, some of the loads will be off when the moment is a maximum. In such a case, it will be well to remember that the section of maximum moment will usually be close to the center, the position of the loads, when the moment at the center is a maximum, may be found as explained in Art. 5. The position of the center of gravity of the loads on the beam for this condition may be calculated, and the loads placed according to the principle just given, placing them first so that the center of the span is half way between the center of gravity and the nearest load, then the next nearest, etc., until a position is found that satisfies all the conditions for maximum moment. If, when the loads occupy the required position, there are more or less on the beam than those for which the center of gravity was computed, it will be necessary to compute anew the location of the center of gravity of the loads then on the beam, and rearrange them. In case more than one position of the loads satisfies the conditions for the maximum bending

*Let the distance from the center of gravity of the loads to $C$ be represented by $f$, Fig 6; then, $x_0 + a = l - f$, which may be called $c$; also, $x_0 = c - a$, and $a x_0 = a(c - a) = a c - a^2 = \left(\frac{c}{2}\right)^2 - \left(a - \frac{c}{2}\right)^2$, and this is a maximum when the negative term is zero, that is, when $a - \frac{c}{2} = 0$, whence $a = \frac{c}{2} = \frac{l - f}{2} = \frac{x_0 + a}{2}$, and, therefore, $x_0 = a$

moment, it will be necessary to compute the bending moment for each position in order to find the greatest moment.

**8. Value of the Moment.**—In Art. 7 it was stated that the bending moment at $C$ is equal to $\frac{a x_0 \Sigma W}{l} - \Sigma W b$. For the maximum bending moment $M$ under a load, we have, since in this case $x_0 = a$,

$$M = \frac{x_0^2 \Sigma W}{l} - \Sigma W b$$

EXAMPLE 1 —A beam 50 feet long supports the system of moving concentrated loads shown in Fig 4. What is the location of the section of maximum bending moment, and the value of the moment?

SOLUTION —As this system of loads is much shorter than the beam, it may be assumed that all the loads will be on when the moment is greatest The center of gravity may be found by taking moments about $W_6$, its distance from $W_6$ is

$$\frac{5{,}000 \times 30 + 10{,}000 \times 22 + 10{,}000 \times 17 + 8{,}000 \times 12 + 8{,}000 \times 6 + 8{,}000 \times 0}{49{,}000} = 13\ 96 \text{ ft}$$

The center of gravity is 1 96 ft to the left of $W_4$ and 3 04 ft. to the right of $W_3$. As $W_4$ is nearer, it will be tried first

When $W_4$ and the center of gravity are equidistant from the center,

$$x_0 = a = 25 + \frac{1\ 96}{2} = 25\ 98 \text{ ft}$$

and $$\frac{a}{l} \times \Sigma W = \frac{25\ 98}{50} \times 49{,}000 = 25{,}500 \text{ lb.}$$

$W_1 + W_2 + W_3 = 25{,}000$ lb , $W_1 + W_2 + W_3 + W_4 = 33{,}000$ lb. As one of these sums is less and the other greater than 25,500 lb , the moment is a maximum under $W_4$.

The bending moment is

$$\frac{25\ 98^2 \times 49{,}000}{50} - (5{,}000 \times 18 + 10{,}000 \times 10 + 10{,}000 \times 5) = 421{,}500 \text{ ft -lb}$$

When $W_3$ and the center of gravity are equidistant from the center,

$$x_0 = a = 25 - \frac{3\ 04}{2} = 23\ 48 \text{ ft}$$

and $$\frac{a}{l} \times \Sigma W = \frac{23\ 48}{50} \times 49{,}000 = 23{,}000 \text{ lb.}$$

$W_1 + W_2 = 15{,}000$ lb , $W_1 + W_2 + W_3 = 25{,}000$ lb As one of these sums is less and the other greater than 23,000 lb , the moment is a maximum under $W_3$

The bending moment is

$$\frac{23\ 48^2 \times 49{,}000}{50} - (5{,}000 \times 13 + 10{,}000 \times 5) = 425{,}300 \text{ ft -lb.}$$

If any other loads are tried, it will be found that they do not satisfy the condition for maximum moment. The section of maximum moment is, therefore, 23.48 ft. from the left end, and the bending moment at that section is 425,300 ft.-lb. Ans.

EXAMPLE 2.—A beam 25 feet long supports the system of moving loads shown in Fig. 4. What is the location of the section of maximum moment, and the value of the moment?

SOLUTION.—As the system of loads is longer than the beam, some of the loads will be off the span when the moment is a maximum. In example 2, Art. 6, it was shown that the condition for maximum moment at the center of this span for the same load is satisfied when $W_3$ is at the center, and when $W_4$ is at the center, it will, therefore, be unnecessary to repeat the demonstration here.

When $W_3$ is at the center, $W_2$, $W_3$, $W_4$, and $W_5$ are on the beam, and the distance of the center of gravity of these loads from $W_5$ is

$$\frac{10{,}000 \times 16 + 10{,}000 \times 11 + 8{,}000 \times 6 + 8{,}000 \times 0}{36{,}000} = 8.83 \text{ ft.}$$

The center of gravity is 2.83 ft. to the left of $W_4$, and 2.17 ft. to the right of $W_3$. As $W_3$ is nearer, it may be tried first.

$$x_0 = a = 12.5 - \frac{2.17}{2} = 11.42 \text{ ft.}$$

Then, $$\frac{a}{l} \times \Sigma W = \frac{11.42}{25} \times 36{,}000 = 16{,}400 \text{ lb.}$$

As this is greater than $W_2$ and less than $W_2 + W_3$, the moment is a maximum under $W_3$, and is equal to

$$\frac{11.42^2 \times 36{,}000}{25} - 10{,}000 \times 5 = 137{,}800 \text{ ft.-lb.}$$

When $W_4$ and the center of gravity are equidistant from the center

$$x_0 = a = 12.5 + \frac{2.83}{2} = 13.92 \text{ ft.}$$

and $$\frac{a}{l} \times \Sigma W = \frac{13.92}{25} \times 36{,}000 = 20{,}000 \text{ lb.}$$

As this is less than $W_2 + W_3 + W_4$, and equal to $W_2 + W_3$, th moment is a maximum under $W_4$, and is equal to

$$\frac{13.92^2 \times 36{,}000}{25} - (10{,}000 \times 10 + 10{,}000 \times 5) = 129{,}000 \text{ ft.-lb.}$$

The fact that $\frac{a}{l} \times \Sigma W$ is equal to $W_2 + W_3$ in this case means that after $W_3$ passes $C$, the moment neither increases nor decreases unt $W_4$ reaches $C$, after which it decreases.

When $W_4$ is at the center, $W_2$, $W_3$, $W_4$, $W_5$, and $W_6$ are on the bea and the center of gravity of these loads is

$$\frac{10{,}000 \times 22 + 10{,}000 \times 17 + 8{,}000 \times 12 + 8{,}000 \times 6 + 8{,}000 \times 0}{44{,}000} = 12.14 \text{ ft}$$

from $W_6$, or .14 ft. to the left of $W_4$; then $C$ will be .07 ft. to the rig

of the center, and for all practical purposes it may be assumed to be at the center The moment at the center under $W_4$ has already been found (example 2, Art 6) to be equal to 128,000 ft -lb It is unnecessary to try any other positions

From the foregoing, it is seen that the maximum moment occurs under $W_3$, when that load is 11 42 ft from the left end, and the moment is equal to 137,800 ft -lb Ans

**9. Comparison.**—In example 2, Art. **6**, it was found that the maximum moment at the center of a 25-foot beam for the same loading as in the preceding example is about 1 3 per cent. less than the actual maximum moment just found, and in example 1, Art. **6**, it was found that the maximum moment at the center of a 50-foot beam for the same loading is about .5 per cent. less than the actual maximum moment just found In any case, the actual maximum moment is very little larger than that at the center, and, for all spans above 50 feet in length, with the ordinary conditions of loading, the latter is sufficiently accurate for all practical purposes. For spans up to about 50 feet, it may be desirable to compute the actual maximum.

**10. Special Cases.**—The following principles are but special cases of the general principle stated in Art. **8**:

1. *The maximum moment in a beam supporting two unequal loads will occur under the heavier load when that load and the center of gravity of the two loads are equidistant from the center of gravity of the beam.*

2. *The maximum moment in a beam supporting two equal loads will occur under either load when the two loads are on opposite sides of the center and one of the loads is at a distance from the center equal to one-fourth the distance between the loads.*

3. *In case a beam that supports two loads is but little longer than the distance between the loads, it may happen that one of the loads placed at the center of the beam will cause a greater moment than if the two loads are arranged as described in the first principle.*

EXAMPLE 1 —A beam 24 feet long supports two moving loads 6 feet apart, the left-hand load is 8,000 pounds, and the right-hand load is 4,000 pounds What is the location of the section of maximum moment, and what is the value of the moment?

SOLUTION —The center of gravity is

$$\frac{8{,}000 \times 6}{12{,}000} = 4 \text{ ft}$$

from the right-hand load  Then the section of maximum moment is 1 ft to the left of the center, therefore, $a = x_0 = 11$ ft , and the moment is equal to

$$\frac{11^2 \times 12{,}000}{24} = 60{,}500 \text{ ft -lb} \quad \text{Ans.}$$

EXAMPLE 2 —A beam 25 feet long supports two equal moving loads of 10,000 pounds each, 6 feet apart  What is the location of the section of maximum moment, and what is the value of the moment?

SOLUTION —As the beam supports two equal moving loads, the second principle applies in this case  One-fourth the distance between the loads is $6 \div 4 = 1\ 5$ ft , and the section of maximum moment is therefore under either load when that load is 1 5 ft from the center of the beam  At this section,

$$a = x_0 = \frac{25}{2} - 1\ 5 = 11 \text{ ft.},$$

and the moment is equal to

$$\frac{11^2 \times 20{,}000}{25} = 96{,}800 \text{ ft -lb} \quad \text{Ans.}$$

EXAMPLE 3.—A beam 8 feet long supports two moving loads 5 feet apart  The left-hand load is 6,000 pounds, and the right-hand load is 4,000 pounds  What is the location of the section of maximum moment, and what is the value of that moment?

SOLUTION —The distance of the center of gravity of the two loads from the right-hand load is equal to

$$\frac{6{,}000 \times 5}{6{,}000 + 4{,}000} = 3 \text{ ft.}$$

and the distance from the left-hand load is $5 - 3 = 2$ ft  Applying the first principle for a maximum moment, the load of 6,000 lb will be $2 - 2 = 1$ ft to the left of the center of the beam, and $\frac{8}{2} - 1 = 3$ ft from the left end of the beam  That is, $a = x_0 = 3$ ft., and the moment is

$$\frac{3^2 \times 10{,}000}{8} = 11{,}250 \text{ ft -lb.}$$

As the beam is but little longer than the distance between the loads, according to the third principle it may happen that the maximum moment will occur when one of the loads is at the center of the beam. When the load of 6,000 lb is at the center, the load of 4,000 lb will be off the beam, and the maximum moment will be equal to

$$\frac{4^2 \times 6{,}000}{8} = 12{,}000 \text{ ft.-lb.}$$

As this value is greater than the one just found, the section of maximum moment is at the center, and the moment is 12,000 ft -lb. Ans.

### EXAMPLES FOR PRACTICE

1 A beam 60 feet long supports the system of moving concei trated loads shown in Fig 5 (*a*) What is the location of the sectio of maximum moment? (*b*) What is the value of the maximui moment?

Ans $\begin{cases} (a) \text{ At 28 90 ft. from the left en} \\ (b) \text{ 1,378,400 ft.-lb.} \end{cases}$

2 A beam 30 feet long supports the system of moving concer trated loads shown in Fig. 5 (*a*) What is the location of the sectio of maximum moment? (*b*) What is the value of that moment?

Ans $\begin{cases} (a) \text{ At 14 6 ft from the left en} \\ (b) \text{ 430,500 ft -lb} \end{cases}$

3 A beam 30 feet long supports two moving loads 8 feet apart If the left-hand load is 30,000 pounds, and the right-hand loa 25,000 pounds, find. (*a*) the location of the section of maximun moment, (*b*) the value of the moment

Ans $\begin{cases} (a) \text{ At 13 18 ft from the left enc} \\ (b) \text{ 318,500 ft -lb} \end{cases}$

4. Two loads 6 feet apart are moving over a beam 10 feet long If the left-hand load is 20,000 pounds, and the right-hand load is 10,000 pounds (*a*) what is the location of the section of maximum moment? (*b*) what is the value of that moment?

Ans $\begin{cases} (a) \text{ At the center} \\ (b) \text{ 50,000 ft -lb.} \end{cases}$

### MAXIMUM SHEAR AT A GIVEN SECTION

**11. Single Moving Load.**—Let $AB$, Fig. 7 (*a*), be a

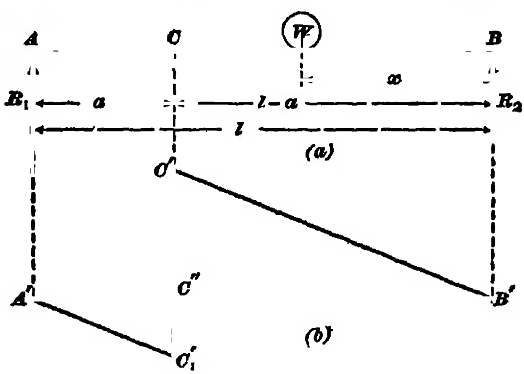



Fig 7

simple beam supporting the moving load $W$, and let it be required to find the position of the load for which the shear

at the section $C$, at a distance $a$ from the left end, is a maximum When $W$ is at any point to the right of $C$, at a distance $x$ from the right end, the shear at $C$ is positive and equal to the left-hand reaction $\frac{Wx}{l}$; when $W$ is at any point to the left of $C$, the shear at $C$ is negative, and equal to $\frac{Wx}{l} - W = -\frac{W(l-x)}{l}$. That is, as $W$ passes on the beam at $B$ and moves toward $C$, the shear at $C$ increases uniformly until $W$ is at $C$; as $W$ passes $C$, the shear suddenly changes from positive to negative; and, as $W$ approaches $A$, the shear uniformly approaches zero. The shear at $C$ is, then, a maximum when $W$ is at $C$.

**12. Influence Line for Shear.**—The variation in shear at $C$ as $W$ moves over the beam may be represented graphically, as in Fig. 7 ($b$). The horizontal line $A' B'$ having been drawn at any convenient distance from $A B$, the vertical line $C'' C'$ is laid off above $A' B'$, equal, by scale, to $\frac{W(l-a)}{l}$, and $C'' C_1'$ is laid off below $A' B'$ equal to $\frac{Wa}{l}$, then, $B' C'$ represents the variation in shear at $C$ as $W$ moves from $B$ to $C$, and $C_1' A'$ represents the variation as $W$ moves from $C$ to $A$. If the ordinates $C'' C'$ and $C'' C_1'$ are made equal to the shear at $C$ due to a load of unity, the line $B' C' C_1' A'$ is called the **influence line** for the shear at $C$.

**13. Any Number of Loads.**—Let $A B$, Fig. 8 ($a$), be a beam supporting a system of moving concentrated loads, and let it be required to find the position of the loads when they cause maximum shear at the section $C$ at a distance $a$ from the left end. As the loads come on the beam at $B$ and move toward the left, the shear at $C$ increases until $W_1$ reaches $C$; it is then equal to $R_1$, and may be denoted by $V_1$ If the sum of the loads then on the beam is $\Sigma W$, and the distance from their center of gravity to the right end is $x_0$, then $R_1 = V_1 = \frac{x_0}{l} \times \Sigma W$. When $W_1$ passes $C$, the shear suddenly decreases, and, when $W_1$ is at a distance $k$ to the

left of $C$, is equal to $\frac{x_0 + k}{l} \times \Sigma' W - W_1$. As the loads move farther to the left, $k$, and, therefore, the shear at $C$, increase until $W_2$ is at $C$. If $d_1$ is the distance from $W_1$ to

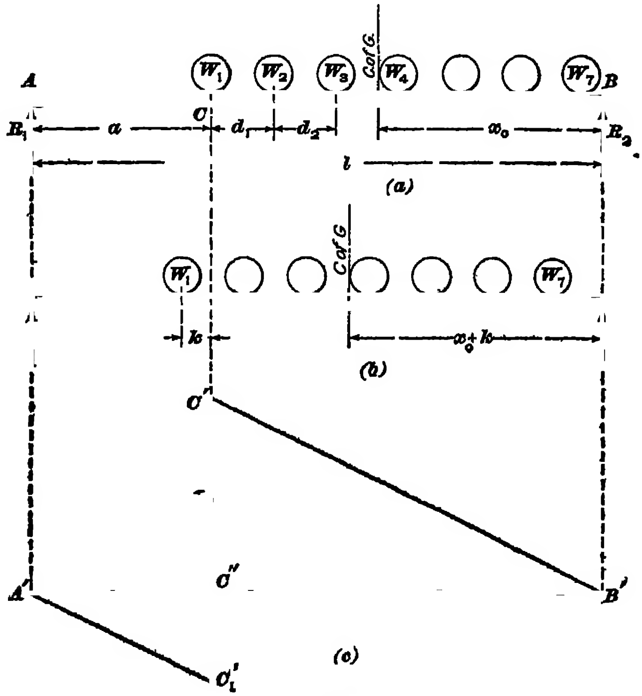
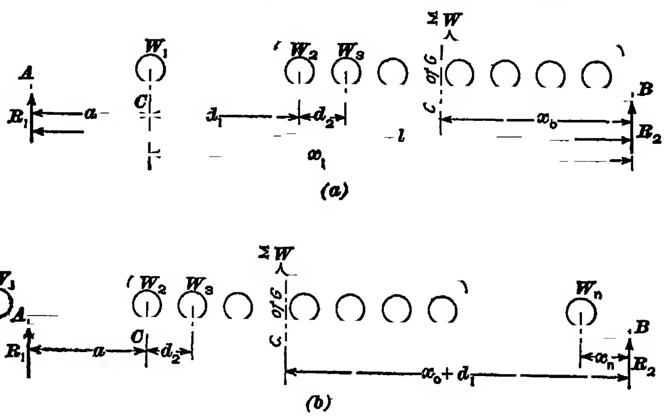


FIG 8

$W_2$, the shear when $W_2$ is at $C$, which may be called $V_2$, is equal to $\frac{x_0 + d_1}{l} \times \Sigma' W - W_1$, the difference between $V_2$ and $V_1$ being $\frac{d_1}{l} \times \Sigma' W - W_1$. If this difference is negative, $V_2$ is less than $V_1$, and the shear is a maximum when $W_1$ is at $C$; if positive, $V_2$ is greater than $V_1$. Suppose, in this case, that the shear is greater when $W_2$ is at $C$ than when $W_1$ is at $C$. Then, when $W_2$ passes $C$, the shear again suddenly decreases, and, when $W_2$ is at a distance $k$ to the left of $C$, is

equal to $\frac{x_0 + d_1 + k}{l} \times \Sigma W - (W_1 + W_2)$; as the loads move to the left, $k$ increases, and, therefore, the shear increases until $W_3$ is at $C$. If $d_2$ is the distance from $W_2$ to $W_3$, the shear at $C$ when $W_3$ is at $C$, which may be called $V_3$, is equal to $\frac{x_0 + d_1 + d_2}{l} \times \Sigma W - (W_1 + W_2)$, the difference between $V_3$ and $V_2$ being $\frac{d_2}{l} \times \Sigma W - W_2$. If this difference is negative, $V_3$ is less than $V_2$, and the shear is a maximum when $W_2$ is at $C$, if positive, $V_3$ is greater than $V_2$. By proceeding in this way, it will be found that, when some load $W_n$ is at $C$, the shear $V_n$ is less than the shear $V_{n-1}$ when the preceding load $W_{n-1}$ is at $C$. Then, the shear at $C$ is a maximum when $W_{n-1}$ is at $C$. By trying the first two or three loads of a system successively at the section, the position of the loads for maximum shear is readily found. As all loads to the left of the section decrease the shear, there will usually be very few, if any, in that position.

**14. Special Conditions.**—In case some of the loads pass off the beam or others come on as the loads move to




(a)

(b)

FIG 9

the left, the value of $\Sigma W$ will change The manner of treating such cases will be shown by the following two illustrations.

Starting with $W_1$ at $C$, Fig. 9 ($a$), let it be assumed that, when the loads move a distance $d_1$ to the left, $W_1$ passes off the beam and $W_n$ comes on [Fig. 9 ($b$)]. Taking $\Sigma W$ as the sum of all the loads, exclusive of $W_1$ and $W_n$, the increase in shear is $\frac{d_1}{l} \times \Sigma W + \frac{W_n x_n}{l}$. The load $W_1$ caused a positive shear at $C$ equal to $\frac{W_1 x_1}{l}$ when it was at $C$; its effect after the loads move a distance $d_1$ to the left is zero; then the decrease in shear is $\frac{W_1 x_1}{l}$ The total increase is, therefore, $\frac{d_1}{l} \times \Sigma W + \frac{W_n x_n}{l} - \frac{W_1 x_1}{l}$; if this expression is negative, the shear decreases, and is a maximum when $W_1$ is at $C$; if positive, the shear increases, and is greater when $W_2$ is at $C$ than when $W_1$ is at $C$.

Starting with $W_2$ at $C$, Fig. 10 ($a$), let it be assumed that $W_1$ is still on the beam but passes off when the loads

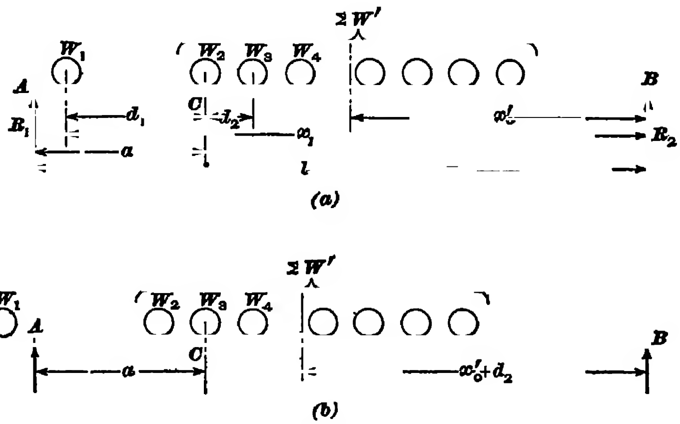



FIG. 10

move to the left, and that, when $W_2$ is at $C$, $W_1$ is still on the beam, no loads coming on at $B$ [Fig. 10 ($b$)]. Here, $\Sigma W'$ will be taken as the sum of all the loads exclusive of $W_1$. The shear when $W_2$ is at $C$ is $\frac{x_0'}{l} \times \Sigma W' + \frac{W_1 x_1}{l} - W_1$, and,

when $W_2$ is at $C$, it is $\frac{x_0' + d_2}{l} \times \Sigma W' - W_2$, the difference being $\frac{d_2}{l} \times \Sigma W' - W_2 + \frac{W_1(l - x_1)}{l}$. If this difference is negative, the shear is decreased; if positive, it is increased.

The student should familiarize himself with the principles just outlined, and not try to remember the expressions as formulas, as each case that arises will probably be somewhat different from those here assumed.

**15. Value of the Shear.**—When the position of the loads has been determined, it is easy to calculate the value of the shear If $Wx$ represents the moment of any one of the loads about the right end, and $W_l$ the sum of all the loads on the left of the section, the shear is equal to

$$\frac{\Sigma Wx}{l} - W_l$$

**16. Section of Maximum Shear.**—The shear is a maximum at the section where the value of $\frac{\Sigma Wx}{l} - W_l$ is greatest; this is just to the right of the left end, and for all practical purposes may be taken at the left end, at that section, $W_l$ is zero and $\Sigma Wx$ is a maximum.

EXAMPLE —A beam 40 feet long supports the system of moving loads shown in Fig 4 What is the maximum shear (*a*) at the left end? (*b*) at a point 10 feet from the left end?

SOLUTION —(*a*) When $W_1$ is at the left end, all the loads are on the beam, when they move to the left, $W_1$ passes off Then, if the loads move 8 ft until $W_2$ is at the end, the increase in the shear is $\frac{8}{40} \times 44{,}000 = 8{,}800$ lb , and the decrease is

$$\frac{5{,}000 \times 40}{40} = 5{,}000 \text{ lb}$$

The shear is therefore greater when $W_2$ is at the end If the loads move 5 ft more to the left until $W_3$ is at the end, the increase in the shear is $\frac{5}{40} \times 34{,}000 = 4{,}250$ lb , and the decrease is $10{,}000 \times \frac{40}{40} = 10{,}000$ lb The shear is therefore greater when $W_2$ is at the end, and its value is

$$\frac{\Sigma Wx}{l} = (10{,}000 \times 40 + 10{,}000 \times 35 + 8{,}000 \times 30 + 8{,}000 \times 24 + 8{,}000 \times 18) \div 40 = 33{,}150 \text{ lb. Ans}$$

(*b*) When $W_1$ is at $C$, 10 ft. from the left end, $W_8$ is at the right end, and, since it will come on the span if the loads move to the left, it may be considered as a part of $\Sigma W$ Then, if the loads move 8 ft. to the left, the increase in shear is $\frac{8}{40} \times 49{,}000 = 9{,}800$ lb , and the decrease is 5,000 lb If the loads move 5 ft farther to the left, the increase in shear is

$$\frac{5}{40} \times 44{,}000 + \frac{2}{40} \times 5{,}000 = 5{,}750 \text{ lb},$$

and the decrease is 10,000 lb Therefore, the shear is a maximum when $W_3$ is at $C$, and its value is

$$\frac{\Sigma W x}{l} - W_1 = (5{,}000 \times 38 + 10{,}000 \times 30 + 10{,}000 \times 25 + 8{,}000 \times 20 + 8{,}000 \times 14 + 8{,}000 \times 8) \div 40 - 5{,}000 = 21{,}900 \text{ lb} \quad \text{Ans.}$$

### EXAMPLES FOR PRACTICE

1 A beam 50 feet long supports the system of moving concentrated loads shown in Fig. 5 What is the maximum shear (*a*) at the left end? (*b*) at a section 5 feet from the left end? (*c*) at a section 15 feet from the left end? (*d*) at the center of the beam?

Ans { (*a*) 92,400 lb; (*b*) 78,900 lb; (*c*) 50,600 lb; (*d*) 26,400 lb

2. A beam 40 feet long supports the system of moving concentrated loads shown in Fig 5 What is the maximum shear (*a*) at the left end? (*b*) at the center?

Ans { (*a*) 81,750 lb; (*b*) 23,000 lb

## CONCENTRATED-LOAD MOMENTS AND SHEARS IN TRUSSES

### MAXIMUM MOMENT AT A JOINT

**17. Joint on the Loaded Chord.**—In order to find the maximum moment at a joint of a truss, due to a system of moving concentrated loads, it is necessary first to find the position occupied by the loads for which the moment at the joint under consideration is a maximum. For this purpose, it will be well to consider the manner in which concentrated loads are transmitted to the panel points and abutments. Let $W$, Fig. 11, be any load moving over a bridge $A B$, when it is at a floorbeam, it is wholly transmitted to one panel

point; when it is between two floorbeams, it is distributed between the two adjacent panel points. In the latter case, the amount that goes to each panel point may be found by resolving $W$ into two components acting vertically through $N$ and $M$, whose values are, respectively, $W_N = W \times \frac{m}{p}$ and $W_M = W \times \frac{n}{p}$. To find the reaction at either end, $W_N$ and $W_M$ may be multiplied by their respective distances from the other end, and the sum of the products divided by the length of the span, as, however, $W$ is the resultant of $W_N$ and $W_M$, the moment of $W$ about any point (such as $Wx$ about $B$) is the same as the sum of the moments of $W_N$ and $W_M$ about the same point. The resultant moment of these components about $B$ is $W_N(x - n) + W_M(x + m) = W\frac{m}{p} \times (x - n) + W\frac{n}{p} \times (x + m) = Wx$. The reactions and bending moments at the panel points can, therefore, be found without computing the panel loads.

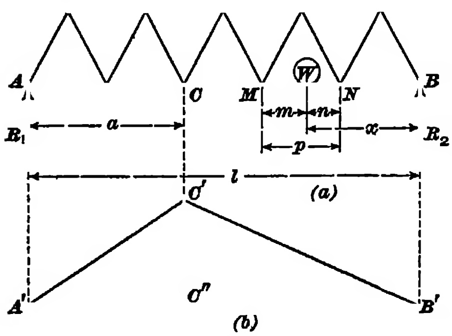


FIG 11

The load in the end panel is transmitted partly to the first panel point of the truss and partly to the abutment: the former is supported by the truss; the latter goes directly to the abutment from the stringers, or, if there is an end floorbeam, this transmits it to the end joint of the truss. In either case, it simply increases the reaction, and causes no stresses in the members of the truss (except the vertical end post of a deck truss), and for this reason the half-panel load at the end was omitted in calculating moments and shears (*Stresses in Bridge Trusses*, Parts 1, 2, and 3). In the present

case, this portion of the load will be different for each different position of the loads, and it will be more convenient to compute the reaction for all the loads on the span, including those in the end panel, in exactly the same way as has already been done in the case of a simple beam. Then, *the maximum moment at any joint of the loaded chord of a truss is the same as the maximum moment at the corresponding point of a simple beam of the same span similarly loaded.* The broken line $A' C' B'$, Fig. 11 (*b*), is the influence line for the moment at $C$.

**18. Moment on Unloaded Chord.**—In the Pratt and other types of simple trusses, in which the joints of the unloaded chord are vertically opposite those of the loaded chord, the moment at any joint of the unloaded chord is the same as that at the opposite joint of the loaded chord. In the Warren and other types of trusses in which the joints of the unloaded chord are not vertically opposite those of the loaded chord, the calculation of moments at the joints of the unloaded chord requires special consideration.

**19. Single Moving Load.**—The variation in moment at a joint of the unloaded chord of a Warren truss can best

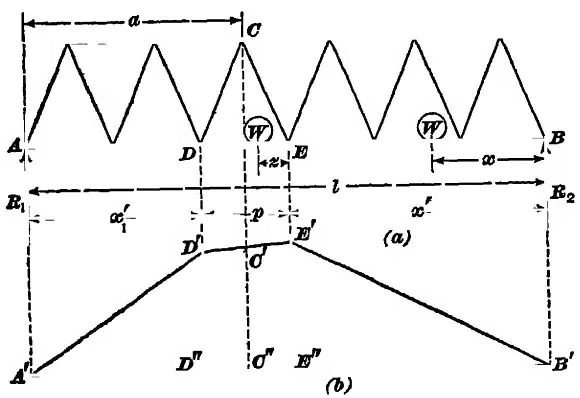



FIG 12

be illustrated by considering the effect of a single moving load, such as $W$, Fig. 12 (*a*). As $W$ moves from $B$ to $E$,

the moment at $C$ is equal to $R_1 \times a$; if $x$ is the distance of $W$ from $B$ when $W$ is between $B$ and $E$, $R_1$ is equal to $\frac{Wx}{l}$, and, as $x$ increases uniformly, the moment at $C$, which is $\frac{Wx}{l} \times a$, increases uniformly until $W$ reaches $E$. The variation in moment at $C$ as $W$ moves from $B$ to $E$ is represented by $B' E'$, Fig 12 (*b*) When $W$ is between $E$ and $D$, a portion of it, $W_D$, is transmitted to $D$, and the remainder, $W_E$, to $E$; then, the moment at $C$ is equal to $R_1 \times a - W_D \times \frac{p}{2}$ It has been shown that $R_1 \times a$ increases uniformly as $W$ moves toward the left; if $z$ represents the distance of $W$ from $E$, then $W_D$ is equal to $\frac{Wz}{p}$, and, as $z$ increases uniformly, $W_D$, and therefore $W_D \times \frac{p}{2}$, must increase uniformly, hence, the difference between $R_1 \times a$ and $W_D \times \frac{p}{2}$ must change at a uniform rate; that is, the influence line from $E$ to $D$ must be straight. The variation in moment at $C$ as $W$ moves from $E$ to $D$ is represented by $E' D'$, Fig. 12 (*b*). When $W$ is at the left of $D$, it is more convenient to consider the right-hand reaction; if $x_1$ denotes the distance from $A$ to $W$, the moment at $C$ is equal to $\frac{Wx_1}{l} \times (l-a)$. As $x_1$ changes uniformly, the moment at $C$ decreases uniformly as $W$ moves from $D$ to $A$. The variation in moment at $C$ as $W$ moves from $D$ to $A$ is represented by $D' A'$, Fig. 12 (*b*).

**20. Influence Line for the Moment at *C*.**—If $x_1'$ represents the distance of $D$ from $A$ and $x'$ the distance of $E$ from $B$, the influence line for the moment at $C$ may be drawn by laying off to scale $E' E''$ equal to $1 \times \frac{x'}{l} \times a$, and $D' D''$ equal to $1 \times \frac{x_1'}{l} \times (l-a)$, and connecting the points so found with each other and with $B'$ and $A'$, respectively, forming the influence line $A' D' E' B'$. It may be shown, in

a similar manner, that the influence line between two consecutive joints of the loaded chord of a truss is always a straight line. In the present case, it is evident that the moment at $C$ is a maximum when the single load is at $E$, and is represented by $C' C''$, in any case, *the moment at a joint of the unloaded chord of the simple Warren truss, due to a single moving load, is a maximum when the load is at one of the panel points at the end of the panel opposite the joint*

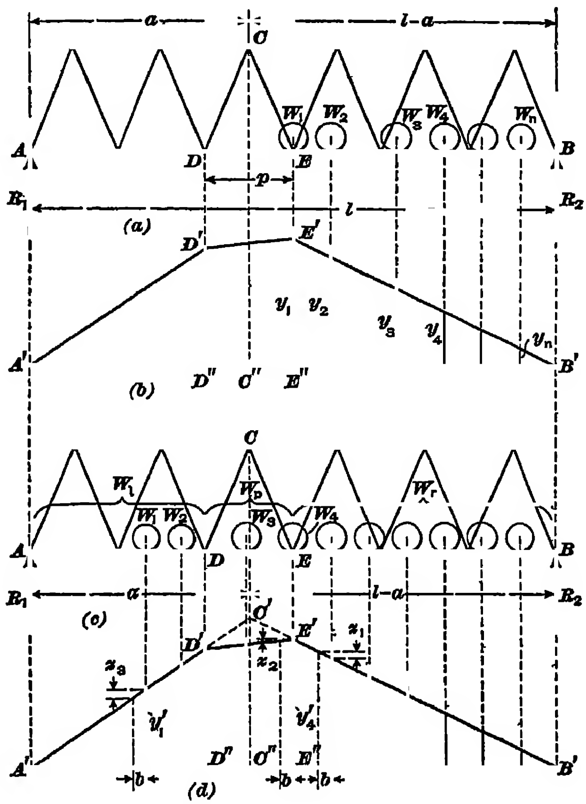



FIG. 13

**21. Any Number of Loads.**—Let $AB$, Fig. 13 ($a$), be a Warren truss over which a system of concentrated loads is moving, and let it be required to determine the position of

the loads when they cause maximum moment at $C$, at a distance $a$ from the left end. Let $A'D'E'B'$, Fig. 13 ($b$), be the influence line for the moment at $C$. As the loads move on the span at $B$, and $W_1$ approaches $E$, the moment at $C$ increases until $W_1$ reaches $E$; if $y_1$, $y_2$, etc. represent the ordinates to the influence line at $W_1$, $W_2$, etc., the moment at $C$ is equal to $W_1 y_1 + W_2 y_2 + \quad W_n y_n$. Now, if the loads move a small distance to the left, $W_1$ will pass $E$; $y_2, y_3$, etc. will each increase by an amount that may be called $z_1$, and $y_1$ will decrease by an amount $z_2$; then, the change in moment is equal to $(W_2 + W_3 + \quad W_n) z_1 - W_1 z_2$, and the moment is increased or decreased according as this is positive or negative.

Suppose that the moment increases until $W_4$ reaches $E$, and that in this position $W_1$ and $W_2$ are to the left of $D$; $W_3$ is between $D$ and $E$, and the other loads are to the right of $E$, as shown in Fig. 13 ($c$). If the loads move a small distance $b$ to the left, the change in moment is equal to

$$(W_5 + W_6 + \quad . W_n) z_1 - [(W_3 + W_4) z_2 + (W_1 + W_2) z_3]$$

and the moment is increased or decreased according to whether the value of this change in moment is positive or negative. If the moment is a maximum when $W_4$ is at $E$, a portion of $W_4$ may be considered on the left and a portion on the right of $E$, so that

$$W_r z_1 = W_p z_2 + W_l z_3 \qquad (1)$$

letting $W_r$ represent the sum of all the loads on the right of $E$, including a portion of the load at $E$, $W_p$, the sum of all the loads in the panel $DE$, including a portion of the load at $E$; and $W_l$, the sum of all the loads to the left of $D$.

In Fig. 13 ($d$), if $A'D'$ and $B'E'$ are produced, they will meet at $C'$ at distances $a$ and $l - a$ from $A'$ and $B'$, respectively. By similar triangles, the following values can be readily found:

$$z_1 = b \times \frac{C'C''}{C''B'} = b \times \frac{C'C''}{l-a}; \; z_2 = b \times \frac{C'C''}{A'C''} = b \times \frac{C'C''}{a},$$

$$z_2 = b \times \frac{E'E'' - D'D''}{D''E''} = \frac{b}{p}\left(C'C''\frac{l-a-\frac{p}{2}}{l-a} - C'C''\frac{a-\frac{p}{2}}{a}\right)$$

$$= b \times C'C'' \times \frac{\left(\frac{l}{2} - a\right)}{a(l-a)}$$

Substituting these values in equation (1),

$$W_r \times b \times \frac{C'C''}{l-a} = W_p \times b \times C'C'' \times \frac{\frac{l}{2} - a}{a(l-a)} + W_l \times b\frac{C'C''}{a};$$

whence $$W_r = \frac{W_p l}{2a} - W_p + W_l \times \frac{l}{a} - W_l$$

and $$\frac{W_r + W_p + W_l}{l} = \frac{\frac{W_p}{2} + W_l}{a}$$

or, denoting by $\Sigma W$ the sum $W_r + W_p + W_l$ of all the loads on the span,

$$\frac{\Sigma W}{l} = \frac{\frac{W_p}{2} + W_l}{a}, \text{ and } W_l + \frac{W_p}{2} = \frac{a}{l} \times \Sigma W$$

From this, the following principle may be given for finding the maximum moment at a joint of the unloaded chord of a Warren truss: *The maximum moment due to a system of moving concentrated loads obtains when one of the loads is at the next panel point of the loaded chord to the right or left of the joint considered. The load at the panel joint must be such that, when considered part on one side and part on the other side of the panel point, the sum of one-half the loads in the panel opposite the given joint and the loads on the left of the panel, divided by the length of truss to the left of the joint, is equal to the average intensity of load on the span*

**22. Value of the Moment.**—As the influence line, Fig. 13 (*b*), is straight between $D$ and $E$, and as $C$ is midway between $D$ and $E$, the ordinate at $C$ is equal to one-half the sum of the ordinates at $D$ and $E$. It can be shown that, for any number of loads, the moment varies uniformly between panel points. The moment $M_C$ at $C$ is equal to

one-half the sum of the moments $M_D$ and $M_E$ at $D$ and $E$, respectively; that is,

$$M_C = \frac{M_D + M_E}{2}$$

EXAMPLE.—What is the maximum moment due to the system of concentrated loads shown in Fig. 4, at the second joint $C$ of the upper chord of a six-panel through Warren truss, assuming the span to have a length of 60 feet?

SOLUTION.—As the system of loads is shorter than the truss, it may be assumed that all the loads will be on when the moment is a maximum. Then, $a = 15$ ft., $l = 60$ ft., and

$$\frac{a}{l} \times \Sigma W = \frac{15}{60} \times 49{,}000 = 12{,}250 \text{ lb.}$$

Let $D$ and $E$ be the panel points to the left and right of $C$, respectively. When $W_2$ is at $E$, if all of $W_2$ is considered on the left of the point $E$, we have

$$W_l + \frac{W_p}{2} = \frac{15{,}000}{2} = 7{,}500 \text{ lb.}$$

As this is less than 12,250, the moment is not a maximum.

When $W_3$ is at $E$, if all of $W_3$ is considered on the right of the point $E$, we have

$$W_l + \frac{W_p}{2} = 5{,}000 + \frac{10{,}000}{2} = 10{,}000 \text{ lb.}$$

If all of $W_3$ is considered on the left of $E$,

$$W_l + \frac{W_p}{2} = 5{,}000 + \frac{20{,}000}{2} = 15{,}000 \text{ lb}$$

As one value is less and the other greater than 12,250, the moment is a maximum.

When $W_4$ is at $E$, $W_2$ is at $D$, and, if $W_4$ and $W_2$ are considered on the right of $E$ and $D$, respectively, giving the smallest value of $W_l + \frac{W_p}{2}$, then,

$$W_l + \frac{W_p}{2} = 5{,}000 + \frac{20{,}000}{2} = 15{,}000 \text{ lb}$$

As this is greater than 12,250, the moment is not a maximum. Therefore, the only position that causes a maximum moment at $C$, is when $W_3$ is at $E$. We have, then, for the reaction $R_1$ and the moments at the points $D$, $E$, and $C$, respectively,

$$R_1 = (5{,}000 \times 53 + 10{,}000 \times 45 + 10{,}000 \times 40 + 8{,}000 \times 35 + 8{,}000 \times 29 + 8{,}000 \times 23) \div 60 = 30{,}180 \text{ lb}$$

$$M_D = 30{,}180 \times 10 - 5{,}000 \times 3 = 286{,}800 \text{ ft.-lb.},$$

$$M_E = 30{,}180 \times 20 - 5{,}000 \times 13 - 10{,}000 \times 5 = 488{,}600 \text{ ft.-lb.};$$

$$M_C = \frac{286{,}800 + 488{,}600}{2} = 387{,}700 \text{ ft.-lb.} \quad \text{Ans.}$$

EXAMPLE FOR PRACTICE

Let the eight-panel through Warren truss represented in Fig 14 support the system of moving concentrated loads represented in Fig. 5.

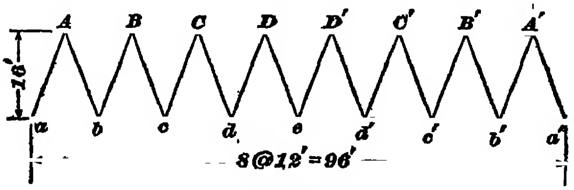



FIG 14

What is the maximum moment. (*a*) at joint *c*? (*b*) at joint *e*? (*c*) at joint *B*?

Ans. (*a*) 2,032,500 ft -lb.
(*b*) 2,680,000 ft.-lb.
(*c*) 1,589,400 ft -lb.

MAXIMUM SHEAR IN A PANEL

**23. Introduction.**—In *Stresses in Bridge Trusses*, Part 1, it was explained that the shear due to a uniform load in any panel of a truss is uniform between the panel points; in the case of concentrated loads, they are transmitted to the trusses at the panel points, and, therefore, the shear at all points in any panel is the same. The position of the joints of the unloaded chord with respect to those of the loaded chord has no effect in the computation for shear

**24. Influence Line for Shear in a Panel.**—Let $AB$, Fig. 15 (*a*), be a truss that supports a load of unity that moves from $B$ to $A$, and let it be required to construct the influence line for the shear in panel $CD$. As the load moves from $B$ to $D$, the shear is positive and equal to $R_1$, and increases uniformly as the load moves toward $D$; as the load moves from $D$ to $C$, the shear decreases uniformly, and is equal to $R_1$ minus the portion that goes to $C$, it becomes zero for some position of the load between $D$ and $C$, then it is negative and increases numerically until the load is at $C$, when the shear is $-R_2$. As the load moves from $C$ to $A$, the shear changes from $-R_2$ to zero; that is, it increases algebraically, although its numerical value decreases from $R_2$

to zero. The broken line $B'D'C'A'$, Fig. 15 (*b*), is the influence line for shear in the panel $CD$, the portion $B'D'$ being parallel to the portion $C'A'$, as can be easily shown. The ordinates $D'D''$ and $C'C''$ are equal, respectively, to $R_1$ and $R_2$, the former of these reactions being for the position $D$ of the load, the latter for the position $C$. The point $E$ where

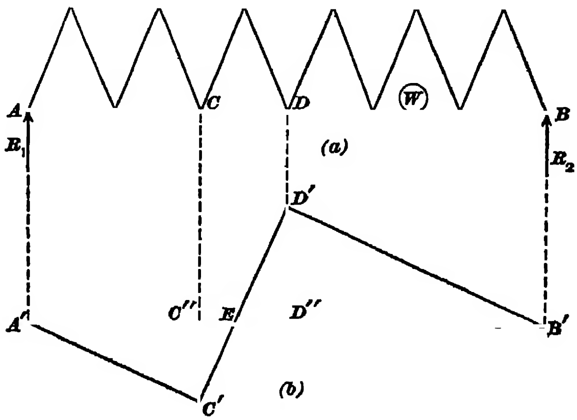



FIG 15

$C'D'$ crosses $A'B'$ is where the load causes no shear in the panel $CD$. In this case, the portions of the load that go to $C$ and $D$ are equal to $R_1$ and $R_2$, respectively, since the shear equals $R_1$ minus the portion going to $C$, and, if the shear is equal to zero, then $R_1$ equals the portion going to $C$. It can be shown that the influence line for shear in any panel is straight between the panel points.

**25. Any Number of Loads.**—Let Fig. 16 (*a*) represent a truss over which a system of concentrated loads is moving, and let it be required to determine the position of the loads when they cause maximum positive shear in the panel $CD$. The influence line is shown in Fig 16 (*b*). When the shear is a maximum, there will be few or no loads in the panel $CD$, and probably none to the left of $C$, but in order to generalize the discussion, it will be assumed that there are loads to the left of $C$ as well as in the panel $CD$. Let $W_l$, $W_p$, and $W_r$

represent, respectively, the sum of the loads to the left of $C$, in the panel $CD$, and to the right of $D$; and let $\Sigma W$ be the sum of all the loads on the span. If, when the loads move a short distance $b$ to the left, the changes in the ordinates to the influence line are represented by $z_1$, $z_2$, and $z_3$, Fig. 16 ($b$),

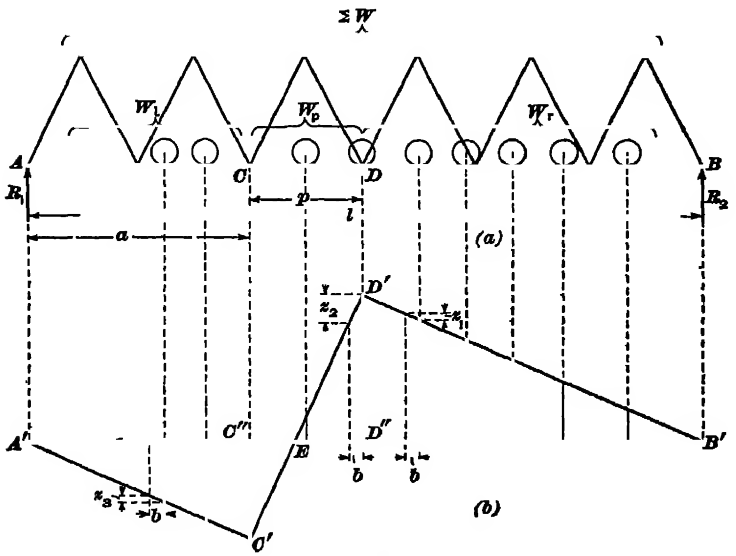



FIG 16

the change in shear will be $W_l z_3 + W_r z_1 - W_p z_2$. The shear will be a maximum when a load is at $D$ such that, if a portion of it is considered to the left and a portion to the right of $D$, the following equation obtains.

$$W_l z_3 + W_r z_1 = W_p z_2 \qquad (1)$$

From similar triangles in Fig. 16 ($b$), the following values are obtained:

$$z_1 = b \times \frac{D'D''}{D''B'} = b \times \frac{D'D''}{l - a - p}$$

$$z_3 = b \times \frac{C'C''}{A'C''} = b \times \frac{D'D''}{l - a - p}$$

$$z_2 = b \times \frac{D'D'' + C'C''}{C''D''} = \frac{b}{p}\left(D'D'' + a \times \frac{D'D''}{l - a - p}\right)$$

Substituting these values in equation (1), multiplying by $\frac{l-a-p}{b \times D' D''}$, and reducing,

$$W_l + W_r + W_p = W_p \times \frac{l}{p};$$

whence
$$\frac{\Sigma W}{l} = \frac{W_p}{p}$$

and
$$W_p = \frac{p}{l} \times \Sigma W$$

Hence the following principle: *The maximum positive shear in any panel of a truss, due to a system of moving concentrated loads, obtains when one of the loads is at the joint to the right of the panel considered. The load at the panel point must be such that, if a portion of it is considered on each side of the joint, the average intensity of load in the panel is equal to the average intensity of load on the span*

**26. Value of the Shear.**—The shear is equal to $R_1$ minus the portion of the loads in the panel that goes to the left of the panel, minus the sum of all the loads to the left of this joint If $Wx$ is the moment of any load about the right end, $R_1$ is equal to $\frac{\Sigma Wx}{l}$. If $Wz$ is the moment of any load in the panel about the joint at the right of the panel, the portion that goes to the joint at the left of the panel is equal to $\frac{\Sigma' Wz}{p}$. The shear is, therefore,

$$V = \frac{\Sigma Wx}{l} - \frac{\Sigma' Wz}{p} - W_l$$

EXAMPLE —The six-panel through Warren truss represented in Fig. 17 supports the system of moving concentrated loads shown in

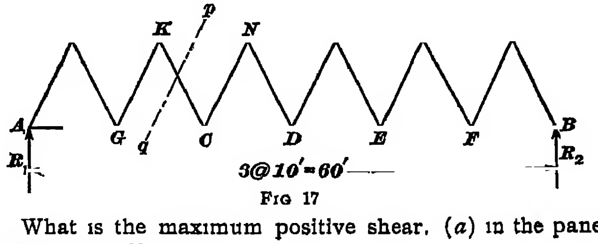



FIG 17

Fig 4 What is the maximum positive shear, (a) in the panel CD? (b) in the panel DE?

SOLUTION.—(*a*) When $W_1$ is at $D$, $W_4$ is at $B$, and may be considered as off the bridge, then,

$$W_p = \frac{10}{60} \times 41{,}000 = 6{,}833 \text{ lb}$$

As $W_1$ is 5,000 lb., or less than 6,833 lb., the shear is not a maximum. When $W_2$ is at $D$, all the loads are on the span, then,

$$W_p = \frac{10}{60} \times 49{,}000 = 8{,}167 \text{ lb}$$

$W_1 = 5{,}000$ lb., and $W_1 + W_2 = 15{,}000$ lb. As one is less and the other greater than $W_p$, the shear is a maximum when $W_2$ is at $D$. Then,

$$V = (5{,}000 \times 38 + 10{,}000 \times 30 + 10{,}000 \times 25 + 8{,}000 \times 20 + 8{,}000 \times 14 + 8{,}000 \times 8) \div 60 - \frac{5{,}000 \times 8}{10} = 13{,}930 \text{ lb} \quad \text{Ans.}$$

(*b*) When $W_1$ is at $E$, $W_5$ and $W_6$ are off the span, then,

$$W_p = \frac{10}{60} \times 33{,}000 = 5{,}500 \text{ lb.}$$

As this is greater than $W_1$, the shear is not a maximum. When $W_2$ is at $E$, $W_6$ is off the span, then

$$W_p = \frac{10}{60} \times 41{,}000 = 6{,}833 \text{ lb}$$

As this is greater than $W_1$ and less than $W_1 + W_2$, the shear is a maximum. Then,

$$V = \frac{5{,}000 \times 28 + 10{,}000 \times 20 + 10{,}000 \times 15 + 8{,}000 \times 10 + 8{,}000 \times 4}{60} - \frac{5{,}000 \times 8}{10} = 6{,}030 \text{ lb} \quad \text{Ans.}$$

### EXAMPLE FOR PRACTICE

The eight-panel through Warren truss represented in Fig. 14 supports the system of moving concentrated loads represented in Fig. 5. What is the maximum positive shear (*a*) in the panel *b c*? (*b*) in the panel *c d*? (*c*) in the panel *e d'*?

Ans. (*a*) 81,460 lb; (*b*) 63,340 lb; (*c*) 27,090 lb

## MAXIMUM FLOORBEAM AND PANEL LOADS

**27. Maximum Floorbeam Load.**—The load that comes on a floorbeam is equal to the sum of the reactions on the stringers in the two adjacent panels. This is equal to twice the reaction from one stringer if the panels are equal and uniform loads are considered, but for concentrated loads the reactions will usually be different, and it is necessary to determine the position of the loads when they cause the maximum load on a floorbeam.

Let Fig. 18 ($a$) represent two adjacent panels of a floor system, of lengths $a$ and $a'$, respectively, and let it be required to determine the maximum reaction at $C$ as a system of concentrated loads moves from $B$ to $A$. The stringers may be assumed to be simply supported at $A$, $B$, and $C$, so that the reactions at these points may be computed as usual. For example, the reaction at $C$ due to $W$ is $\frac{Wx}{a'}$. A load at $C$ is supported directly by the floorbeam. The line $B'C'A'$, Fig 18 ($b$), is the influence line for the reaction at $C$, and, as this has the same form as the influence line for the moment at $C$ in a beam of the length $AB$, it can be shown, in a manner similar to that employed in Art. 5, that the condition for maximum moment at the section $C$ in a beam of length $AB$ is also the condition for maximum reaction at $C$. That is: *The load on a floorbeam that comes from two adjacent panels is a maximum when such a load is at the floorbeam that, if considered part on each side, the average intensity of load on the panel to the left is equal to the average intensity of load on the panel to the right, and also equal to the average intensity of load on the two panels taken together.*

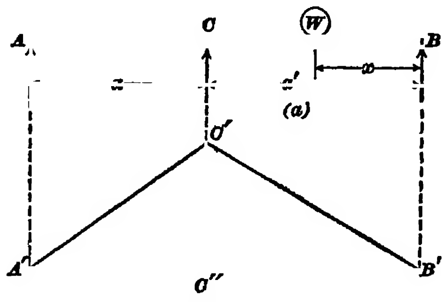


Fig 18

In the ordinary case, the panel lengths $a$ and $a'$ are equal, and the principle just stated then reduces to the following:

*The load on a floorbeam, when the panels on each side of it are equal, is a maximum when such a load is at the floorbeam that, if considered part on each side, the load in the panel to the left will be equal to the load in the panel to the right, and equal to one-half the sum of the loads in the two panels.*

**28. Maximum Panel Load.**—The maximum panel load can be found from the maximum floorbeam load. In almost every case, the former will be equal to one-half the latter.

EXAMPLE —What is the maximum panel load in a truss that supports the system of loads shown in Fig 4, the panels being 15 feet long?

SOLUTION —In determining maximum moments, it has been seen that the condition is usually satisfied when one of the heavier loads is at the section In this case, then, it will be unnecessary to try $W_1$ at $C$ When $W_2$ is at $C$, the total load in both panels is 33,000 lb , as one-half of this is greater than the sum of $W_1$ and $W_2$, this position does not give a maximum

When $W_3$ is at $C$, the total load is 41,000 lb As one-half of 41,000 is greater than the sum of $W_1$ and $W_2$, and less than the sum of $W_1$, $W_2$, and $W_3$, this position fulfils the conditions for a maximum

When $W_4$ is at $C$, the total load is equal to 44,000 lb As one-half of 44,000 is greater than the sum of $W_2$ and $W_3$, and less than the sum of $W_2$, $W_3$, and $W_4$, this position fulfils the conditions for a maximum.

When $W_3$ is at $C$, the panel load is

$$\frac{\dfrac{5{,}000 \times 2 + 10{,}000 \times 10 + 8{,}000 \times 4 + 8{,}000 \times 10}{15} + 10{,}000}{2} = 12{,}400 \text{ lb.}$$

When $W_4$ is at $C$, the panel load is

$$\frac{\dfrac{10{,}000 \times 5 + 10{,}000 \times 10 + 8{,}000 \times 3 + 8{,}000 \times 9}{15} + 8{,}000}{2} = 12{,}200 \text{ lb.}$$

The panel load is, therefore, greatest when $W_3$ is at $C$, it is equal to 12,400 lb. Ans

### EXAMPLES FOR PRACTICE

1 If the system of concentrated loads shown in Fig 4 moves across a bridge, what is the maximum load that comes on a floorbeam supporting the adjacent ends of two panels 12 and 15 feet long, respectively?
Ans 23,300 lb

2 What is the maximum panel load on a truss that supports the system of loads shown in Fig. 5, when the length of panels is 20 feet?
Ans. 34,380 lb.

3 The adjacent ends of two spans 40 and 60 feet long, respectively, rest on the same pier What is the maximum load on the pier when two systems of loads, each like that shown in Fig 5, move over the spans, assuming the first (left) load of the second system to be 7 feet to the right of the last load of the first system? Ans 145,750 lb.

# STRESSES DUE TO CONCENTRATED LOADS

## PARALLEL-CHORD TRUSSES

### SINGLE-SYSTEM TRUSSES

**29.** From the preceding principles, the live-load stresses in the members of any parallel-chord truss with a single system of web, and in the members of Warren trusses with secondary verticals, can be readily determined. As the maximum live-load stresses are usually desired, it is necessary to find the maximum live-load moments or shears, as explained in the preceding pages.

For example, to find the maximum live-load stress in the diagonal $KC$ of the truss $AB$, Fig. 17, the shear in the panel $GC$ is considered. The live-load tension in $KC$ is greatest when the positive live-load shear in the panel $GC$ (or on the plane of section $pq$) is a maximum; the live-load compression is greatest when the negative live-load shear in the panel $GC$ is a maximum. As the maximum positive live-load shear in the panel $EF$ is equal to the maximum negative live-load shear in the panel $GC$, the maximum live-load tension and compression in the diagonal $KC$ are equal, respectively, to the product of csc $H$ and the maximum positive shears in the panels $GC$ and $EF$, found as explained in Art. **25** for the system of loads under consideration.

The live-load stress in the chord member $KN$, Fig. 17, is equal to the live-load moment at $C$ divided by the height of the truss, and is greatest when the moment at $C$ is a maximum. The maximum moment at $C$ for the system of loads under consideration can be found as explained in Art. **17.**

Similarly, the stress in $GC$ can be found from the moment at $K$ found as explained in Arts. **21** and **22.**

In the following pages, whenever stresses are mentioned, live-load stresses are meant, unless dead-load stresses are specified

The methods of finding the stresses in some of the members of the Baltimore truss and in the members of multiple-system trusses will now be discussed.

### THE BALTIMORE TRUSS

**30.** In the Baltimore truss, Fig. 19 (*a*), the maximum stresses in the upper-chord members can be found from the maximum moments at the opposite joints; the maximum stresses in the lower halves of the main diagonals can be found from the maximum shears in the respective panels; and the maximum stresses in the subverticals and substruts can be found from the maximum panel loads. There remains to be considered the maximum stresses in the lower-chord members, in the upper halves of the main diagonals, in the counters, and in the main verticals.

**31. Lower Chord.**—The maximum stress in the end panel of the lower chord is equal to the horizontal component of the maximum stress in the lower half of the end post, which is found from the maximum shear in the end panel. The maximum stress in any other part of the lower chord, such as $GD$, Fig. 19 (*a*), is found from the moment at the center of moments in the upper chord, in this case joint $E$. The plane $q$ that cuts $GD$ and two other members is to the right of the panel load at $G$, while the center of moments is to the left of this panel load; the moment of the load at $G$ is then opposite in sign to that of the loads to the left of $C$. On this account, the influence line for the moment at $E$ is different from the influence lines that have already been discussed, being composed of three lines $B_1D_1$, $D_1G_1$, and $G_1A_1$, Fig. 19 (*b*). Now, if $W_l$ denotes the sum of the loads in the length $AG$, $W_p$ the sum of the loads in the

length $GD$ (the panel cut by the plane $q$), and $\Sigma W$ the total load on the span $AB$, it may be shown, in a manner similar to that employed in Art 21, that *the moment at E is a maxi-*

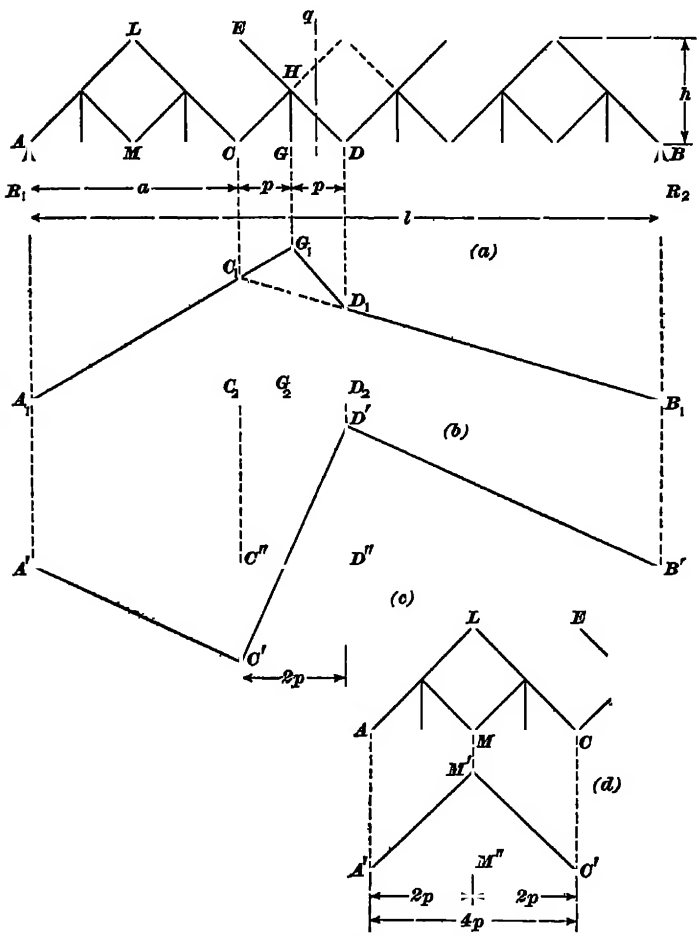



FIG 19

*mum when such a load is at G that, if considered part on one side and part on the other, the difference between the loads to the left of G and those in panel GD, divided by the length of the truss to the left of the center of moments is equal to the average*

*intensity of the load on the span.* That is, the stress in the lower chord is a maximum when

$$\frac{W_l - W_p}{a} = \frac{\Sigma W}{l}$$

or when

$$W_l - W_p = \frac{a}{l} \times \Sigma W$$

**32. Upper Half of Main Diagonals.**—The influence line for the vertical component of the stress in the upper half of a main diagonal, such as $EH$, Fig. 19 ($a$), may be drawn by computing the vertical component of the stress due to a load of unity that moves over the span. This line is composed of the three straight lines $B'D'$, $D'C'$, and $C'A'$, Fig 19 ($c$); this is exactly the same as the influence line for shear in the panel $CD$ of a Pratt truss having the same length and half as many panels as the truss represented in Fig 19 ($a$). It can be shown (see Art. 25) that *the stress in $EH$ is a maximum when there is such a load at $D$ that, if considered part on each side, the average intensity of load in the double panel $CD$ is equal to the average intensity of load on the span.* That is,

$$\frac{W_{2p}}{2p} = \frac{W}{l};$$

whence

$$W_{2p} = \frac{2p}{l} \times \Sigma W$$

The vertical component of the stress in $EH$ can be found, as usual, by subtracting from the shear in panel $CG$ one-half the panel load at $G$, both being computed when the loads are in a position that satisfies the condition just stated; the result is the same as though the shear were computed in the panel $CD$ of the Pratt truss already mentioned.

**33. Intermediate Verticals.**—The stress in any intermediate vertical, such as $EC$, Fig. 19 ($a$), is a maximum when the stress in the adjacent diagonal is a maximum, and is equal to the vertical component of the maximum stress in that diagonal.

**34. Hip Vertical.**—The influence line for the stress in $LM$ is represented in Fig 19 ($d$), and has the same form as

the influence line for the moment at the center of a beam having a span equal to four panel lengths. Therefore, *the stress in the hip vertical is a maximum when such a load is at its foot that, if considered part on each side, the average intensity of the load in the two panels to the left is equal to the average intensity of the load in the two panels to the right.* It can be shown that the stress in $LM$ is equal to the maximum panel load for a truss having a panel length equal to $2p$.

**35. Counters.**—When the upper half of a counter is in action, the lower half of the main diagonal in the same panel is out of action, and the stress in the counter may be found, as usual, from the shear. The vertical component of the stress in the lower half of a counter, such as $CH$, Fig. 19 $(a)$, is equal to the maximum negative shear in the panel $CD$ of a Pratt truss of the same span and half the number of panels.

EXAMPLE.—The Baltimore truss shown in Fig. 20 is one truss of a bridge that supports the system of concentrated loads followed by a uniform load represented in Fig. 21. What is the maximum stress: $(a)$ in the upper-chord member $CE$? $(b)$ in the lower-chord member $de$? $(c)$ in the diagonal $CD$? $(d)$ in the vertical $Cc$? $(e)$ in the counter $eF$? It is assumed that one-half the load is carried by one truss.

SOLUTION.—$(a)$ The stress in $CE$ is equal to the moment at $e$ divided by 30. It can be found, by trial, that the first position that satisfies the condition for a maximum moment is when $W_8$ is at $e$. In this case, $\frac{a}{l} = \frac{4}{12} = \frac{1}{3}$. The numerical work can be arranged conveniently in tabular form as follows:

| Position of Load | $\Sigma W$ Pounds | $\frac{a}{l} \times \Sigma W$ Pounds | $W_l$ Pounds |
|---|---|---|---|
| $W_8$ at $e$ | 748,000 | 249,300 | Between 222,000 and 244,000 |
| $W_9$ at $e$ | 768,000 | 256,000 | Between 244,000 and 266,000 |
| $W_{10}$ at $e$ | 800,000 | 266,700 | Between 266,000 and 284,000 |
| $W_{11}$ at $e$ | 814,000 | 271,300 | Between 266,000 and 306,000 |
| $W_{12}$ at $e$ | 794,000 | 264,700 | Between 266,000 and 306,000 |

As the values of $\frac{a}{l} \times \Sigma W$ for $W_9$, $W_{10}$, and $W_{11}$ lie between the extreme values of $W_l$, respectively, these three positions satisfy the

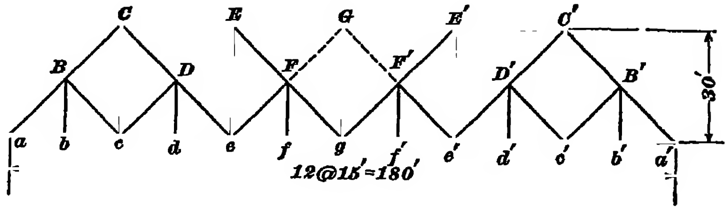

C
E
G
E'
C'
B
D
F
F'
D'
B'
30'
a
b
c
d
e
f
g
f'
e'
d'
c'
b'
a'
12@15'=180'

Fig 20

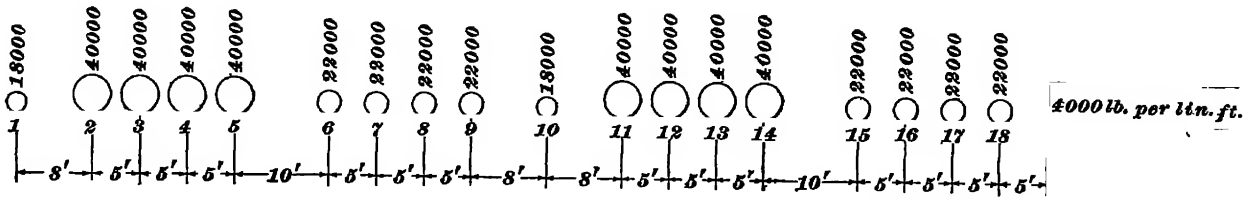

18000
40000
40000
40000
40000
22000
22000
22000
22000
18000
40000
40000
40000
40000
22000
22000
22000
22000
4000 lb. per lin. ft.
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
8' 5' 5' 5' 10' 5' 5' 5' 8' 8' 5' 5' 5' 10' 5' 5' 5' 5'

Fig 21

condition for a maximum, and it is necessary to compute the moment for each position to find the greatest

When $W_9$ is at $e$, $R_1$ = 385,300 lb , and the moment at $e$ is

$$385{,}300 \times 60 - (18{,}000 \times 48 + 160{,}000 \times 32\ 5 + 88{,}000 \times 7\ 5) = 16{,}394{,}000 \text{ ft -lb}$$

When $W_{10}$ is at $e$, $R_1$ = 420,144 lb , and the moment at $e$ is

$$420{,}144 \times 60 - (18{,}000 \times 56 + 160{,}000 \times 40\ 5 + 88{,}000 \times 15\ 5) = 16{,}356{,}640 \text{ ft -lb}$$

When $W_{11}$ is at $e$, $R_1$ = 438,011 lb , and the moment at $e$ is

$$438{,}011 \times 60 - (160{,}000 \times 48\ 5 + 88{,}000 \times 23\ 5 + 18{,}000 \times 8) = 16{,}308{,}660 \text{ ft -lb}$$

The greatest of these moments occurs when $W_9$ is at $e$ Then, the stress in $CE$ is

$$\frac{16{,}394{,}000}{2 \times 30} = 273{,}200 \text{ lb , compression} \quad \text{Ans}$$

(*b*) The stress in $de$ is equal to the moment at $C$ divided by 30 It will be found, by trial, that the conditions for maximum moment at $C$ are satisfied when $W_8$, $W_9$, $W_{10}$, and $W_{13}$, respectively, are at $d$ The results of the calculations are as follows

| Position of Load | $\Sigma W$ Pounds | $\frac{a}{l} \times \Sigma W$ Pounds | $W_l - W_p$ Pounds |
|---|---|---|---|
| $W_4$ at $d$ | 708,000 | 118,000 | Between 18,000 and 76,000 |
| $W_5$ at $d$ | 728,000 | 121,300 | Between 76,000 and 134,000 |
| $W_6$ at $d$ | 768,000 | 128,000 | Between 112,000 and 134,000 |
| $W_7$ at $d$ | 788,000 | 131,300 | Between 134,000 and 178,000 |
| $W_9$ at $d$ | 810,000 | 135,000 | Between 186,000 and 230,000 |
| $W_{10}$ at $d$ | 802,000 | 133,700 | Between 110,000 and 146,000 |
| $W_{11}$ at $d$ | 754,000 | 125,700 | Between 26,000 and 66,000 |
| $W_{12}$ at $d$ | 734,000 | 122,300 | Between 26,000 and 106,000 |
| $W_{13}$ at $d$ | 754,000 | 125,700 | Between 106,000 and 164,000 |
| $W_{14}$ at $d$ | 752,000 | 125,300 | Between 142,000 and 200,000 |

Since the value of $\frac{a}{l} \times \Sigma W$ lies between the extreme values of $W_l - W_p$ when $W_8$, $W_9$, $W_{10}$, and $W_{13}$, respectively, are at $d$, the conditions for maximum moment at $C$ are fulfilled for each of these positions, and it is necessary to calculate the moment for each position to find the greatest moment

The moment at $C$ is equal to $R_1$ multiplied by 30, minus the moment of all the loads to the left of $c$ about $C$, plus the moment of the panel load at $d$ about $C$, as explained in *Stresses in Bridge Trusses*, Part 2.

When $W_8$ is at $d$, $R_1$ is 343,744 lb, and the panel load at $d$ is 87,333 lb Then, the moment at $C$ is

$$343{,}744 \times 30 - 18{,}000 \times 8 + 87{,}333 \times 15 = 11{,}478{,}320 \text{ ft -lb.}$$

When $W_9$ is at $d$, $R_1$ is 385,300 lb, and the panel load at $d$ is 57,333 lb Then, the moment at $C$ is

$$385{,}300 \times 30 - [(18{,}000 \times 18) + (40{,}000 \times 10) + (40{,}000 \times 5)] + 57{,}333 \times 15 = 11{,}495{,}000 \text{ ft -lb}$$

When $W_{10}$ is at $d$, $R_1$ is 429,544 lb, and the panel load at $d$ is 55,200 lb Then, the moment at $C$ is

$$429{,}544 \times 30 - (120{,}000 \times 23 + 22{,}000 \times 8 + 22{,}000 \times 3) + 55{,}200 \times 15 = 10{,}712{,}320 \text{ ft -lb}$$

When $W_{12}$ is at $d$, $R_1$ is 386,011 lb, and the panel load at $d$ is 106,667 lb Then, the moment at $C$ is

$$386{,}011 \times 30 - [(88{,}000 \times 18.5) + (18{,}000 \times 3)] + 106{,}667 \times 15 = 11{,}498{,}330 \text{ ft -lb}$$

The last value found is the greatest, so the moment at $C$ is greatest when $W_{12}$ is at $d$ The maximum stress in $de$ is, then,

$$\frac{11{,}498{,}330}{2 \times 30} = 191{,}640 \text{ lb, tension} \quad \text{Ans}$$

($c$) The stress in $CD$ is a maximum when the average intensity of load in the double panel $ce$ is equal to the average intensity of load on the span In this case, $\frac{2p}{l} = \frac{1}{6}$

When $W_3$ is at $e$, $\Sigma W = 628{,}000$ lb, $\frac{2p}{l} \times \Sigma W = 104{,}700$ lb., $W_{2p}$ is between 58,000 lb and 98,000 lb

When $W_4$ is at $e$, $\Sigma W = 648{,}000$ lb, $\frac{2p}{l} \times \Sigma W = 108{,}000$ lb., $W_{2p}$ is between 98,000 lb and 138,000 lb

Then, the stress is a maximum when $W_4$ is at $e$, in this position, $R_1 = 267{,}300$ lb

The loads at $c$ and $d$, due to the loads in the panels $cd$ and $de$, are 3,600 lb and 54,400 lb, respectively Then, the vertical component of the stress in $CD$ is

$$\frac{267{,}300 - 3{,}600 - \frac{54{,}400}{2}}{2} = 118{,}250 \text{ lb.}$$

and the stress in $CD$ is

$$118{,}250 \times \frac{\sqrt{\overline{15}^2 + \overline{15}^2}}{15} = 167{,}200 \text{ lb, tension} \quad \text{Ans}$$

($d$) The stress in $Cc$ is a maximum when the average intensity of load in the length $ac$ is equal to the average intensity of load in the length $ce$ There will probably be more loads in the length $ae$ when the second set of heavy loads is in that position, so it is unnecessary to

try $W_2$, $W_3$, etc. at $c$. The results of the calculations for several positions are as follows:

| Position of Load | $\Sigma W$ Pounds | $\frac{a}{l} \times \Sigma W$ Pounds | $W_l$ Pounds |
|---|---|---|---|
| $W_{11}$ at $c$ | 266,000 | 133,000 | Between 84,000 and 124,000 |
| $W_{12}$ at $c$ | 266,000 | 133,000 | Between 102,000 and 142,000 |
| $W_{13}$ at $c$ | 266,000 | 133,000 | Between 120,000 and 160,000 |
| $W_{14}$ at $c$ | 266,000 | 133,000 | Between 138,000 and 178,000 |

The condition for maximum stress in $Cc$ is satisfied when $W_{12}$ is at $c$ and when $W_{13}$ is at $c$.

When $W_{12}$ is at $c$, the stress in $Cc$ is 82,050 lb.

When $W_{13}$ is at $c$, the stress in $Cc$ is 82,750 lb , tension Ans

($e$) The maximum stress in $eF$ is the same as that in $e'F'$ when the right end of the truss is loaded, this occurs when the average intensity of load in the double panel $ge'$ is equal to the average intensity of load on the span.

When $W_2$ is at $e'$, $\Sigma W = 324{,}000$ lb , $\frac{2p}{l} \times \Sigma W = 54{,}000$ lb , $W_{2p}$ is between 18,000 lb and 58,000 lb.

When $W_3$ is at $e'$, $\Sigma W = 364{,}000$ lb , $\frac{2p}{l} \times \Sigma W = 60{,}700$ lb , $W_{2p}$ is between 58,000 lb and 98,000 lb

Both positions fulfil the conditions for maximum stress

When $W_2$ is at $e'$, $R_1$ for one truss is 34,500 lb , and the load at $f'$ is 4,800 lb The vertical component of the stress in $e'F'$ is

$$34{,}500 - 4{,}800 + \frac{4{,}800}{2} = 32{,}100 \text{ lb}$$

When $W_3$ is at $e'$, $R_1$ for one truss is 39,450 lb , and the load at $f'$ is 14,450 lb. The vertical component of the stress in $e'F'$ is

$$39{,}450 - 14{,}450 + \frac{14{,}450}{2} = 32{,}200 \text{ lb.}$$

Then, the stress in $e'F'$ is

$$32{,}200 \times \frac{\sqrt{\overline{15}^2 + \overline{15}^2}}{15} = 45{,}500 \text{ lb., tension. Ans.}$$

### MULTIPLE-SYSTEM TRUSSES

**36. Influence Lines.**—Fig. 22, ($b$) and ($c$), shows the influence lines for the stresses in the chord member $FH$, and the web member $HE$, respectively, of the Whipple truss $AB$ shown in Fig. 22 ($a$); they are typical influence

›r the stresses in the members of multiple-system

awing the influence line for the stress in a member of ple-system truss, the truss is separated into simple ;, as explained in *Stresses in Bridge Trusses*, Part 2,

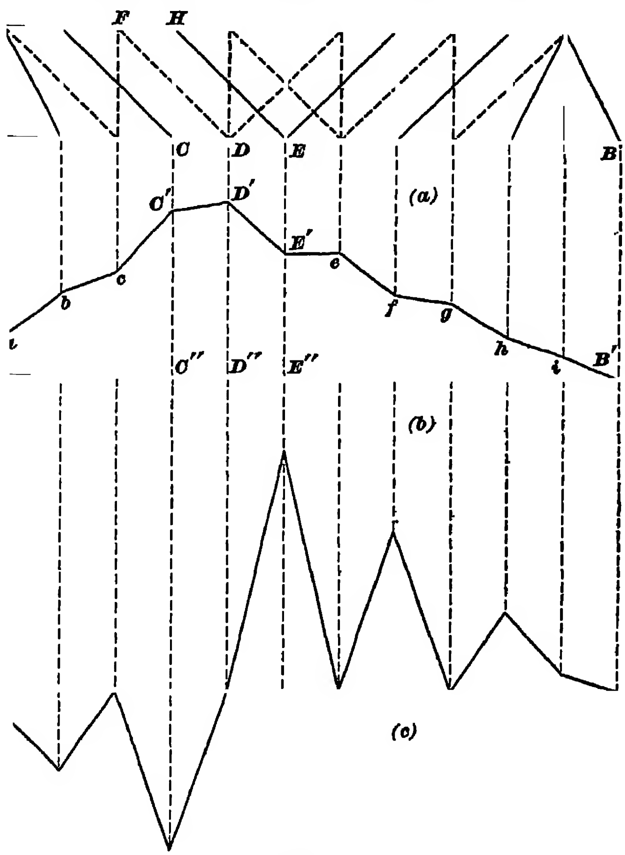



FIG 22

oad of unity is placed successively at the different ›ints. The stress in the member under consideration lated for each position of the load, and is plotted to ›rtically under the panel point as an ordinate (such ′, $D''D'$, $E''E'$, etc.), to a horizontal line $A'B'$. The

line $A' a b c C' D' E'$, etc. connecting the points so located i the influence line for the stress in the member. As these lines are very irregular, the principles that govern the conditions for the maximum stress in a member are very complicated and not adapted to practical use. If desired however, the stress in a member can be found, by the method of influence lines, by plotting to scale the influence line for the member, laying off the series of loads successively in several positions, one of which will probably cause the maximum stress, and scaling the ordinate to the influence line under each load for each position of the loads. Then the algebraic sum of the products of all the loads on the span, and their respective ordinates for any position, will equal the stress in the member.

In order to find the maximum stress in a member, it will be well to determine first the positions the loads occupy when the moment or shear is a maximum at the section of a simple beam (having the same span as the truss) corresponding to the point of the influence line where the ordinate is greatest. For example, for the maximum stress in $HE$, the loads may first be placed in the position they occupy when the shear is a maximum at the section $E$ of a beam of the length $AB$. With the loads in this position, the stress may be calculated as just described, then, with the loads moved successively in each direction until other loads come at the desired point, the stress may be calculated for several positions until it is seen that the stress decreases by moving the loads any farther in either direction The maximum stress will usually be found after two or three trials. This method is simple in application and gives results that are sufficiently accurate if the influence line is constructed and the ordinates scaled very carefully.

**37. Panel Concentrations.**—The method of concentration by panels consists in determining, in the same way as in the preceding article, the probable position the loads occupy when the stress in a member is a maximum, and, when the loads are in that position, calculating the amount that goes

to each panel point of the truss With these panel loads, the stresses may be found in the same way as in *Stresses in Bridge Trusses*, Part 2. By repeating this process for two or three positions, the maximum stress will be found. In finding stresses in web members, it is only necessary to compute the panel loads for the system to which the member belongs.

EXAMPLE —Let the double Warren truss represented in Fig 23 support one-half of the system of moving concentrated loads represented in Fig 21 When $W_{11}$ is at the center of the span, what are the panel concentrations?

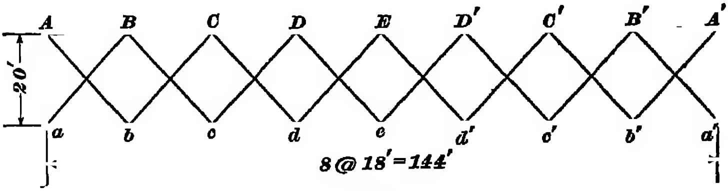



FIG 23

SOLUTION —

| PANEL POINT | PANEL LOAD, IN POUNDS |
|---|---|
| $a$ | $\frac{40{,}000 \times 2 + 18{,}000 \times 10}{2 \times 18} = 7{,}200$ |
| $b$ | $\frac{40{,}000 \times (5 + 10 + 15 + 16) + 18{,}000 \times 8}{2 \times 18} = 55{,}100$ |
| $c$ | $\frac{22{,}000 \times (3 + 8 + 13) + 40{,}000 \times (3 + 8 + 13)}{2 \times 18} = 41{,}300$ |
| $d$ | $\frac{18{,}000 \times 8 + 22{,}000 \times (16 + 15 + 10 + 5)}{2 \times 18} = 32{,}100$ |
| $e$ | $\frac{22{,}000 \times 2 + 18{,}000 \times 10 + 40{,}000 \times (18 + 13 + 8 + 3)}{2 \times 18} = 52{,}900$ |
| $d'$ | $\frac{40{,}000 \times (5 + 10 + 15) + 22{,}000 \times (1 + 6 + 11)}{2 \times 18} = 44{,}300$ |
| $c'$ | $\frac{22{,}000 \times (7 + 12 + 17 + 14) + 36{,}000 \times 4\ 5}{2 \times 18} = 35{,}100$ |
| $b'$ | $\frac{22{,}000 \times 4 + 36{,}000 \times 13\ 5 + 72{,}000 \times 9}{2 \times 18} = 33{,}900$ |
| $a'$ | $\frac{72{,}000 \times 9}{2 \times 18} = 18{,}000$ |

### EXAMPLE FOR PRACTICE

If the double Warren truss represented in Fig 23 supports one-half of the system of moving concentrated loads shown in Fig 21, what are the panel loads when $W_{12}$ is at $e$?

| | Panel Point | Panel Load, in Pounds | Panel Point | Panel Load, in Pounds |
|---|---|---|---|---|
| Ans | $a$ | 17,500 | $d'$ | 37,400 |
| | $b$ | 59,300 | $c'$ | 34,100 |
| | $c$ | 37,900 | $b'$ | 35,100 |
| | $d$ | 30,400 | $a'$ | 18,000 |
| | $e$ | 60,300 | | |

## INCLINED-CHORD TRUSSES

**38.** By means of the principles already discussed, the stresses in inclined-chord trusses can be found in all the members that depend simply on the moment or shear. For example, the stresses in the chord members can be found directly from the moments; the stress in the end post, from the shear in the end panel, and the stress in the hip vertical, from the maximum panel load. The stresses in the intermediate verticals and diagonals require separate consideration.

### THE CURVED-CHORD TRUSS

**39. Maximum Stress in a Diagonal.**—Let it be required to determine the position of a system of moving concentrated loads when the stress in the diagonal $ED$ of the truss represented in Fig. 24 ($a$) is a maximum. The influence line for the stress in $ED$ is composed of the three straight lines $B'D'$, $D'C'$, and $C'A'$, and is drawn by computing the stress in $ED$ due to a load of unity that moves over the span; the ordinate $D''D'$ represents the stress in $ED$ when the load is at $D$; etc. Using the same method of proof as in previous cases, it can be shown that the stress in $ED$ is a maximum when $\frac{W_r}{a'} + \frac{W_l}{a} \times \frac{CF}{FD} = \frac{W_p}{FD}$, $W_r$, $W_l$, and $W_p$, representing, as usual, the sum of all the loads to the right of $D$, to the left of $C$, and in the panel $CD$, respectively, there being a load at $D$. Under ordinary conditions, there

will be few loads in the panel $CD$ and no loads to the left of $C$. Making $W_l$ equal to zero, the preceding equation reduces to

$$\frac{W_r}{a'} = \frac{W_p}{FD};$$

or

$$\frac{\Sigma W - W_p}{a'} = \frac{W_p}{FD},$$

whence

$$W_p = \frac{FD}{a' + FD}\Sigma W = \frac{FD}{FB}\Sigma W,$$

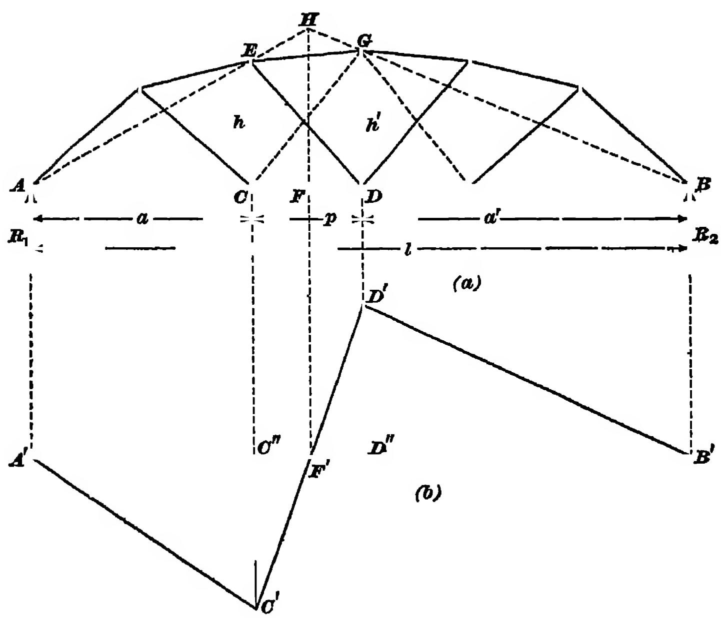



FIG. 24

which is a convenient formula to use when the point $F$ has been located.

To locate the point $F$, consider that, when a load is at $F$, the stress in $ED$ due to that load is zero, then the stress in $CD$ is equal to the horizontal component of the stress in $EG$. The stress in $CD$ is $1 \times \frac{FB}{l} \times \frac{a}{h}$, the horizontal component of the stress in $EG$ is $1 \times \frac{AF}{l} \times \frac{a'}{h'}$; then,

$$\frac{FB}{l} \times \frac{a}{h} = \frac{AF}{l} \times \frac{a'}{h'},$$

whence
$$\frac{AF}{FB} = \frac{a\,h'}{a'\,h},$$

that is,
$$\frac{l - FB}{FB} = \frac{a\,h'}{a'\,h},$$

and
$$FB = l \times \frac{a'\,h}{a\,h' + a'\,h}$$

Also,
$$FD = FB - a'$$

For all ordinary purposes, it is sufficiently accurate to locate the point $F$ graphically, laying out the truss carefully to scale and drawing the lines $AE$ and $BG$ through the joints $A$ and $E$, $B$ and $G$, respectively. Then $F$ lies vertically under their intersection $H$ From similar triangles, in Fig 24 ($a$),

$$\frac{FH}{AF} = \frac{h}{a}, \text{ and } \frac{FH}{FB} = \frac{h'}{a'};$$

whence, as before,
$$\frac{AF}{FB} = \frac{a\,h'}{a'\,h}$$

Example —Let the eight-panel truss represented in Fig 25 support one-half of the system of moving concentrated loads represented in

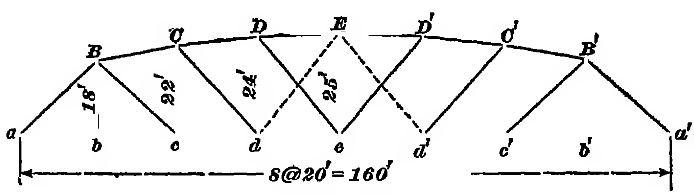



Fig 25

Fig 21, and let it be required to find the maximum stress in the diagonal $Cd$

Solution —In this truss, $Fa'$ (the point $F$ is not shown) corresponds to $FB$ in Fig 24 Also, $l = 160$, $h = 22$, $h' = 24$, $a = 40$, $a' = 100$ Therefore,

$$Fa' = 160 \times \frac{100 \times 22}{40 \times 24 + 100 \times 22} = 111\ 39 \text{ ft.}$$

and $Fd = 11\ 39$ ft Then, $\frac{Fd}{Fa'} = \frac{11\ 39}{111\ 39} = 1023$ or 1, nearly

When $W_2$ is at $d$, $\Sigma W = 532{,}000$ lb, $\frac{Fd}{Fa'} \times \Sigma W = 53{,}200$ lb, $W_p$ is between 18,000 lb and 58,000 lb Therefore, the stress is a maximum

when $W_2$ is at $d$ $R_1$ for one truss is 95,225 lb, the panel load at $c$ is 3,600 lb The shear in the panel $cd$ is 95,225 − 3,600 = 91,625 lb. The vertical component in $CD$ is

$$\frac{95{,}225 \times 60 - 3{,}600 \times 20}{24} \times \frac{2}{20} = 23{,}500 \text{ lb.}$$

Then, the stress in $Cd$ is

$$(91{,}625 - 23{,}500) \times \csc Cdc = 68{,}125 \times \frac{\sqrt{22^2 + 20^2}}{22} = 92{,}000 \text{ lb, tension.}$$

Ans

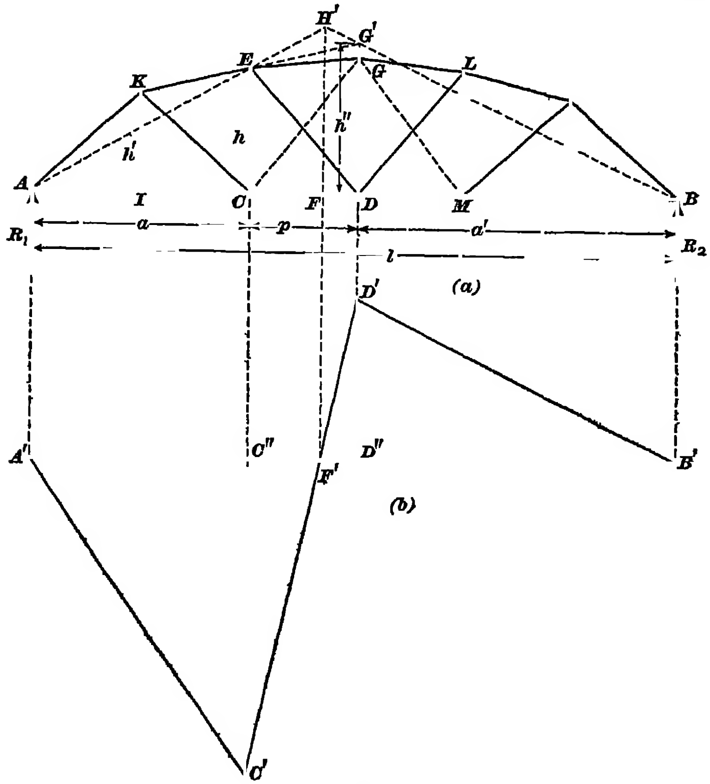



FIG 26

**40. Maximum Stress in a Vertical.**—Let it be required to determine the position of a system of moving concentrated loads when the stress in the vertical $EC$ of the truss represented in Fig. 26 is a maximum. The influence line for the stress in $EC$, Fig. 26, is similar to that for the

diagonal $ED$, and it may be shown that the same conditions determine the position for the maximum stress in the vertical as in the diagonal, the only difference being in the location of the point $F$. Then, as in the case of the diagonal,

$$\frac{W_r}{a'} + \frac{W_l}{a} \times \frac{CF}{FD} = \frac{W_p}{FD}$$

and, when $W_l = 0$,

$$W_p = \frac{FD}{FB} \times \Sigma W$$

To locate the point $F$, consider that, when a load is at $F$, the stress in $EC$ due to that load is equal to zero, and the vertical component of the stress in $KE$ is equal to the shear on a plane of section that cuts $KE$, $EC$, and $CD$; that is, to the shear in the panel $CD$.

The shear is equal to $1 \times \frac{FB}{l} - 1 \times \frac{FD}{p}$, and the vertical component in $KE$ is equal to

$$1 \times \frac{FB}{l} \times \frac{a}{h} \times \frac{h - h'}{p}$$

Then, $$\frac{FB}{l} - \frac{FD}{p} = \frac{FB}{l} \times \frac{a}{h} \times \frac{h - h'}{p};$$

whence, substituting for $FD$ its equivalent $FB - a'$, and solving for $FB$,

$$FB = l \times \frac{a'h}{h(a + a') + a(h - h')}$$

Now, if $KE$ is produced to meet the vertical $GD$ produced at $G'$, $G'D - CE = CE - KI$, or $h'' - h = h - h'$. Substituting $h'' - h$ for $h - h'$ in the foregoing expression, and reducing, the following equation is obtained.

$$FB = l \times \frac{a'h}{ah'' + a'h}$$

which is also the value of $FB$ for the diagonal $ED$ in a truss having the height $h''$ at $D$. Hence, $F$ may be found by drawing the lines $AE$ and $BG'$ to their intersection $H'$ and drawing the vertical $H'F$, as in the case of the diagonal.

EXAMPLE.—What is the maximum stress in the vertical $Cc$ of the truss represented in Fig. 25, due to the system of loads represented in Fig. 21?

SOLUTION.—In this case, $h = 22$ ft. and $h' = 18$ ft.; therefore, $h'' = 26$ ft.

$$Fa' = 160 \times \frac{100 \times 22}{40 \times 26 + 100 \times 22} = 108.64 \text{ ft.}, \; Fd = 8.64 \text{ ft.};$$

$$\frac{Fd}{Fa'} = \frac{8.64}{108.64} = .08, \text{ nearly}$$

When $W_2$ is at $d$, $\Sigma W = 532{,}000$ lb., $\frac{FD}{FB} \times \Sigma W = 42{,}560$ lb., and $W_p$ is between 18,000 lb. and 58,000 lb. Then, the stress is a maximum when $W_2$ is at $d$. $R_1 = 95{,}225$ lb., and the load at $c$ = 3,600 lb. $BC$ and $cd$ intersect 70 ft. to the left of $a$. Then, the stress in $Cc$ is

$$\frac{95{,}225 \times 70 - 3{,}600 \times 110}{100} = 57{,}000 \text{ lb., compression. Ans.}$$

**41. Minimum Stress in a Vertical.**—The minimum stresses in all the verticals not adjacent to panels containing counters can be found from the principles already explained. The minimum stress in any other vertical is tension; it occurs when the two diagonals that meet the vertical at one end are in action, and is equal to the algebraic sum of the vertical components of the stresses in the chord members that meet the vertical at the other end. The minimum stress (so called) in the center vertical obtains when the moment at the center of the span is a maximum. The minimum stress in the other verticals can best be found by trial. For example, for the minimum stress in $EC$ ($LM$), Fig. 26 ($a$), the loads should extend over as great a portion of the right end of the truss as possible without throwing the counter $GM$ out of action. This position can be ascertained by trying several loads successively at $M$ until it is found that any further movement of the loads causes $GM$ to go out of action. The last position of the loads before $GM$ goes out of action is the position for which the stress is a minimum.

## THE PETIT TRUSS

**42.** As in the case of the Baltimore truss, the stresses in the members of the upper chord and in the end panels of the lower chord of a Petit truss can be found from the bending moments at the opposite joints; the loading causing the maximum stresses in the remaining members of the

lower chord can be found by means of the formula

$$W_l - W_p = \frac{a}{l} \times \Sigma W$$

explained in Art **31**; the stresses in the subverticals and short diagonals can be found from the panel load, and the stress in the hip vertical, from the panel load for a truss having panels twice the length of the Petit truss. There remain to be considered the methods of finding the maximum stresses in the main diagonals, verticals, and counters.

**43. Lower Half of Main Diagonal.**—The stress in the lower half of the end post can be found from the shear in the end panel. The maximum stress in the lower half of any other main diagonal, such as $GD$, Fig. 27, can be found

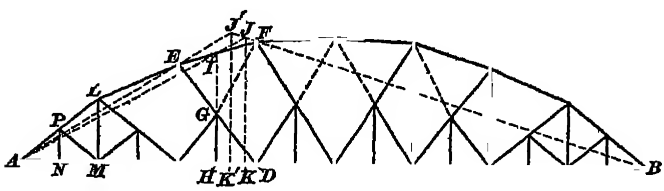



FIG 27

in precisely the same way as in a diagonal of a simple curved-chord truss. For example, for the member $GD$, $GH$ may be produced to meet $EF$ at $I$, the lines $AI$ and $BF$ produced to their intersection at $J$, and the vertical $JK$ drawn, intersecting the lower chord at $K$. Then, when there are no loads to the left of $H$ (as is usually the case when the stress in $GD$ is a maximum), the stress in $GD$ is a maximum when $W_p = \frac{KD}{KB} \times \Sigma W$, $KB$ being equal to

$$l \times \frac{BD \times HI}{AH \times DF + BD \times HI}$$

**44. Upper Half of Main Diagonal.**—The influence line for the upper half of a main diagonal, such as $EG$, is the same as that for the diagonal $ED$ of a simple curved-chord truss of the same dimensions as that represented in Fig. 27, but having twice the panel length. Then, for $EG$, the lines $AE$ and $BF$ may be drawn to their intersection $J'$

and the vertical $J'K'$ drawn to its intersection with the lower chord. The stress in $EG$ is a maximum when $W_{2p} = \frac{K'D}{K'B} \times \Sigma W$. The influence line for the upper half of the end post $PL$ is the same as that for the shear in the end panel of the simple curved-chord truss shown in Fig. 24, therefore, the stress in $PL$ is a maximum when there is a load at $M$ and the shear in the double panel $AM$ (considering $NP$ and $PM$ to be omitted) is a maximum.

**45. Intermediate Verticals.**—The stress in an intermediate vertical, such as $EC$, Fig 28, is a maximum under the same conditions as the stress in the member $EC$ of the simple curved-chord truss of the same dimensions shown in

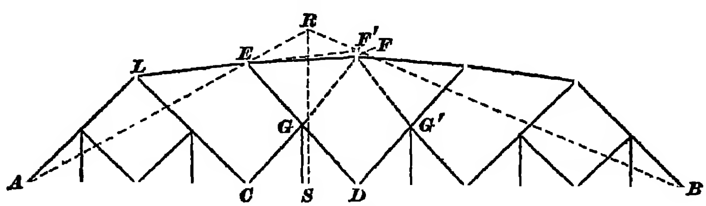



FIG 28

Fig 26. If $LE$ is produced to meet $FD$ in $F'$, the lines $AE$ and $BF'$ drawn to their intersection $R$, and the vertical $RS$ drawn, the stress in $EC$ is a maximum when $W_{2p} = \frac{SD}{SB} \times \Sigma W$, and similarly for other intermediate verticals. The stress in the center vertical is a minimum when the moment at the center of the truss is a maximum; that in the other intermediate verticals may be found most readily by trial in the same way as for the simple curved-chord truss

**46. Counters.**—When $GF$, Fig. 29, is in action as a counter, $GD$ is out of action Considering $G'F'$ instead, the line $G'H'$ may be produced vertically to its intersection with $E'F'$ at $I'$, the lines $BI'$ and $AF'$ drawn to their intersection $J''$, and the vertical $J''K''$ to its intersection with the lower chord. Then, the stress in $G'F'$ is a maximum when there is a load at $H'$, and $W_p = \frac{K''H'}{K''B} \times \Sigma W$. When $CG$

is in action as a counter, $EG$ is in action as a subtie, and the counter $CGF$ is not straight In finding the position the loads occupy when the stress in $CG$ is a maximum, it may be assumed that $CF$ is straight, and the position of the loads found on this assumption. Then, locating the point $K'''$ in the usual way, the stress in $C'\,G'$ (considering the right end

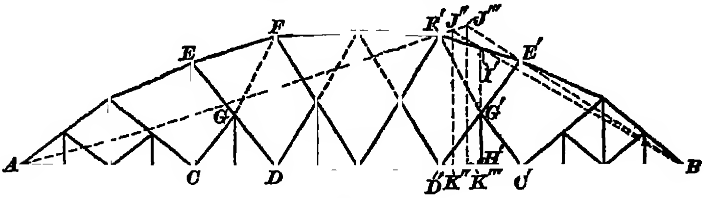



Fig 29

instead of the left) is a maximum when there is a load at $C'$ and $W_{s,p} = \frac{K'''\,C'}{K'''\,B} \times \Sigma\, W$. The ratio $\frac{K'''\,C'}{K'''\,B}$ is not strictly correct, but will invariably determine the correct loading In calculating the stress from this loading, the exact formula given in *Stresses in Bridge Trusses*, Part 3, must be used

## STANDARD SYSTEMS AND METHODS OF COMPUTATION

### STANDARD SYSTEMS OF CONCENTRATED LOADS

**47. Loads Used in Computations.**—The principles used in calculating stresses due to moving concentrated loads find their most frequent applications in connection with railway bridges. Specifications for such bridges require them to be designed to support certain loads, the loading most used consisting of two locomotives coupled together and followed by a train of cars. The actual weight used by different railroads differ, they are usually based on the heaviest rolling stock in use, an allowance being made to provide for a probable increase in weight in the future As the loads on the wheels of a heavily loaded train are very nearly equal, it is customary to consider the train as a uniform

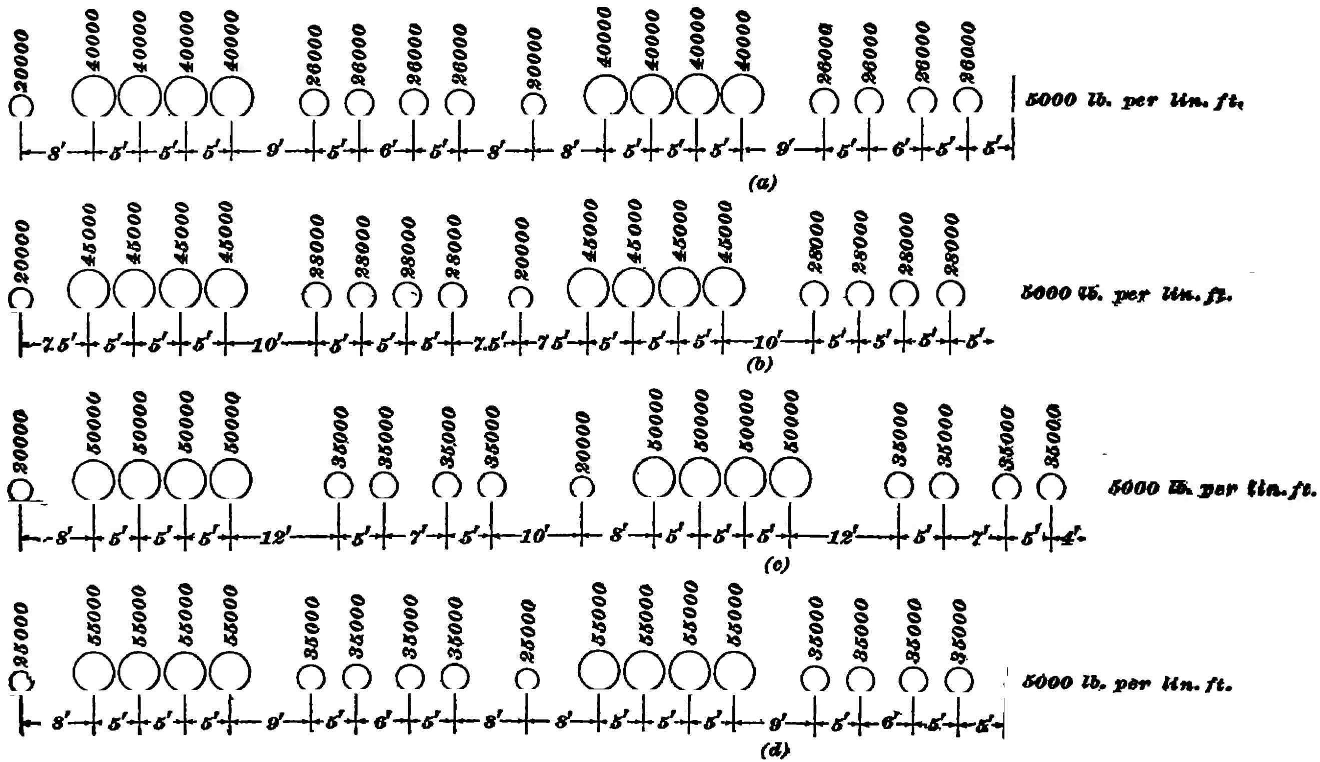

20000 40000 40000 40000 40000 26000 26000 26000 26000 20000 40000 40000 40000 40000 26000 26000 26000 26000
5000 lb. per lin. ft.
8' 5' 5' 5' 9' 5' 6' 5' 8' 8' 5' 5' 5' 9' 5' 6' 5' 5'
(a)
20000 45000 45000 45000 45000 28000 28000 28000 28000 20000 45000 45000 45000 45000 28000 28000 28000 28000
5000 lb. per lin. ft.
7.5' 5' 5' 5' 10' 5' 5' 5' 7.5' 7.5' 5' 5' 5' 10' 5' 5' 5' 5'
(b)
30000 50000 50000 50000 50000 35000 35000 35000 35000 30000 50000 50000 50000 50000 35000 35000 35000 35000
5000 lb. per lin. ft.
8' 5' 5' 5' 12' 5' 7' 5' 10' 8' 5' 5' 5' 12' 5' 7' 5' 4'
(c)
35000 55000 55000 55000 55000 35000 35000 35000 35000 25000 55000 55000 55000 55000 35000 35000 35000 35000
5000 lb. per lin. ft.
8' 5' 5' 5' 9' 5' 6' 5' 8' 8' 5' 5' 5' 9' 5' 6' 5' 5'
(d)

FIG 30

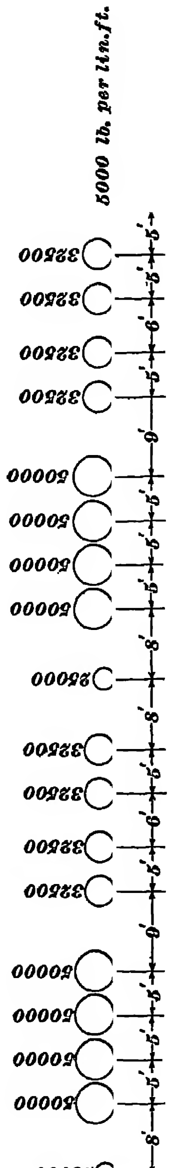


FIG 31

load per linear foot. In Fig. 30 are shown the loadings prescribed by several of the leading American railroad companies

**48. Standard Loadings.**—Several attempts have been made to secure the adoption of certain standard systems for use in computation The best known among these, and probably the most widely used, were devised by Theodore Cooper. Each of his systems consists of two typical locomotives and a train of cars, the spacing of the wheels being very nearly equal to the actual spacing of the wheels of the heaviest locomotives in use. One of his systems is represented in Fig 31, the loads shown at the different wheels being the loads on the axles, one-half of which goes to each wheel, this system is known as E50 on account of the fact that the weight on each driver axle (2, 3, 4, etc ) is equal to 50,000 pounds Other systems are known as E40, E30, etc , the weight on each driver axle being equal, respectively, to 40,000, 30,000 pounds, etc. The spacing of the wheels of the different systems is the same, and the loads of one system may be derived from those of another by the use of a simple multiplier. For example, if each axle load and the uniform load per linear foot of E50 is multiplied by .8 $\left(=\frac{40}{50}\right)$, the result is the system known as E40

The convenience of this method is apparent, as the moments, shears, and stresses due to one system may be found from those due to any other, by the use of

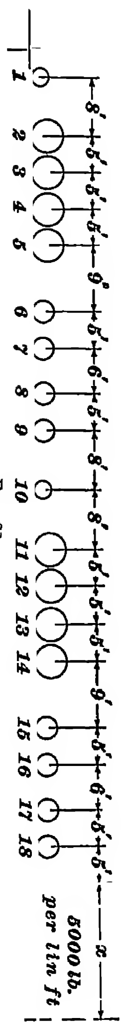



Fig 32

| C | D | E | F |
|---|---|---|---|
| Weights on Axles Pounds | Sums of Distances From First Axle Feet | Sums of Weights on Axles From *1* to *N* Inclusive Pounds | Moments About Axles of all Preceding Weights Foot-Pounds |
| 25,000 | 0 | 25,000 | 0 |
| 50,000 | 8 | 75,000 | 200,000 |
| 50,000 | 13 | 125,000 | 575,000 |
| 50,000 | 18 | 175,000 | 1,200,000 |
| 50,000 | 23 | 225,000 | 2,075,000 |
| 32,500 | 32 | 257,500 | 4,100,000 |
| 32,500 | 37 | 290,000 | 5,387,500 |
| 32,500 | 43 | 322,500 | 7,127,500 |
| 32,500 | 48 | 355,000 | 8,740,000 |
| 25,000 | 56 | 380,000 | 11,580,000 |
| 50,000 | 64 | 430,000 | 14,620,000 |
| 50,000 | 69 | 480,000 | 16,770,000 |
| 50,000 | 74 | 530,000 | 19,170,000 |
| 50,000 | 79 | 580,000 | 21,820,000 |
| 32,500 | 88 | 612,500 | 27,040,000 |
| 32,500 | 93 | 645,000 | 30,102,500 |
| 32,500 | 99 | 677,500 | 33,972,500 |
| 32,500 | 104 | 710,000 | 37,360,000 |
| | 109 | 710,000 | 40,910,000 |
| $5,000\,x$ | | | |
| | $109 + x$ | $710,000 + 5,000\,x$ | $40,910,000 + 710,000\,x + 2,500\,x^2$ |

the multiplier by means of which the first system is derived from the second. The system that is used depends on the amount and class of traffic. For a great many railroads, E40 is heavier than the heaviest traffic; while for some others it has been deemed advisable to use E60 in the design of some of their bridges.

**49. Moment Diagram.**—When a large number of calculations are to be made from the same system, the work may be shortened by tabulating certain quantities that continually arise. For example, Fig. 32 represents what is called a moment diagram, computed in this case for E50. Opposite *A* are given the numbers of the wheels, the latter being numbered consecutively from the left end; *B* is the spacing of the wheels; *C*, the weight that comes on each axle; *D*, the distance of any wheel from the first, *E*, the sum of all the loads from the first up to and including the one over which the sum is placed, and *F*, the sum of the moments of all the preceding loads about any wheel The moment about any wheel may be found from the moment about the preceding wheel by adding to the latter the product of the sum of all the loads up to and including that about which the moment is known and the distance between the two wheels. For example, the moment of *1* to *8* about *9* is

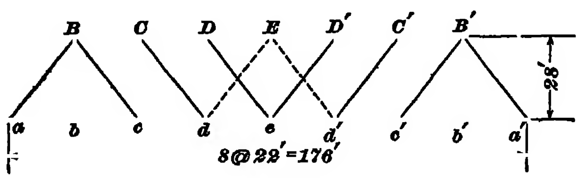



FIG 33

equal to 8,740,000 foot-pounds. To find the moment about *10*, consider that the lever arm of each load from *1* to *9* is increased by 8 feet, and that the moment is increased by the product of 8 feet and the sum of all the loads from *1* to *9*, inclusive. Then, the moment about *10* is equal to

$$8{,}740{,}000 + 355{,}000 \times 8 = 11{,}580{,}000 \text{ foot-pounds}$$

The use of the diagram will be illustrated by an example.

EXAMPLE.—The truss represented in Fig. 33 supports one-half of a bridge that supports an E50 loading. What is the maximum stress: (*a*) in the diagonal $Bc$? (*b*) in the chord member $CD$?

SOLUTION.—(*a*) The stress in $Bc$ is a maximum when the shear in the panel $bc$ is a maximum, that is, when $W_p = \frac{p}{l} \times \Sigma W$. In this case, $\frac{p}{l} = \frac{1}{8}$.

When $W_3$ is at $c$, $a'$ is 132 ft. to the right of $c$ and 145 ft. to the right of $W_1$, that is, $145 - 109 = 36$ ft. to the right of the head of the uniform load. Then,

$$W = 710{,}000 + 5{,}000 \times 36 = 890{,}000 \text{ lb.}$$

and $\frac{p}{l} \times \Sigma W = 111{,}250$ lb.

The load at $b$ is equal to

$$\frac{\text{moment of } 1 \text{ and } 2 \text{ about } c}{2 \times 22} = \frac{575{,}000}{2 \times 22} = 13{,}070 \text{ lb.}$$

The left reaction is equal to the moment of loads *1* to *18* and 36 ft. of the uniform load about $a'$, divided by the length of span. Then, $R_1$ is equal to

$$\frac{40{,}910{,}000 + 710{,}000 \times 36 + 2{,}500 \times 36 \times 36}{2 \times 176} = 198{,}040 \text{ lb. for one truss}$$

The stress in $Bc$ is

$$(198{,}040 - 13{,}070) \times \frac{\sqrt{22^2 + 28^2}}{28} = 235{,}300 \text{ lb., tension. Ans.}$$

(*b*) The stress in $CD$ is a maximum when the moment at $d$ is a maximum.

When $W_9$ is at $d$, $\Sigma W = 710{,}000 + 5{,}000 \times 49 = 955{,}000$ lb., and $\frac{a}{l} \times \Sigma W = 358{,}125$ lb. As $W_l$ is less than 355,000 lb., this position does not give a maximum.

When $W_{10}$ is at $d$, $\Sigma W = 710{,}000 + 5{,}000 \times 57 = 995{,}000$ lb., and $\frac{a}{l} \times \Sigma W = 373{,}125$ lb. As $W_l$ is between 355,000 and 380,000 lb., this position gives a maximum.

When $W_{11}$ is at $d$, $\Sigma W = 710{,}000 + 5{,}000 \times 65 = 1{,}035{,}000$ lb., and $\frac{a}{l} \times \Sigma W = 388{,}125$ lb. As $W_l$ is between 380,000 and 430,000 lb., this position gives a maximum.

When $W_{12}$ is at $d$, $\Sigma W = 710{,}000 - 25{,}000 + 5{,}000 \times 70 = 1{,}035{,}000$ lb., and $\frac{a}{l} \times \Sigma W = 388{,}125$ lb. As $W_l$ is greater than 405,000 lb., this position does not give a maximum.

It is now necessary to compute the moments when $W_{10}$ and $W_{11}$, respectively, are at $d$.

When $W_{10}$ is at $d$,

$$R_1 = \frac{40{,}910{,}000 + 710{,}000 \times 57 + 2{,}500 \times 57 \times 57}{2 \times 176} = 254{,}240 \text{ lb.}$$

The moment at $d$ is

$$254{,}240 \times 66 - \frac{11{,}580{,}000}{2} = 10{,}990{,}000 \text{ ft.-lb.}$$

When $W_{11}$ is at $d$,

$$R_1 = \frac{40{,}910{,}000 + 710{,}000 \times 65 + 2{,}500 \times 65 \times 65}{2 \times 176} = 277{,}340 \text{ lb}$$

The moment at $d$ is

$$277{,}340 \times 66 - \frac{14{,}620{,}000}{2} = 10{,}994{,}000 \text{ ft -lb.}$$

Then the stress in $CD$ is

$$10{,}994{,}000 - 28 = 392{,}600 \text{ lb., compression. Ans.}$$

## APPROXIMATE METHODS BY THE USE OF EQUIVALENT LOADS

**50. Introduction.**—It is very laborious to calculate the stresses in some types of trusses—such as multiple-system and subdivided-panel trusses—if the actual concentrated loads are used On this account, it is sometimes desirable to substitute for these loads some simpler system of loading that will cause, as nearly as possible, the same stresses. Such loadings are called **equivalent loads.** The two principal kinds of equivalent loads will now be considered.

The difference between the results obtained from equivalent loads and those from the actual wheel loads are relatively greater for short spans than for long spans. For spans up to 75 or 100 feet, the actual wheel loads should be used; and for *all* spans, the stresses in such members as *hip verticals*, *short verticals*, and *diagonals*, and in *floor members* should be calculated from the *actual wheel loads.*

**51. Equivalent Uniform Load.**—The method of equivalent uniform loads is most useful in computing moments and chord stresses; it consists in substituting for the actual wheel loads a uniform load per linear foot that will cause, as nearly as possible, the same stresses. The best way to compute this equivalent load is to calculate first the maximum moment on the span at a section (usually

| Location of Load | $\Sigma W$ Pounds | $\frac{a}{l} \times \Sigma W$ Pounds | $W_l$ Pounds |
|---|---|---|---|
| $W_4$ at $C$ | 710,000 + 5,000 × 29 = 855,000 | 213,750 | Less than 175,000 |
| $W_5$ at $C$ | 710,000 + 5,000 × 34 = 880,000 | 220,000 | Between 175,000 and 225,000 |
| $W_6$ at $C$ | 710,000 + 5,000 × 43 = 925,000 | 231,250 | Between 225,000 and 237,500 |
| $W_7$ at $C$ | 710,000 + 5,000 × 48 = 950,000 | 237,500 | Greater than 257,500 |
| $W_{11}$ at $C$ | 710,000 − 225,000 + 5,000 × 75 = 860,000 | 215,000 | Less than 205,000 |
| $W_{12}$ at $C$ | 710,000 − 225,000 + 5,000 × 80 = 885,000 | 221,250 | Between 205,000 and 255,000 |
| $W_{13}$ at $C$ | 710,000 − 257,500 + 5,000 × 85 = 877,500 | 219,375 | Greater than 222,500 |

called the **quarter point**) midway between the center and the end, due to the actual wheel loads, and then find the uniform load over the entire span that will cause the same moment. If $w$ represents the uniform load per linear foot over the entire span, the moment at the quarter point is equal to $\frac{3}{32} w l^2$; if $M_c$ represents the moment at the quarter point due to the actual wheel loads, then

$$\frac{3}{32} w l^2 = M_c$$

and
$$w = \frac{32}{3} \times \frac{M_c}{l^2}$$

It will usually be found that the moment at the center due to this uniform load is slightly greater than that due to the actual wheel loads, and that the moment near the end is slightly less. If the value of $w$ were found from the maximum moment at the center, all the moments would be too small.

EXAMPLE.—What equivalent uniform load may be used in computing chord stresses for a span of 160 feet with an E50 loading?

SOLUTION.—It is first necessary to compute the maximum moment at a section 40 ft from the left end due to E50. The numerical work can be most conveniently arranged as shown in the accompanying table.

The moment at the quarter point $C$ is a maximum when $W_5$, $W_6$, and $W_{12}$, respectively, are at the section.

When $W_8$ is at $C$,

$$R_1 = \frac{40{,}910{,}000 + 710{,}000 \times 34 + 2{,}500 \times 34 \times 34}{160} = 424{,}625 \text{ lb.}$$

The moment at $C$ is

$$424{,}625 \times 40 - 2{,}075{,}000 = 14{,}910{,}000 \text{ ft -lb.}$$

When $W_6$ is at $C$,

$$R_1 = \frac{40{,}910{,}000 + 710{,}000 \times 43 + 2{,}500 \times 43 \times 43}{160} = 475{,}391 \text{ lb.}$$

The moment at $C$ is $475{,}391 \times 40 - 4{,}100{,}000 = 14{,}915{,}640$ ft -lb.

When $W_{12}$ is at $C$, the wheels *1* to *5* are off the span; and, in using the diagram to get moments about point $C$ and the right end of the span, it is necessary to deduct the moment of these loads as follows· The reaction of $R_1$ is

$$\frac{(40{,}910{,}000 + 710{,}000 \times 80 + 2{,}500 \times 80 \times 80) - (2{,}075{,}000 + 225{,}000 \times 166)}{160}$$

$$= 464{,}281 \text{ lb}$$

The moment at $C$ is

$$(464{,}281 \times 40) - \{16{,}770{,}000 - [2{,}075{,}000 + (225{,}000 \times 46)]\}$$

$$= 14{,}226{,}240 \text{ ft.-lb}$$

The moment is greatest when $W_6$ is at $C$, then,

$$w = \frac{32}{3} \times \frac{14{,}915{,}640}{160 \times 160} = 6{,}214\ 9, \text{ or, say, } 6{,}215 \text{ lb per lin ft. Ans.}$$

**52. Uniform Load and Concentrated Loads.**—The method of finding an equivalent system composed of a uniform load and several concentrated loads is most useful in computing shears and web stresses; it consists in substituting for the actual wheel loads a combination of a uniform load, equal to the specified train load, and a single concentrated load for each locomotive, equal to the difference in weight between the locomotive and the weight of a portion of the train the same length as the locomotive, acting simultaneously with the uniform load. For example, in Figs. 31 and 32, wheels *1* to *9* constitute one locomotive, the distance between the end wheels, usually called the **wheel base**, being 48 feet; the second locomotive starts with wheel *10* at a distance of 56 feet from wheel *1*, the corresponding wheel of the first locomotive, the latter distance is spoken of as the **length**. The weight of one locomotive, found by adding the loads on all the wheels from *1* to *9*, is 355,000 pounds; the weight of an equal length, 56 feet, of train, is $56 \times 5{,}000 = 280{,}000$ pounds; the

difference, 75,000 pounds, is taken as a concentrated load acting simultaneously with the uniform load of 5,000 pounds per foot, and is called the **locomotive excess**. When there are two locomotives, there are two excesses, in this case, they are 56 feet apart, the distance between corresponding points of the two locomotives. The equivalent load for one locomotive is shown in Fig. 34; for convenience in calculation, the concentrated load may be located anywhere with respect to the uniform load. In using this method, it is convenient to find first the maximum shears or stresses due to the uniform load, as though it were the only load on the truss, and then those due to the concentrated loads alone, placed so that the shear or stress under consideration will be a maximum; the sum of the results will be the shear or stress desired

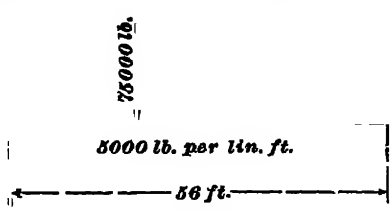



FIG 34

EXAMPLE —Using the equivalent load found in the preceding paragraph, find the maximum shear in the panel $bc$ of the truss in Fig 33.

SOLUTION —Considering first the uniform load alone, the panel load for one truss is $\frac{5,000 \times 22}{2} = 55,000$ lb The shear in the panel $bc$ due to a uniform load is a maximum when the joints $c$ to $b'$ are loaded, and, as there are then no panel loads to the left of the panel $bc$, the shear is equal to the left reaction, or to

$$55,000 \times \frac{(1+2+3+4+5+6)}{8} = 144,400 \text{ lb}$$

The shear in the panel $bc$ due to two concentrated loads 56 ft apart is greatest when one of the loads is at $c$ and the other 56 ft to the right of $c$, and, as there are then no loads to the left of $c$, the shear in the panel $bc$ is equal to the left reaction, or to

$$\frac{75,000 \times (76 + 132)}{2 \times 176} = 44,300 \text{ lb}$$

The shear in the panel $bc$, due to a combination of the uniform load with the locomotive excesses acting simultaneously, is

$$144,400 + 44,300 = 188,700 \text{ lb. Ans}$$

**EXAMPLES FOR PRACTICE**

1 What is the maximum moment at the center of a span 160 feet in length due to Cooper's E50? Ans 19,885,000 ft -lb

2 What is the maximum moment at the center of a span 160 feet in length, due to the equivalent uniform load of 6,215 pounds per linear foot found in the example in Art **51**? Ans 19,888,000 ft -lb

3 What is the difference between the moments found in examples 1 and 2? Ans 3,000 ft -lb , or 015 per cent

4 Find the locomotive excess for the system of concentrated loads shown in Fig 21 Ans 42,000 lb

## GRAPHIC METHOD FOR CONCENTRATED-LOAD MOMENTS AND SHEARS

**53. Equilibrium Polygon for Maximum Moment.** In order to determine by the graphic method the maximum moment at a given section of a span due to a system of moving concentrated loads, it is necessary to find first the correct position of the loads by the principles that have already been explained When this has been found, the value of the moment can be ascertained by constructing the equilibrium polygon and multiplying the proper intercept by the normal ray, as explained in *Graphic Statics*. When a large number of calculations are to be made for the same system of loads, the work can be considerably shortened by constructing an equilibrium polygon for the complete system of loads, in the case of two engines and train, the polygon may be drawn for a sufficient length of the uniform load representing the train, to provide for the longest span for which calculations are to be made.

Fig 35 represents an equilibrium polygon for Cooper's E50, together with 100 feet of uniform train load In constructing the polygon for the uniform load, the latter was considered as concentrated at points 5 feet apart; each concentration being 25,000 pounds, and the first being taken 2.5 feet from the head of the uniform load. This portion of the polygon is in reality a curve tangent to the strings of the equilibrium polygon half way between their intersections

with the assumed concentrations. In practice, the polygon is made as large as possible; if great care is taken in scaling all distances, the results will be sufficiently close

**54. Method of Procedure.**—If it is known what position the loads occupy when the moment is a maximum at a given section *C*, the value of the moment can be found by laying off the span to the same scale as the spacing of the loads, placing section *C* at the proper load and drawing verticals through the ends of the span. These verticals are the lines of action of the reactions; then, if the closing line is drawn between their intersections with the strings of the equilibrium polygon, and a vertical line is drawn through *C*, the moment at *C* is equal to the intercept of this vertical multiplied by the normal ray.

If it is not known what position the loads occupy when the moment is a maximum, the span may be laid off with the section *C* at several of the loads successively, and the preceding process repeated until the maximum intercept is found

EXAMPLE —Let it be required to find the maximum moment at the quarter point of a span 140 feet long due to Cooper's E50

SOLUTION —As it is known that the span will be quite fully loaded, it may be assumed that there will be several loads to the left of the quarter point The span may be laid off, to the same scale as the spacing of the loads, on the edge of a straight strip of paper, and the quarter point marked, this strip may then be laid horizontally on the diagram, Fig 35, so that *C* is at one of the loads, for instance $W_4$, verticals through *A* and *B* cut the equilibrium polygon at $A_4$ and $B_4$, respectively, then $A_4 B_4$ is the closing line for this position A vertical through *C* (the line of action of $W_4$) gives the intercept $C_4' C_4''$, and the moment at *C* is equal to $C_4' C_4'' \times N$ Next, *C* may be placed at $W_5$, then at $W_6$, etc , until the maximum intercept is found In this case, the intercept increases until $W_8$ is at *C*, then decreases and increases again until $W_{12}$ is at *C* The intercept is a maximum when $W_8$ is at *C*, and is equal, by scale, to 23 52 ft The normal ray is equal to 500,000 lb , by construction Then, the maximum moment at *C* is

$$23.52 \times 500{,}000 = 11{,}760{,}000 \text{ ft-lb} \quad \text{Ans}$$

**55. Maximum Shear at a Given Section.**—The maximum shear at a given section of a beam, due to a

system of moving concentrated loads, can be found in almost the same way as the moment, by trying several loads at the section successively, and drawing the closing line of the equilibrium polygon and the ray parallel to that line in the force polygon for each position of the loads. Then, the reactions and the shear at any section can be scaled directly from the load line

EXAMPLE —Let it be required to find the maximum shear at the quarter point of a beam 80 feet in length, due to Cooper's E50

SOLUTION —The span may be laid off with the quarter point at $W_2$, and verticals drawn at the ends of the span, intersecting the equilibrium polygon at $A_2$ and $B_2$, the closing line $A_2B_2$ is then drawn, Fig 35 Then, if $O r$ is drawn parallel to $A_2B_2$, the left reaction is equal to $0\text{-}r$, and the shear at the quarter point $1\text{-}r$ = 179,000 lb Next, the span may be laid off with the quarter point at $W_3$, the left reaction is equal to $0\text{-}r'$, and the shear at the quarter point is $0\text{-}r' - 0\text{-}2 = 2\text{-}r'$ = 159,000 lb As the shear when $W_2$ is at the quarter point is greater than this, the maximum shear is 179,000 lb Ans.

# STRESSES IN BRIDGE TRUSSES
## (PART 5)

## TRANSVERSE FORCES

**1. Introduction.**—All the outer forces considered in *Stresses in Bridge Trusses*, Parts 2, 3, and 4, are vertical forces. As explained in Part 1, bridges are subject to the action of horizontal or transverse forces—principally wind pressure and centrifugal force—that must be resisted by lateral trusses or bracing lying in planes that are not vertical. The magnitudes of these transverse forces and the stresses caused by them will now be discussed.

### WIND PRESSURE

**2. Intensity of Wind Pressure.**—Records of the intensity of wind pressure have been kept in various places, and from them it has been deduced that the maximum pressure during heavy gales may be as high as 50 pounds per square foot of exposed surface; in some instances, even greater pressures have been recorded, but they are probably of so rare occurrence that they may be safely neglected. The best bridge engineers of the present time consider it sufficient to provide for a pressure of 50 pounds per square foot over the entire exposed area of railroad and highway bridges, or, in the case of railroad bridges, an alternative pressure, if it produces greater stresses in the lateral system, of 30 pounds per square foot against both the exposed surface of a train of cars and the exposed area of the

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bridge. The weight of a train of empty cars is approximately 900 pounds per linear foot, and the standard distance from center to center of rails is about 4 9 feet Taking moments about the top of a rail, as shown in Fig. 1, the moment of stability of an empty car, or the resistance to overturning, is

$$900 \times \frac{4.9}{2} = 900 \times 2\ 45 = 2{,}205 \text{ foot-pounds per linear foot}$$

The exposed area of the side of a car will be assumed to extend from about 2 feet to 12 feet above the top of the rail; then, the exposed area per linear foot is $10 \times 1 = 10$ square feet, the center of which is 7 feet above the top of the rail, as shown in Fig 1 Then, if the wind pressure per square foot, uniformly distributed over the side of the car, that will just overturn it is represented by $p$, the total pressure $P$ per linear foot is 10 $p$, and, taking moments about the top of the rail, the overturning moment is $10p \times 7$ foot-pounds per linear foot. Placing this equal to the moment of stability just found, and solving for $p$, we have

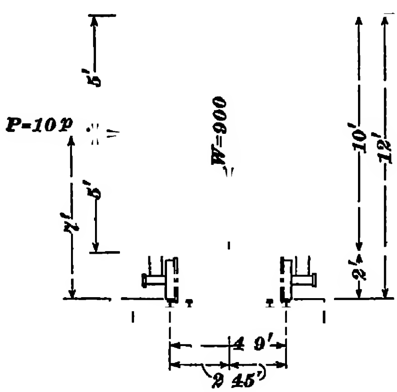


Fig 1

$$p = \frac{2{,}205}{10 \times 7} = 31.5 \text{ pounds per square foot}$$

It is assumed, therefore, that when the wind pressure is greater than about 30 pounds per square foot it will be impossible to operate trains, the higher pressure of 50 pounds per square foot is assumed to act against the unloaded bridge if it causes greater stresses in the members of the lateral system than does a pressure of 30 pounds per square foot. In case a train of cars is caught on a bridge by a sudden gust of wind, the pressure may be greater than 30 pound- per square foot, but, as this condition will probably last for

a very short time, it is assumed that no harm will result from it For all practical purposes, the pressure on a train of cars may be taken equal to 300 pounds per linear foot, acting 7 feet above the top of the rail, although other values and distances are used by some engineers.

In referring to the direction of the wind, it will be convenient to speak of the *windward truss* and the *leeward truss*, the former being the one on the side from which the wind is blowing, and the latter being the one on the other side. For example, if the bridge is located north and south, and the wind is blowing from east to west, the east truss is the windward truss, and the west truss is the leeward truss.

**3. Exposed Area.**—The exposed area of an unloaded bridge is usually taken equal to twice the exposed area of one truss; it is found by multiplying the length of each truss member by its greatest width, as seen in elevation, multiplying the sum of the products by 2, and adding the product to the area of the floor, as seen in elevation. The exposed area of a loaded through bridge is usually taken equal to the sum of the exposed area of one truss and that of the elevation of the floor, as it is assumed that the leeward truss is sheltered by the train of cars In deck bridges, the exposed area of truss and floor is taken the same as for an unloaded bridge, since the train of cars does not shelter the leeward truss. In highway bridges, the pressure on the exposed area of the loads is usually small compared with that on the exposed area of the bridge, and hence in practice is quite customarily neglected, or else a small amount is added to the pressure on the loaded chord. In railroad bridges, the pressure against the exposed area of a train of cars is relatively large, and is usually taken into account. The pressure against the trusses is taken as a fixed load covering the whole length of span, that against the train is taken as a moving load covering such portion of the span as will cause maximum stresses in the members of the lateral system.

There are other methods of computing the wind pressure on bridges, such as allowing a fixed amount per linear foot

of each chord, but the method just outlined is most generally used and is believed to be superior to any other. For this reason, it will be used in this and succeeding Sections.

**4. Lateral System.**—The wind pressure against the exposed surface of a bridge is transmitted to the joints of the trusses by the members themselves, and is resisted by bracing, called **lateral bracing**, connecting the main trusses and lying in the planes of the upper and lower chords, respectively, and by vertical or inclined bracing, called **sway bracing** or **transverse bracing**, connecting opposite posts in the two trusses. The sway bracing connecting the end posts is called the **portal bracing**. The combined system formed by all these bracings is usually called the **lateral system**. The pressure against the exposed surfaces of the loads is transmitted to the lateral system of the loaded chord by means of the floor.

### LATERAL BRACING

**5. Description.**—The lateral bracing usually consists of Pratt trusses lying in the planes of the chords of the main trusses. In parallel-chord bridges, the lateral trusses lie in horizontal planes, in inclined-chord bridges, the lateral trusses of the inclined chord lie in the several planes of the inclined members, and the panel lengths of the lateral trusses are equal to the actual lengths of the respective chord members For the purpose of determining the stresses, however, it is customary and sufficiently accurate for all ordinary spans to consider the panels of the lateral truss equal in length to those of the main trusses, the stresses are then found in the same way as for parallel-chord trusses. In Fig. 2, (*b*) is the main truss; (*a*), the upper lateral truss, and (*c*), the lower lateral truss, *b* being the distance center to center of trusses. The chords $BC$, $CD$, etc. of the lateral trusses are the respective chord members of the main trusses; the transverse members $CC_1$, $DD_1$, etc. are compression members and connect the opposite joints of the two main trusses; the diagonals $CB_1$, $DC_1$,

etc. are considered to be tension members, although in the best modern practice they are built to resist both tension and compression. In a through bridge, the floorbeams act as the transverse struts for the lower lateral system; in a deck bridge having a floor system, the floorbeams act as transverse struts for the upper lateral system.

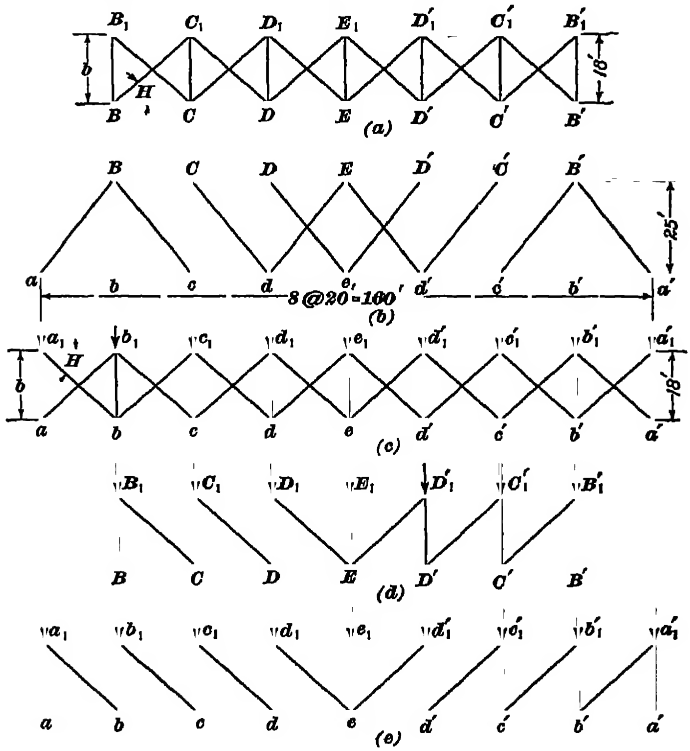


FIG 2

There are two diagonals in each panel of the lateral trusses in order to resist the wind pressure from either direction; when one set of diagonals is in action, the stresses in the others are assumed to be zero. Assuming that the wind panel loads act as shown in Fig. 2 (*c*), the diagonals shown in (*d*) and (*e*) will be in action. The loads on the lower lateral truss are transmitted directly to the abutments or other

supports; the loads on the upper lateral truss are transmitted to the upper joints of the end posts, which, with the assistance of the portal bracing, transmit them to the supports.

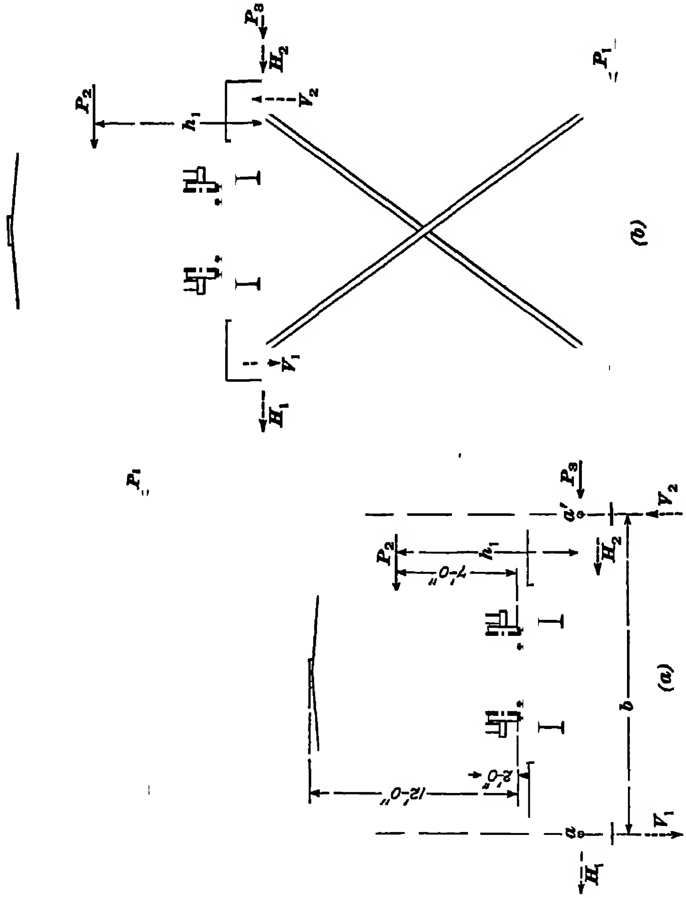


FIG 3

6. **Calculation of Wind Stresses.**—In Fig. 3, (*a*) is the cross-section of a through bridge, and (*b*) the cross-section of a deck bridge; $P_1$, $P_2$, and $P_3$ are the wind panel loads on the unloaded chord, train of cars, and loaded chord, respectively. It will be assumed that the lateral trusses of the unloaded chords for the two types of bridges here

represented are in the planes of the top and bottom, respectively, of the chord members. It will also be assumed that the lateral trusses of the loaded chords lie in the planes of the bottoms of the floorbeams. The pressure $P_2$ will evidently cause stresses in the lateral trusses of the loaded chord; it also tends to overturn the train, thereby increasing the amount of the live load that goes to the leeward truss and decreasing the load on the windward truss. The pressure may be resolved into the components $H_1$ and $V_1$ at $a$, Fig. 3 ($a$), and $H_2$ and $V_2$ at $a'$, $a$ and $a'$ being the intersections of the lateral truss of the loaded chord with the main vertical trusses. Then, $P_2 = H_1 + H_2$

It is impossible to compute the actual values of $H_1$ and $H_2$ by the equations of equilibrium, but these values may be assumed equal to each other and to $\frac{P_2}{2}$. In actual practice, however, $P_2$ is frequently added to $P_1$, and the total wind panel load is assumed to be applied at the windward side. Taking moments about $a'$, we have

$$V_1 b = P_2 h_1;$$

whence

$$V_1 = \frac{P_2 h_1}{b}$$

Similarly,

$$V_2 = \frac{P_2 h_1}{b} = V_1$$

The component $V_2$ simply decreases the panel loads and, therefore, the stresses in the windward truss, if only the maximum stresses are sought, it need not be further considered; $V_1$ increases the stresses in the leeward truss, and this increase may be found by multiplying the maximum live-load stresses due to vertical loads by the ratio of $V_1$ per panel to the live panel load; for, under any condition of loading, the live panel load on the leeward truss may be increased by the overturning effect of the wind. Then, if $W''$ is the live panel load, the wind stresses in the members of the vertical truss are equal to $\frac{P_2 \frac{h_1}{b}}{W''}$ multiplied by the maximum live-load stresses. The stresses in the lateral

trusses may be found in the same way as for a vertical Pratt truss, as explained in *Stresses in Bridge Trusses*, Part 2. The overturning effect due to the wind pressure on the upper chord will be discussed later in connection with the portal bracing.

EXAMPLE.—Assume that the eight-panel through bridge shown in Fig. 2 (*b*) is subject to a wind pressure of 400 pounds per linear foot on the upper chord, and a fixed, or dead, wind pressure of 200 pounds per linear foot, and a live, or moving, wind pressure of 300 pounds per linear foot, on the lower chord. What are the maximum wind stresses in all the members (*a*) of the upper lateral truss? (*b*) of the lower lateral truss? (*c*) What is the amount by which each panel load of the leeward truss must be increased, assuming that the distance from the center of the side of a car to the lower lateral truss is 11 feet, and the distance from center to center of the trusses is 18 feet?

SOLUTION.—Since the panel length of the bridge is 20 ft., Fig. 2 (*b*), and the distance center to center of trusses is 18 ft., the length of a diagonal of the lateral truss is

$$\sqrt{20^2 + 18^2} = 26.91 \text{ ft., and } \csc H = \frac{26.91}{18}$$

(*a*) The wind panel load for the upper lateral truss is $400 \times 20 = 8{,}000$ lb.; the reactions are each

$$\frac{8{,}000 \times 5}{2} = 20{,}000 \text{ lb}$$

Then, the stresses in the members are as follows [Fig. 2 (*d*)]

| MEMBER | STRESS, IN POUNDS |
|---|---|
| $B_1C$ | $20{,}000 \times \dfrac{26.91}{18} = -29{,}900$ |
| $C_1C$ | $+20{,}000$ |
| $C_1D$ | $(20{,}000 - 8{,}000) \times \dfrac{26.91}{18} = -17{,}900$ |
| $D_1D$ | $(20{,}000 - 8{,}000) = +12{,}000$ |
| $D_1E$ | $(20{,}000 - 8{,}000 - 8{,}000) \times \dfrac{26.91}{18} = -5{,}980$ |
| $EE_1$ | $= +8{,}000$ |
| $BC$ | $= 0$ |
| $B_1C_1$ | $\dfrac{20{,}000 \times 20}{18} = +22{,}200$ |
| $CD$ | $\dfrac{20{,}000 \times 20}{18} = -22{,}200$ |
| $C_1D_1$ | $\dfrac{20{,}000 \times 40 - 8{,}000 \times 20}{18} = +35{,}600$ |
| $DE$ | $\dfrac{20{,}000 \times 40 - 8{,}000 \times 20}{18} = -35{,}600$ |
| $D_1E_1$ | $\dfrac{20{,}000 \times 60 - 8{,}000 \times (20 + 40)}{18} = +40{,}000$ |

(*b*) The dead wind panel load for the lower lateral truss is $200 \times 20 = 4{,}000$ lb., the reactions are each

$$\frac{4{,}000 \times 7}{2} = 14{,}000 \text{ lb}$$

The live wind panel load is $300 \times 20 = 6{,}000$ lb., each live reaction for full load on the lower lateral truss is

$$\frac{7 \times 6{,}000}{2} = 21{,}000 \text{ lb}$$

The dead- and live-load stresses are found as explained in *Stresses in Bridge Trusses*, Part 2, in the present case they are found together. The combined stresses, in pounds, are as follows [Fig. 2 (*e*)].

The combined stress in $a_1 b$ is

$$(21{,}000 + 14{,}000) \times \frac{26.91}{18} = -52{,}300$$

The combined stress in $b_1 b$ is

$$21{,}000 + 14{,}000 = +35{,}000$$

The combined stress in $b_1 c$ is

$$\left[(14{,}000 - 4{,}000) + \frac{6{,}000 \times \frac{6 \times (6+1)}{2}}{8}\right] \times \frac{26.91}{18} = -38{,}500$$

The combined stress in $c_1 c$ is

$$\left[(14{,}000 - 4{,}000) + \frac{6{,}000 \times \frac{6 \times (6+1)}{2}}{8}\right] = +25{,}800$$

The combined stress in $c_1 d$ is

$$\left[(14{,}000 - 4{,}000 - 4{,}000) + \frac{6{,}000 \times \frac{5 \times (5+1)}{2}}{8}\right] \times \frac{26.91}{18} = -25{,}800$$

The combined stress in $d_1 d$ is

$$\left[(14{,}000 - 4{,}000 - 4{,}000) + \frac{6{,}000 \times \frac{5 \times (5+1)}{2}}{8}\right] = +17{,}250$$

The combined stress in $d_1 e$ is

$$\left[(14{,}000 - 4{,}000 - 4{,}000 - 4{,}000) + \frac{6{,}000 \times \frac{4 \times (4+1)}{2}}{8}\right] \times \frac{26.91}{18} = -14{,}200$$

The combined stress in $e_1 e$ is

full panel load $= 4{,}000 + 6{,}000 = +10{,}000$

The combined stress in $a b$ is 0

The combined stress in $a_1 b_1$ is

$$\frac{35{,}000 \times 20}{18} = +38{,}900$$

The combined stress in $b c$ is

$$\frac{35{,}000 \times 20}{18} = -38{,}900$$

The combined stress in $b_1 c_1$ is

$$\frac{35{,}000 \times 40 - 10{,}000 \times 20}{18} = +66{,}700$$

The combined stress in $c d$ is

$$\frac{35{,}000 \times 40 - 10{,}000 \times 20}{18} = -66{,}700$$

The combined stress in $c_1 d_1$ is

$$\frac{35{,}000 \times 60 - 10{,}000 \times (20 + 40)}{18} = +83{,}300$$

The combined stress in $d e$ is

$$\frac{35{,}000 \times 60 - 10{,}000 \times (20 + 40)}{18} = -83{,}300$$

The combined stress in $d_1 e_1$ is

$$\frac{35{,}000 \times 80 - 10{,}000 \times (20 + 40 + 60)}{18} = +88{,}900$$

(*c*) The live panel load (which is the wind pressure on the train) was found in (*b*) to be 6,000 lb Since the trusses are 18 ft apart, and the distance from the center of the wind pressure on the train to the lower lateral truss is 11 ft , the increase in the vertical load per panel on the leeward truss is

$$\frac{6{,}000 \times 11}{18} = 3{,}670 \text{ lb.}$$

## EXAMPLES FOR PRACTICE

1 A ten-panel deck bridge with vertical end posts has a span length of 180 feet and a distance from center to center of trusses of 18 feet If the wind pressure on a train of cars is 300 pounds per linear foot, and on the upper chord is 300 pounds per linear foot, what is the maximum stress in the diagonal of the second panel of the upper lateral truss? Ans − 54,200 lb

2 If the center of the wind pressure on the cars on the bridge referred to in example 1 is 10 feet above the upper lateral truss, what is the amount by which each panel load of the leeward truss must be increased to allow for the overturning effect of the wind? Ans 3,000 lb.

3 If the wind pressure on the lower chord of the bridge referred to in example 1 is 200 pounds per linear foot, what is the maximum stress in the second panel of the leeward chord of the lower lateral truss? Ans. − 16,200 lb

## PORTAL AND OTHER SWAY BRACING

**7. Description.**—It was explained in Art. 5 that the lateral truss of the upper chord simply transmits the wind pressure to the upper joints of the end posts, usually called

the **hip joints**, and that the end posts, with the assistance of the portal bracing, transmit it to the supports. In the through bridge, the portal bracing lies in the plane of the end posts, usually inclined, and is made as deep as possible without encroaching on the headroom required by the loads that cross the bridge. In Fig. 4 are represented the end posts $A\,C$ and $A'\,C'$ of two trusses connected by the standard types of portal bracing, usually called simply

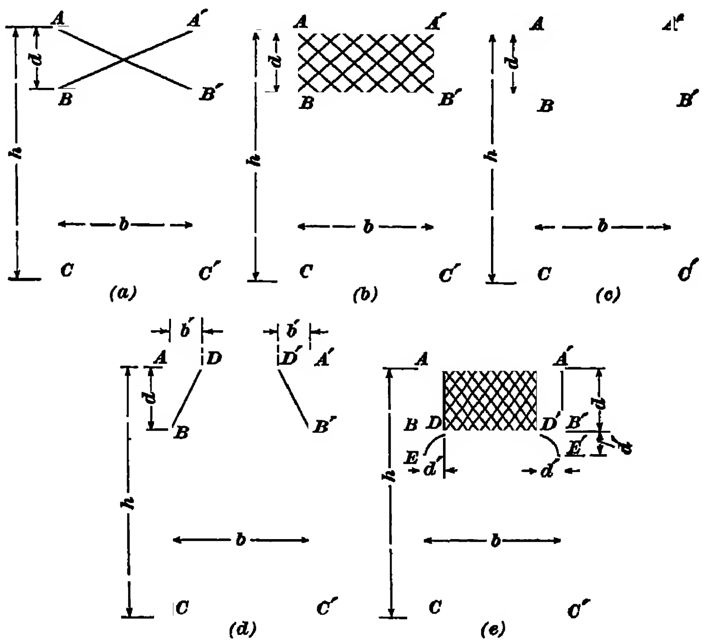


Fig 4

the *portal* (*a*) consists of the transverse members $A\,A'$ and $B\,B'$ and the two diagonals $A\,B'$ and $A'B$; (*b*) consists of the transverse members $A\,A'$ and $B\,B'$ and a lattice web, (*c*) consists of a plate girder; and (*d*) consists of the transverse member $A\,A'$ and the **knee braces** $B\,D$ and $B'\,D'$. In (*e*), the portion $A\,A'\,B'\,B$ may be similar to either of the forms shown in (*b*) and (*c*), the brackets $B\,D\,E$ and $B'\,D'\,E'$ under the portal are frequently added, partly for architectural

effect, and partly to afford additional stiffness. It is almost impossible to state in a general way which type should be used, as this will depend to a great extent on the judgment of the designer; in the analysis of each type, however, the conditions to which that type is best adapted will be mentioned. In all that follows regarding the stresses in the portal bracing, it must be remembered that the portal and all the stresses considered lie in the plane that passes through the center lines of the end posts

**8. Method of Calculation.**—In the analysis of trusses in the preceding articles, it was assumed in each case that the stress in each member was a direct stress, and that each member was hinged or free to turn about the joints at its two ends In the case of portal bracing, the end post, in addition to the direct stresses, is subjected to shearing and bending stresses due to the wind pressure; and, whenever the end posts are considered cut by a plane of section, these additional stresses must be taken into consideration. The portal and end posts can be considered as a structure some members of which are subjected to bending as well as to direct stresses. The only external forces that act on this structure are the wind pressure at the top and the reactions that this pressure causes at the supports $C$ and $C'$ As in the case of the lateral truss, the wind pressure on the portal may, for convenience, be assumed to act as a single force on the windward side; it is equal to one-half the sum of the wind panel loads on the upper chord, including the pressure at the hip joint; in this Section, it will be called $P$ The analytic method is the most convenient for finding the stresses in the portal and end posts, and so the graphic method will not be discussed.

**9. Reactions.**—The reactions at $C$ and $C'$, Fig. 5 ($a$), may be found by applying the conditions of equilibrium to the external forces, considering the reactions to be resolved into components parallel and at right angles, respectively, to the end posts, the components parallel to the end posts will be called the $Y$ components, and those perpendicular to the

end posts the $X$ components. Assuming the components $X$ and $X'$, $Y$ and $Y'$ of the reactions at $C$ and $C'$ to act as shown in Fig. 5, and denoting by $b$ the distance from center

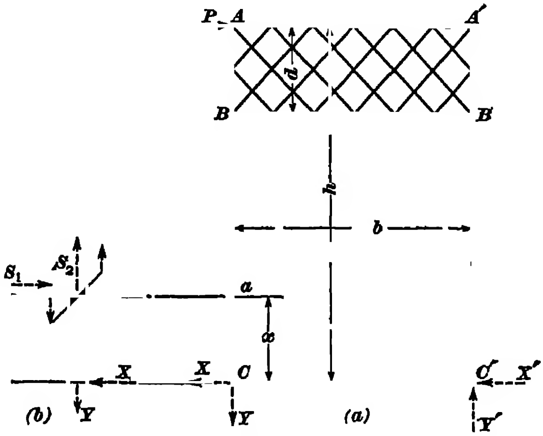

FIG 5

to center of the trusses, and by $h$ the distance from the top of the end post to the bottom, we have, from the equation $\Sigma M = 0$,

$$Y = \frac{P \times h}{b}, \; Y' = \frac{P \times h}{b} = Y,$$

and from the equation $\Sigma X = 0$,

$$P = X + X'$$

It is impossible to calculate the actual values of $X$ and $X'$ by the equations of equilibrium, but in practice it is customary in this case to assume that they are each equal to $\frac{P}{2}$. This is probably quite close to their actual values The reactions as just found are the same, no matter what type of portal is used.

**10. End Posts.**—The stresses in the end posts and the portals depend on the condition of the connections at $C$ and $C'$—whether the end posts are hinged at these points or are firmly fixed in direction. For the present, it will be assumed that they are hinged, and the stresses will be found on this assumption. Later on, the effect of fixing them will be

discussed. As there are no diagonals meeting the end posts at $C$ and $C'$, Fig. 5 ($a$), those posts must be capable of resisting both the $X$ and $Y$ components: the $Y$ components will act along them, causing direct stresses; the $X$ components will act on the portions $B\,C$ and $B'\,C'$, below the portal, as on overhanging beams loaded at $C$ and $C'$, and fixed in direction at $B$ and $B'$, respectively.

The end post $B\,C$ may be considered cut by a plane at the section $a$ at a distance equal to $x$ from $C$, and the portion between $a$ and $C$ considered as a free body, as shown in Fig 5 ($b$). As this portion is in equilibrium, the forces acting on it must be in equilibrium. Let $S_1$ and $S_2$ be, respectively, the horizontal and the vertical component of the stress at $a$. Then, from the equation $\Sigma\,Y = 0$,

$$S_2 = Y = P\frac{h}{b}, \text{ tension in } B\,C$$

From the equation $\Sigma\,X = 0$,

$$S_1 = X = \frac{P}{2}, \text{ shear on section } a$$

Applying the equation $\Sigma\,M = 0$, moments being taken about the center of section $a$, the moment of $Y$ will be zero, and the moment of resistance to be offered by the end post at section $a$ will equal $Xx$, or $\frac{Px}{2}$. Therefore, there is tension in $B\,C$ from $B$ to $C$ equal to $P\frac{h}{b}$, shear at any section between $B$ and $C$ equal to $\frac{P}{2}$, and bending moment at any distance $x$ from $C$ equal to $\frac{Px}{2}$; at $B$, where $x$ is equal to $h - d$, this moment is greatest and equal to $\frac{P}{2}(h - d)$. In like manner, it may be shown that in member $B'\,C'$ there is compression equal to $P\frac{h}{b}$, shear at any point between $B'$ and $C'$ equal to $\frac{P}{2}$, and bending moment at $B'$ equal to $\frac{P}{2}(h - d)$. The fiber stresses caused in the end posts by

these stresses must be added to the stresses due to dead and live loads to get the maximum stresses.

The stresses from $C$ and $C'$ to the lowest point of the portal bracing will be the same as just given, no matter what type of portal is used The portals will now be considered.

**11. The Braced Portal.**—The portal shown in Fig. 4 ($a$) is sometimes called a **braced portal**, and is used when the depth of portal is large compared to the length, thereby

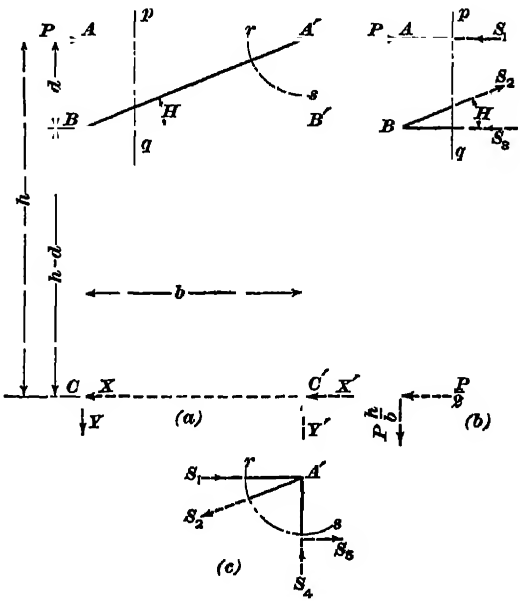

FIG 6

giving the diagonals a good inclination. The latter are usually constructed in such a way that they can resist tension and compression, but it is customary to assume that they act in tension only, when one is in action, the other is assumed to be out of action If the wind comes from the left, $P$ is applied at $A$, and the diagonal $B A'$ is in action, as shown in Fig. 6 ($a$) The stresses in the members may now be found by the method of moments and shears. The portal may be considered cut by a plane, such as $p q$, Fig 6 ($a$), that cuts

$AA'$, $BA'$, and $BB'$, and the portion to the left of the plane considered as a free body, as shown in Fig. 6 (*b*), the forces acting on this portion are $P$ at $A$, $\frac{P}{2}$ and $\frac{Ph}{b}$ at $C$, and the forces $S_1$, $S_2$, and $S_3$ representing the stresses in $AA'$, $BA'$, and $BB'$, respectively, and assumed to act as represented in Fig. 6 (*b*). Applying the equation $\Sigma Y = 0$, we have

$$S_2 \sin H - \frac{Ph}{b} = 0;$$

whence
$$S_2 = \frac{Ph}{b} \csc H$$

As this is positive, the assumed direction of $S_2$ is correct, and the stress in $BA'$ is tension.

The stress in $AA'$, represented by $S_1$, can be found most readily by the method of moments, taking $B$ as the center of moments. Applying the equation $\Sigma M = 0$, we have

$$\frac{P}{2}(h - d) + Pd - S_1 d = 0;$$

whence
$$S_1 = \frac{Ph}{2d} + \frac{P}{2}$$

As this is positive, the assumed direction of $S_1$ is correct, and the stress in $AA'$ is compression.

The stress in $BB'$, represented by $S_3$, can be found most readily by applying the equation $\Sigma X = 0$ to all the $X$ components that act on the portion shown in Fig. 6 (*b*). The $X$ component of $S_2$ is $S_2 \cos H$, or $\frac{Ph}{b} \times \cot H$, and, substituting for $\cot H$ its value $\frac{b}{d}$, $S_2 \cos H$ becomes $\frac{Ph}{d}$. Then,

$$S_3 - \frac{Ph}{d} - P + \left(\frac{Ph}{2d} + \frac{P}{2}\right) + \frac{P}{2} = 0;$$

whence
$$S_3 = \frac{Ph}{2d}$$

As this is positive, the assumed direction of $S_3$ is correct, and the stress in $BB'$ is compression.

At the joint $A$, the two external forces $P$ and $S_1$ are not equal; the difference between them,

$$\left(\frac{Ph}{2d} + \frac{P}{2}\right) - P = \frac{Ph}{2d} - \frac{P}{2},$$

acting to the left, causes shearing stress in the member $AB$. As there are no $Y$ forces at the joint $A$, there is no direct stress in $AB$.

The joint $A'$ may be considered a free body, as represented in Fig 6 $(c)$; the forces acting on it are $S_1$, $S_3$, $S_4$, and $S_5$, the latter representing the shear in $A'B'$. Applying the equation $\Sigma Y = 0$, we have

$$S_4 - S_3 \sin H = 0, \text{ or } S_4 - \frac{Ph}{b} = 0;$$

whence
$$S_4 = \frac{Ph}{b}.$$

As this is positive, the assumed direction of $S_4$ is correct, and the direct stress in $A'B'$ is compression. Applying the equation $\Sigma X = 0$, we have

$$S_5 + S_1 - S_3 \cos H = 0$$

or
$$S_5 + \left(\frac{Ph}{2d} + \frac{P}{2}\right) - \frac{Ph}{d} = 0;$$

whence
$$S_5 = \frac{Ph}{2d} - \frac{P}{2}, \text{ shear in } A'B'$$

which is equal to the shear in $AB$.

When the wind blows from the right, the diagonal $AB'$ will be in action, and $A'B$ out of action; the stresses in the other members of the portal will be the same as those given above; the stresses in the end posts will change in character.

**12. Plate-Girder Portal.**—Fig 4 $(c)$ shows a type of portal composed of the two transverse members $AA'$ and $BB'$, usually made of angles, connected by a solid web or plate. The construction is identical with that of a plate girder. This form of portal is called a **plate-girder portal**, and is especially adapted to cases where the available depth of portal is small compared with the length, as in the case of a shallow truss. In this Section, it will be sufficient to get the expressions for the shears and moments on the girder at various points; the methods of calculating the flange and web stresses from them will be fully discussed in connection with the design of plate girders. If the plate-girder portal shown in Fig. 7 is cut by a plane at

right angles to the flanges $AA'$ and $BB'$ at a distance $x$ from $AC$, and the portion to the left of the section is considered as a free body, it will be seen that the shear on the section is equal to $Y$, which is equal to $P\frac{h}{b}$, and, as this is independent of $x$, the shear will be constant from $AB$ to $A'B'$. When the wind comes from the left, the shear is negative; when from the right, it is positive.

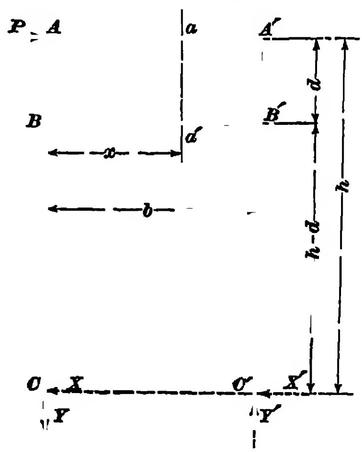

FIG 7

The moment $M$ of the external forces on the left of the section about the point $a$, which may be called the bending moment at $a$ in the upper flange, is equal to

$$Xh - Yx = \frac{Ph}{2} - \frac{Phx}{b}$$

When $x = 0$, $\quad M = \frac{Ph}{2}$

When $x = \frac{b}{2}$, $\quad M = \frac{Ph}{2} - \frac{Ph}{2} = 0$

When $x = b$, $\quad M = \frac{Ph}{2} - Ph = -\frac{Ph}{2}$

The moment $M'$ about the point $a'$, or the bending moment at $a'$ in the lower flange, is equal to

$$X(h-d) - Yx + Pd = \frac{Ph}{2} + \frac{Pd}{2} - \frac{Phx}{b}$$

When $x = 0$, $\quad M' = \frac{Ph}{2} + \frac{Pd}{2}$

When $x = \frac{b}{2}$, $\quad M' = \frac{Pd}{2}$

When $x = b$, $\quad M' = -\frac{Ph}{2} + \frac{Pd}{2}$

When the wind comes from the right, the distance $x$ is the distance from the section to the right end, and the moments are given by the same formulas.

**13. Lattice Portal.**—Fig. 4 ($b$) shows a type of portal composed of the two transverse struts $AA'$ and $BB'$ connected by an open or lattice web, there being several systems of web members. This is by far the most common type in use, especially where a reasonable depth may be had—say, at least, one-sixth the distance from center to center of the end posts The actual number of web systems is usually from two to six or eight, but in some cases more are used, the number depending to a great extent on the depth of the portal No definite rule can be stated for sketching out or selecting the arrangement of web members, which depends somewhat on the judgment of the designer, and is frequently controlled by esthetic considerations.

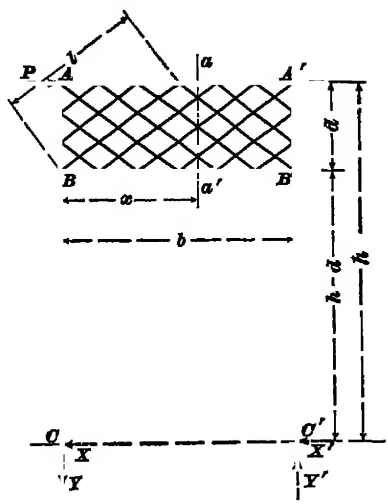


FIG 8

If the portal shown in Fig. 8 is considered cut by a plane $aa'$ at right angles to the flanges $AA'$ and $BB'$, and the portion on the left of the section is considered as a free body, it will be seen that the shear on the section and the moments about the points in the upper and lower flanges at a distance $x$ from the left end are the same as those found for a plate-girder portal. They are, then, as follows:

$$\text{shear} = \frac{Ph}{b}; \text{ moment at } a = \frac{Ph}{2} - \frac{Phx}{b};$$

$$\text{moment at } a' = \frac{Ph}{2} + \frac{Pd}{2} - \frac{Phx}{b}$$

As in the case of the multiple-system truss, it is impossible to calculate the stresses in the members directly by the equations of equilibrium; in this case, it is also impossible to find what proportion of the load goes to each system, so that the

systems cannot be treated independently, as in the case of the truss. For purposes of calculation, however, it is customary to assume that the shear on any section cut by a plane at right angles to the flanges is evenly distributed among the web members cut by the plane; if the number of web members cut is $n$, the $Y$ component in each member is assumed to be $\frac{Ph}{nb}$. This assumption is not absolutely correct, but is probably as close to the true condition of affairs as any assumption that can be made. In the present case, where all the web members have the same inclination, if the length of one member between flanges is $l$, the stress in each diagonal is $\frac{Ph}{nb} \times \frac{l}{d}$. When the wind comes from the left, the shear is negative, and causes tension in those members that slope downwards to the left, and compression in those that slope upwards to the left; when the wind comes from the right, the shear is positive, and the stresses are opposite to the above, but of the same numerical values.

The stresses in the flanges may be calculated most readily by the method of moments. As the $Y$ components in the various web members are equal, and as the inclinations of the members are the same, their $X$ components must be equal also. Then, if a plane is passed through the center of one of the panels of the flanges, it will cut the web members at their intersections. The lever arms, and, therefore, the moments of the $Y$ components of the stresses in the web members, about the points $a$ and $a'$ in the flanges, will be equal to zero. At each intersection, there are two $X$ components, equal and opposite; the sum of their moments about the points $a$ and $a'$ is, therefore, equal to zero The moments of the external forces about $a$ and $a'$ have already been found. Then, the stress in $AA'$ is equal to the moment at $a'$ divided by $d$, and the stress in $BB'$ is equal to the moment at $a$ divided by $d$. The stress in $AA'$ is

$$\frac{\frac{Ph}{2} + \frac{Pd}{2} - \frac{Phx}{b}}{d} = \frac{Ph}{2d} + \frac{P}{2} - \frac{Phx}{bd}$$

and the stress in $BB'$ is

$$\frac{\frac{Ph}{2} - \frac{Phx}{b}}{d} = \frac{Ph}{2d} - \frac{Phx}{bd}$$

The actual stresses will depend on the value of $x$. If the web members divide the flanges into $m$ panels of the same length $p$, the smallest value of $x$ will be $\frac{p}{2}$, and the largest, $mp - \frac{p}{2}$, or $\frac{2m-1}{2}p$. When there is a large number of small panels, the smallest value of $x$ may be taken equal to zero, and the largest equal to $mp$, or $b$. Then:

When $x = 0$,

stress in $AA' = \frac{Ph}{2d} + \frac{P}{2}$, compression at left end

When $x = b$,

stress in $AA' = -\frac{Ph}{2d} + \frac{P}{2}$, tension at right end

When $x = 0$,

stress in $BB' = \frac{Ph}{2d}$, tension at left end

When $x = b$,

stress in $BB' = -\frac{Ph}{2d}$, compression at right end

When the wind comes from the right, $x$ is the distance from the right end, and the stresses in the members are reversed.

**14. Portal With Knee Braces.**—When the depth of the truss is such that there is insufficient room above the traffic for any of the portals that have been previously described, an arrangement similar to that shown in Fig 4 ($d$) is frequently used. The horizontal member $ADD'A'$ is the portal and connects the end posts at the top; the inclined members, or knee braces, $BD$ and $B'D'$ connect the end posts to the under side of the portal, which is usually a plate or lattice girder. The reactions at $C$ and $C'$, and the stresses in the end posts below the points $B$ and $B'$, are the same as found in Art. **10.** The knee braces are subjected to direct stresses of tension and compression; the portal is subjected

to direct stresses of tension and compression, and also to shearing and bending stresses.

**15.** The stress in $BD$, Fig. 9 ($a$), may be found by taking moments about $A$ of all the forces on the left of a plane of section $pq$ cutting $BD$ and $AD$. Denoting the $X$

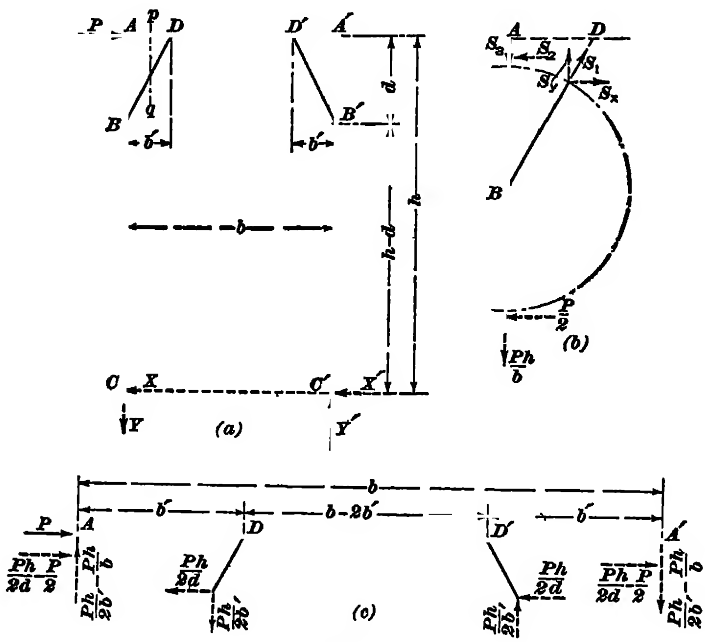

Fig 9

component of the stress in $BD$ by $S_x$, and the length of $BD$ by $l$, and taking moments about $A$, we have

$$\Sigma M = \frac{P}{2}h - S_x d = 0;$$

whence
$$S_x = \frac{Ph}{2d}$$

and the stress in $BD$ is

$$S_x \sec ADB = \frac{Ph}{2d} \times \frac{BD}{AD} = \frac{Ph}{2d} \times \frac{l}{b'}$$

Also,
$$S_y = \frac{Ph}{2b'}$$

Applying the equations $\Sigma X = 0$ and $\Sigma Y = 0$ to all the forces acting at the point $B$, Fig. 9 ($b$), the stresses in $AB$ are found as follows:

$$\Sigma X = \frac{P}{2} - \frac{Ph}{2d} + S_2 = 0;$$

whence $$S_2 = \frac{Ph}{2d} - \frac{P}{2}, \text{ shear in } AB.$$

$$\Sigma Y = \frac{Ph}{b} - \frac{Ph}{2b'} + S_3 = 0;$$

whence $$S_3 = \frac{Ph}{2b'} - \frac{Ph}{b}, \text{ compression in } AB$$

In like manner, it may be shown that the stress in $B'D'$ is compression equal to $\frac{Phl}{2db'}$, and the stresses in $A'B'$ are shear equal to $\frac{Ph}{2d} - \frac{P}{2}$, and tension equal to $\frac{Ph}{2b'} - \frac{Ph}{b}$.

As this style of portal is usually very shallow and rests on top of the chords, it will be sufficiently accurate to ignore the depth of the portal itself in computing the moments.

The $X$ and $Y$ components of the forces that act on the portal are shown in Fig. 9 ($c$); the stresses will now be found.

At the joint $A$, the forces are as follows. the wind pressure $P$, the shear and direct stress in $AB$, and the stresses in $AD$. Denoting the direct stress in $AD$ by $S_4$, and writing the equation $\Sigma X = 0$, we have

$$\Sigma X = S_4 - P - \frac{Ph}{2d} + \frac{P}{2} = 0;$$

whence $$S_4 = \frac{Ph}{2d} + \frac{P}{2}, \text{ compression}$$

Denoting the shear in $AD$ by $S_5$, and writing the equation $\Sigma Y = 0$, we have

$$\Sigma Y = S_5 - \frac{Ph}{2b'} + \frac{Ph}{b} = 0;$$

whence $$S_5 = \frac{Ph}{2b'} - \frac{Ph}{b}, \text{ positive shear}$$

The bending moment at any section at a distance $x$ to the right of $A$ is

$$M = \left(\frac{Ph}{2b'} - \frac{Ph}{b}\right) x$$

At $D$, $x = b'$, and $M_D = \frac{Ph}{2} - \frac{Phb'}{b}$, positive moment.

The forces at the joint $D$ are the stresses in $AD$, $DD'$, and $BD$. Denoting the direct stress in $DD'$ by $S_s$, and writing the equation $\Sigma X = 0$, we have

$$\Sigma X = S_s - \left(\frac{Ph}{2d} + \frac{P}{2}\right) + \frac{Ph}{2d} = 0,$$

whence $$S_s = \frac{P}{2}, \text{ compression}$$

Denoting the shear in $DD'$ by $S_r$, and writing the equation $\Sigma Y = 0$,

$$\Sigma Y = S_r - \frac{Ph}{2b'} + \left(\frac{Ph}{2b'} - \frac{Ph}{b}\right) = 0;$$

whence $$S_r = \frac{Ph}{b}, \text{ negative shear}$$

The bending moment at any section of $DD'$, at a distance $x_1$ from $A$, is

$$M = \left(\frac{Ph}{2b'} - \frac{Ph}{b}\right)x_1 - \frac{Ph}{2b'}(x_1 - b')$$
$$= \frac{Ph}{2} - \frac{Phx_1}{b}, \text{ positive moment}$$

At $D'$, $x_1 = b - b'$, and $M_{D'} = \frac{Phb'}{b} - \frac{Ph}{2}$, positive moment.

In like manner, the stresses in $D'A'$ may be found. They are as follows.

$\frac{Ph}{2b'} - \frac{Ph}{b}$, positive shear; and $\frac{Ph}{2d} - \frac{P}{2}$, tension

**16. Portal With Curved Brackets.**—With the portals shown in Fig. 4 ($b$) and ($c$), the form of knee brace or bracket shown in Fig. 4 ($e$) is frequently used; it serves a double purpose in decreasing the bending on the end post and adding slightly to the architectural appearance of the entrance to the bridge. In this case, the maximum bending moment on the end post occurs at $E$ and $E'$, and is equal to $\frac{P}{2}(h - d - d')$, which is $\frac{Pd'}{2}$ less than when no bracket is used.

The bending moment on the portal where it is in contact with the bracket is less than when no bracket is used. In

calculating the moments at various sections along the portal, the formulas already found in Arts. **12** and **13** may be used, $x$ varying from a minimum value $d'$ to a maximum value $b - d'$ for the plate girder, and from $d' + \frac{p}{2}$ to $b - d' - \frac{p}{2}$ for the lattice portal. The shear between the brackets will be the same as when no brackets are used.

**17. Formulas for Portals.**—The formulas necessary for the designs of the various portals that have been analyzed are shown in Fig. 10 ($a$) to ($e$), with the wind assumed as coming from the left. When the wind comes from the right, the formulas will be the same, but will in each case apply to the corresponding member on the other side on the bridge.

EXAMPLE —In the portals shown in Fig 4, let $h = 32$ feet, $d = 8$ feet, $b = 18$ feet, $b' = 4$ feet, $d' = 3$ feet, and $P = 24{,}000$ pounds, the wind coming from the left What are the wind stresses in the different portals and end posts, using the formulas given in Fig 10?

SOLUTION.—In Fig 10 ($a$), ($b$), ($c$), and ($d$), the stresses in $BC$ are as follows

The tension is $\frac{Ph}{b} = \frac{24{,}000 \times 32}{18} = 42{,}700$ lb.

The shear is $\frac{P}{2} = 24{,}000 \div 2 = 12{,}000$ lb.

The bending moment at $B$ is

$$\frac{P}{2}(h - d) = \frac{24{,}000}{2} \times (32 - 8) = 288{,}000 \text{ ft.-lb.}$$

The stresses in $B'C'$ are the same numerically as those in $BC$; the direct stress of 42,700 lb. in $B'C'$ is compression.

In Fig 10 ($e$), the stresses in $EC$ and $E'C'$ are the same, respectively, as those in $BC$ and $B'C'$, just given, except that the bending moment at $E$ and at $E'$ is equal to

$$\frac{P}{2}(h - d - d') = \frac{24{,}000}{2} \times (32 - 8 - 3) = 252{,}000 \text{ ft -lb}$$

1. The stresses in the members of the braced portal shown in Fig. 10 ($a$) will next be found.

The stresses in $AA'$, $A'B$, and $BB'$ are as follows

in $AA'$, $\frac{Ph}{2d} + \frac{P}{2} = \frac{24{,}000 \times 32}{2 \times 8} + \frac{24{,}000}{2} = 60{,}000$ lb.

in $A'B$, $\frac{Phl}{db} = \frac{24{,}000 \times 32 \times \sqrt{18^2 + 8^2}}{8 \times 18} = 105{,}100$ lb, tension

in $BB'$, $\frac{Ph}{2d} = \frac{24{,}000 \times 32}{2 \times 8} = 48{,}000$ lb, compression

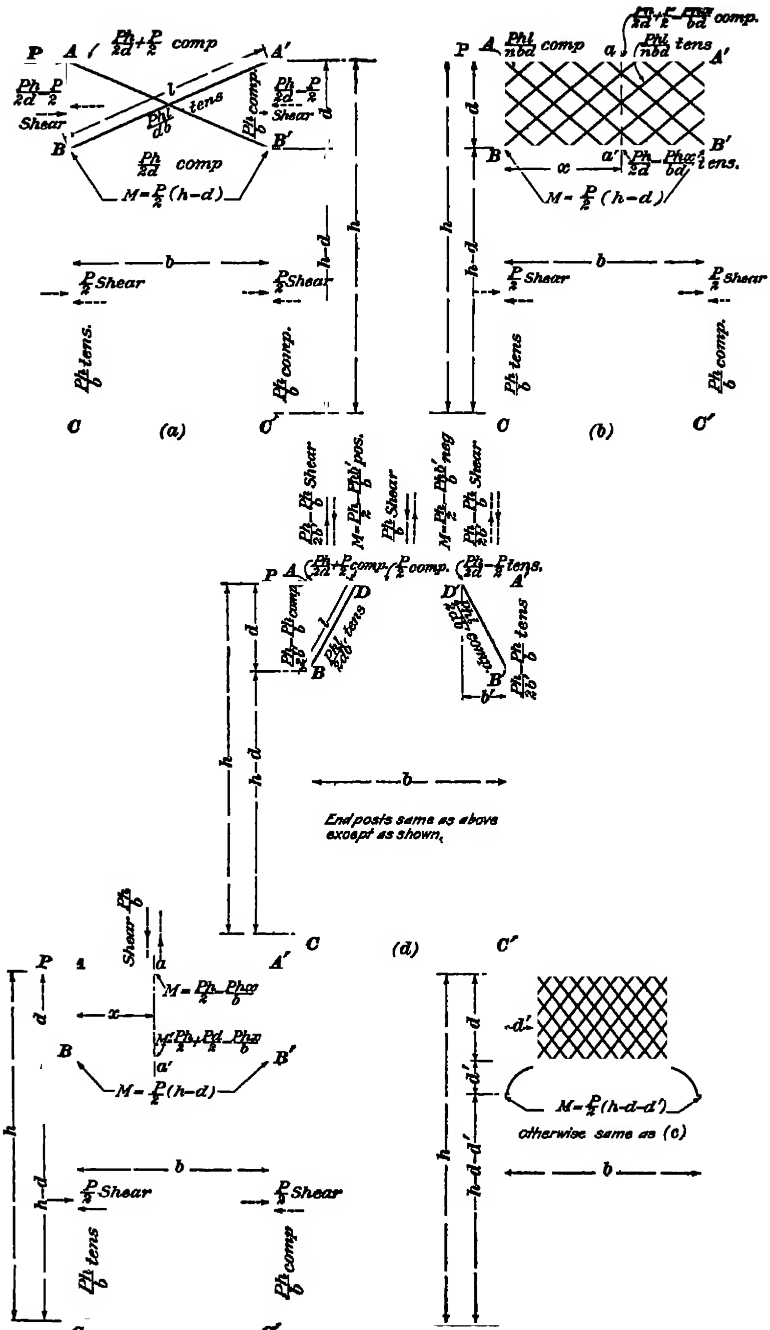
(a)
(b)
(c)
(d)
(e)
End posts same as above except as shown.
otherwise same as (c)

FIG 10

The direct stress in $AB$ is 0.

The shear in $AB$ and $A'B'$ is

$$\frac{Ph}{2d} - \frac{P}{2} = \frac{24{,}000 \times 32}{2 \times 8} - \frac{24{,}000}{2} = 36{,}000 \text{ lb.}$$

The direct stress in $A'B'$ is

$$\frac{Ph}{b} = \frac{24{,}000 \times 32}{18} = 42{,}700 \text{ lb., compression}$$

2 The stresses in the members of the lattice portal shown in Fig 10 ($b$) will next be found

The stress in each web member is equal to

$$\frac{Phl}{nbd} = \frac{24{,}000 \times 32 \times \sqrt{9^2 + 8^2}}{6 \times 18 \times 8} = 10{,}700 \text{ lb.,}$$

tension in those members that slope upwards to the right, and compression in the remaining members.

The stress in $AA'$ is equal to

$$\frac{Ph}{2d} + \frac{P}{2} - \frac{Phx}{bd} = \frac{24{,}000 \times 32}{2 \times 8} + \frac{24{,}000}{2} - \frac{24{,}000 \times 32 \times x}{8 \times 18}$$

$$= 60{,}000 - \frac{16{,}000}{3} x$$

Since there are six panels and $b = 18$ ft., $p = \frac{18}{6} = 3$ ft ; then, at the left end $x = 1.5$ ft and the stress in $AA'$ is

$$60{,}000 - \frac{16{,}000}{3} \times 1.5 = 52{,}000 \text{ lb, compression}$$

At the right end, $x = 16.5$ ft , and the stress in $AA'$ is

$$60{,}000 - \frac{16{,}000}{3} \times 16.5 = -28{,}000 \text{ lb, tension}$$

The stress in $BB'$ is

$$\frac{Ph}{2d} - \frac{Phx}{bd} = \frac{24{,}000 \times 32}{2 \times 8} - \frac{24{,}000 \times 32 \times x}{8 \times 18} = 48{,}000 - \frac{16{,}000}{3} \times x$$

At the left end, the stress in $BB'$ is

$$48{,}000 - \frac{16{,}000}{3} \times 1.5 = 40{,}000 \text{ lb., tension}$$

At the right end, the stress in $BB'$ is

$$48{,}000 - \frac{16{,}000}{3} \times 16.5 = -40{,}000 \text{ lb, compression}$$

3 The shears and moments on the plate-girder portal shown in Fig 10 ($c$) will next be found.

Shear at any section is $\frac{Ph}{b} = 42{,}700$ lb , negative shear.

The moments in the upper flange are: at the left end, $M = \frac{Ph}{2} = 384{,}000$ ft -lb , and, at the right end, $M = -\frac{Pn}{2} = -384{,}000$ ft.-lb.

The moments in the lower flange are: at the left end,

$$M' = \frac{Ph}{2} + \frac{Pd}{2} = 480{,}000 \text{ ft.-lb.}$$

and, at the right end,

$$M' = \frac{Pd}{2} - \frac{Ph}{2} = -288{,}000 \text{ ft -lb}$$

4. The stresses in the portal with knee braces, shown in Fig 10 (*d*), will next be found

The stresses in $BD$ and $B'D'$ are as follows:

in $BD$, $\frac{Phl}{2db'} = \frac{24{,}000 \times 32 \times \sqrt{8^2+4^2}}{2 \times 8 \times 4} = 107{,}300$ lb , tension

in $B'D'$, $\frac{Phl}{2db'} = 107{,}300$ lb , compression

The direct stresses in $AB$ and $A'B'$ are

in $AB$,

$$\frac{Ph}{2b'} - \frac{Ph}{b} = \frac{24{,}000 \times 32}{2 \times 4} - \frac{24{,}000 \times 32}{18} = 53{,}300 \text{ lb , compression}$$

in $A'B'$, $\frac{Ph}{2b'} - \frac{Ph}{b} = 53{,}300$ lb , tension

The direct stresses in $AD$, $DD'$, and $D'A'$ are.

in $AD$, $\frac{Ph}{2d} + \frac{P}{2} = \frac{24{,}000 \times 32}{2 \times 8} + \frac{24{,}000}{2} = 60{,}000$ lb , compression

in $DD'$, $\frac{P}{2} = \frac{24{,}000}{2} = 12{,}000$ lb , compression

in $D'A'$, $\frac{Ph}{2d} - \frac{P}{2} = \frac{24{,}000 \times 32}{2 \times 8} - \frac{24{,}000}{2} = 36{,}000$ lb , tension

The shears in $AD$, $DD'$, and $D'A'$ are as follows.

in $AD$, $\frac{Ph}{2b'} - \frac{Ph}{b} = \frac{24{,}000 \times 32}{2 \times 4} - \frac{24{,}000 \times 32}{18} = 53{,}300$ lb,

in $DD'$, $\frac{Ph}{b} = \frac{24{,}000 \times 32}{18} = 42{,}700$ lb

in $D'A'$, $\frac{Ph}{2b'} - \frac{Ph}{b} = 53{,}300$ lb

The bending moments at $D$ and $D'$ are

at $D$, $\frac{Ph}{2} - \frac{Phb'}{b} = \frac{24{,}000 \times 32}{2} - \frac{24{,}000 \times 32 \times 4}{18} = 213{,}300$ ft -lb , pos

at $D'$, $\frac{Ph}{2} - \frac{Phb'}{b} = 213{,}300$ ft -lb., neg.

5. The stresses in the portal with curved brackets will next be found

The stresses in the web members are the same as found in 2 The stresses in the flanges are different, however, from those there found, because the smallest value of $x$ is 3 75 ft and the largest is 14.25 ft

The stresses in $AA'$ and $BB'$ are as follows.

in $AA'$ (at 3 75 ft to right of left end),

$$60{,}000 - \frac{16{,}000}{3} \times 3\ 75 = 40{,}000 \text{ lb , compression}$$

in $A A'$ (at 3 75 ft. to left of right end),

$$60{,}000-\frac{16{,}000}{3}\times 14\ 25=-16{,}000\text{ lb, tension}$$

in $B B'$ (at 3 75 ft to right of left end),

$$48{,}000-\frac{16{,}000}{3}\times 3\ 75=28{,}000\text{ lb, tension}$$

in $B B'$ (at 3 75 ft to left of right end),

$$48{,}000-\frac{16{,}000}{3}\times 14\ 25=-28{,}000\text{ lb, compression}$$

**18. Fixed End Posts.**—In the preceding discussion, it was assumed that the end posts were hinged at the bottom, or free to turn. When they simply rest on pedestals, this is very near the actual case, and the bending moment at the lower joint is zero. When they are firmly riveted at their lower ends to floorbeams or cross-braces of considerable depth, they offer some resistance to bending at these points, and this bending is opposite in direction to that at the lower connection of the portal. Then there is a point of inflection somewhere between the bottom of the end post and the portal, the actual location of which depends on the nature of the top and bottom connections. For all ordinary cases, in a well-designed bridge, if the end posts are fixed at top and bottom, the point of inflection will be about midway between the lower joint and the lower line of the portal; to allow for lack of sufficient rigidity at the lower joint, it is well to consider the point of inflection never higher than one-half the distance from the lower joint to the lower line of the portal. The actual location of the point of inflection may be found for each case, but as its determination involves the use of advanced mathematics, it will not be given here.

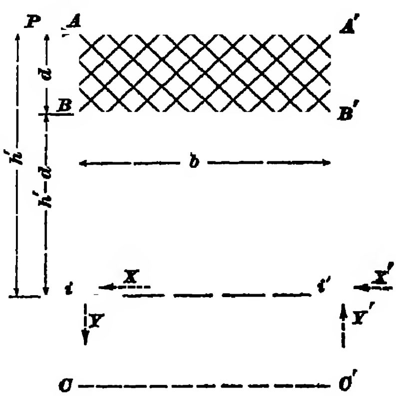


FIG 11

As the bending moment at the points of inflection is zero, the only stresses in the end posts at those points are the direct stresses and shears, the $Y$ and $X$ components. In Fig 11, let $i$ and $i'$ represent these points at a distance $h'$ from the top of the portal. Then, if moments are taken about $i$ and $i'$, it will be found that

$$Y = P\frac{h'}{b} = Y'; \text{ also, } X = \frac{P}{2}$$

As these are the same as though the end posts were supported at the points $i$ and $i'$, the stresses in the portals may be found by means of the formulas already deduced, substituting for $h$, in each case, the distance $h'$. It is clear that the stresses in the end posts and portal are decreased by fixing the end posts at the bottom.

EXAMPLE —In Fig 4 ($b$), suppose that the wind pressure and dimensions are the same as given in the example in Art **17**. What are the wind stresses in the end posts and members of the portal, assuming that the point of inflection is located one-third of the distance from $C$ to $B$, that is, 8 feet above $C$ and $C'$, so that $h' = 24$ feet?

SOLUTION —The stresses will be found by the formulas given in Fig 10 ($b$), substituting $h'$ for $h$. Then, the direct stresses in the end posts are

$$\frac{Ph'}{b} = \frac{24{,}000 \times 24}{18} = 32{,}000 \text{ lb}$$, tension in $B\,C$ and compression in $B'\,C'$

The shear in $B\,C$ and $B'\,C'$ is

$$\frac{P}{2} = \frac{24{,}000}{2} = 12{,}000 \text{ lb.}$$

The moment at $B$ and $B'$ is

$$\frac{P}{2}(h' - d) = 12{,}000 \times 16 = 192{,}000 \text{ ft.-lb.}$$

The stress in each web member is

$$\frac{Ph'l}{nbd} = \frac{24{,}000 \times 24 \times \sqrt{9^2 + 8^2}}{6 \times 18 \times 8} = 8{,}000 \text{ lb.},$$

tension in those members that slope upwards to the right and compression in the others

The stress in $A\,A'$ is

$$\frac{Ph'}{2d} + \frac{P}{2} - \frac{Ph'x}{bd} = \frac{24{,}000 \times 24}{2 \times 8} + 12{,}000 - \frac{24{,}000 \times 24 \times x}{8 \times 18}$$
$$= 48{,}000 - 4{,}000\,x$$

At the left end, $x = 1.5$, and the stress is

$$48{,}000 - 4{,}000 \times 1.5 = 42{,}000 \text{ lb., compression}$$

At the right end, $x = 16.5$, and the stress is

$$48{,}000 - 4{,}000 \times 16.5 = -18{,}000 \text{ lb, tension}$$

The stress in $B B'$ is

$$\frac{Ph'}{2d} - \frac{Ph'x}{bd} = \frac{24{,}000 \times 24}{2 \times 8} - \frac{24{,}000 \times 24 \times x}{8 \times 18} = 36{,}000 - 4{,}000\,x$$

At the left end, $x = 1.5$, and the stress is

$$36{,}000 - 4{,}000 \times 1.5 = 30{,}000 \text{ lb, tension}$$

At the right end, $x = 16.5$, and the stress is

$$36{,}000 - 4{,}000 \times 16.5 = -30{,}000 \text{ lb, compression}$$

It will be seen that in this example all the stresses are less than those found in the example in Art **17**, except the shear in the end posts, which is the same

**19. Wind Effects on the Main Truss.**—The $Y$ components of the stresses in the end posts act directly along the members, and must be combined with the dead- and live-load stresses, in order to get the maximum stresses. On the leeward side, the compression due to the wind must be added to the dead- and live-load compression to get the maximum, on the windward side, the tension due to the wind must be subtracted from the dead- and live-load compression, or probably from the dead-load compression, to get the minimum stress. The latter is of interest only when the tension due to wind is greater than the dead-load compression, in which case the truss must be well bolted down to resist overturning The direct stress in an inclined end post due to the wind causes a stress in the lower chord, as shown in Fig. 12 If $Y$ is the direct wind stress in the end post, then $Y \cos H$ is the wind stress in the lower chord, due to that in the end post, and it is constant from end to end, being tension on the leeward side, and compression on the windward.

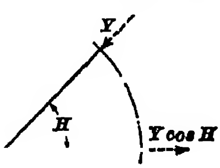


FIG 12

The total wind effects on the chords of the main trusses are as follows:

*Top Chords.*—Stresses due to their positions as chord members of the upper lateral truss, and stresses caused by the increase or decrease in the vertical load due to the overturning effect.

*Bottom Chords*.—Stresses due to their positions as chord members of the lower lateral truss, stresses caused by the increase or decrease in the vertical loads due to the overturning effect, and stresses due to the direct wind stresses in the inclined end posts.

**20. Sway Frames.**—In a deck bridge, the wind pressure that comes to the upper lateral system is transmitted to the abutments or supports by bracing in the planes of the end posts, similar to that shown in Fig. 13. The two transverse members $AA'$ and $BB'$, together with the diagonals $AB'$ and $A'B$, are usually called a **sway frame.** The diagonals are assumed to resist tension only; so, if the wind comes from the left, the member $A'B$ will be in action.

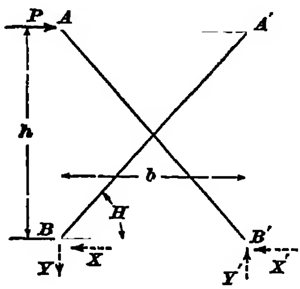

FIG 13

The reactions at $B$ and $B'$, due to the load $P$ at $A$, may be found from the equations of equilibrium. Taking moments about $B$ and $B'$,

$$Y = P\frac{h}{b}, \text{ and } Y' = P\frac{h}{b} = Y$$

In regard to the $X$ component of the reactions, it is evident that there is some at $B$ and some at $B'$; but, as the point of overturning is at $B'$, it is safe to assume $X' = P$ and $X = 0$. Then, the stresses in the members are as follows:

| MEMBER | STRESS |
|---|---|
| $AA'$ | $P$, compression |
| $A'B$ | $P \sec H$, tension |
| $AB'$ | 0 |
| $A'B'$ | $P\frac{h}{b}$, compression |
| $AB$ | 0 |
| $BB'$ | $P$, compression |

**21.** Deck and through bridges are given additional lateral stiffness by means of vertical transverse or sway frames at every panel point or at every other panel point. In the through bridge, these are of the same general form as the

portals already discussed; they are made as deep as conditions will allow In the deck bridge, they are made the same depth as the trusses. As the upper and lower lateral trusses, together with the bracing in the planes of the end posts, form a complete system for transmitting lateral forces to the supports, it is impossible to tell how much load comes on these intermediate frames. The stresses in them are usually found in the same way as in the end sway frames and portals already described, on the assumption that they are supported at the bottom chord, the load at the top joint being taken equal to one-half a top panel load. This simply affords a basis for design, and is not intended to relieve the wind pressure on the upper lateral truss.

EXAMPLE —In Fig 13, let $h = 25$ feet and $b = 18$ feet. If the wind pressure $P$ is equal to 20,000 pounds coming from the left, what are the stresses in $A A'$, $A' B$, and $B B'$?

SOLUTION.—The stress in $A A'$ is $P = 20{,}000$ lb , compression; the stress in $A' B$ is

$$P \sec H = 20{,}000 \times \frac{\sqrt{25^2 + 18^2}}{18} = 34{,}200 \text{ lb , tension;}$$

and the stress in $B B'$ is $P = 20{,}000$ lb , compression. Also, the compression in $A' B'$ is

$$P\frac{h}{b} = 20{,}000 \times \frac{25}{18} = 27{,}800 \text{ lb. Ans.}$$

### EXAMPLES FOR PRACTICE

1 In the portal represented in Fig. 4 (*a*), if $h = 32$ feet, $b = 16$ feet, and $P = 20{,}000$ pounds coming from the left, what is the direct stress in $B C$, assuming the end posts to be hinged at the bottom?

Ans. 40,000 lb., tension

2. If the dimensions and wind pressure are the same as in the preceding example, and $d = 4$ feet, what is the stress in $B B'$?

Ans. 80,000 lb , compression

3. If the dimensions and wind pressure on the portal shown in Fig 4 (*b*) are the same as described in examples 1 and 2, what is the stress in the first panel at the left end of $A A'$?

Ans. 76,700 lb , compression

4. In the portal shown in Fig 4 (*d*), if $h = 24$ feet, $b = 18$ feet, $d = 8$ feet, $b' = 6$ feet, and $P = 12{,}000$ pounds, coming from the right, what is the stress in $B D$? Ans. 30,000 lb., compression

## CENTRIFUGAL FORCE

**22. Value of Centrifugal Force.**—When the track that crosses a bridge is curved, the cars tend to move along the tangent, thereby exerting a lateral thrust, the amount of which can be found from the formula for centrifugal force given in *Kinematics and Kinetics,* namely,

$$F = \frac{W v^2}{g r}$$

In the present case, $W$ is the weight of the train, in pounds per linear foot; $v$, the velocity of the train, in feet per second; $g$, the acceleration due to gravity; $r$, the radius of the curve, in feet, and $F$, the value of the centrifugal force, in pounds per linear foot. In practice, it is customary to express the speed of the train in miles per hour, and the degree of curve in degrees. If $V$ represents the speed in miles per hour, then

$$v = \frac{5{,}280\, V}{60 \times 60} \text{ feet per second}$$

If $D$ represents the degree of curve, the radius $r$ is approximately equal to $\frac{5{,}730}{D}$ feet. Substituting these values in the formula for $F$, we get

$$F = \frac{W \times \left(\frac{5{,}280\, V}{60 \times 60}\right)^2}{32.16 \times \frac{5{,}730}{D}} = .00001167\, V^2 D W$$

The value of the centrifugal force is sometimes given as a percentage of the live load. For instance, for a speed of 50 miles per hour on a curve of 2°, the centrifugal force is

$$.00001167 \times 50^2 \times 2 \times W = .0584\, W,$$

that is, 5.84 per cent. of the live load. If, in this case, $W$ is 4,000 pounds per linear foot, then $F$ is $4{,}000 \times .0584 = 234$ pounds.

**23. Distribution of Load.**—The track on a curve is subject to the action of the weight and centrifugal force of the train. In practice, it is customary to make the outer rail on a curve higher than the inner rail, by an amount sufficient

to cause the resultant of the weight and centrifugal force to be perpendicular to the plane of the track half way between the rails. Under this condition, the loads that come on the rails are equal; but, as the track is curved, and, therefore, not the same distance from the trusses at every point, the loads at the panel points are not equal. Fig. 14 represents the cross-section of a through bridge having the width $b$, and shows the center line $c\,c'$ of the bridge; the surface $T\,T'$ of the track; the plane $a\,a'$ of the lower lateral truss; the line of action, perpendicular to $T\,T'$, of the resultant $Q$ of the weight and the centrifugal force; and the intersections $p$ and $q$, respectively, of the line of action of $Q$ with $T\,T'$ and $a\,a'$. The inclination of $T\,T'$ is much exaggerated to make the explanation clearer.

If $Q$ is resolved at $q$ into its vertical and horizontal components $W$ and $F$, respectively, the force $F$, being horizontal, will be resisted directly by the lower lateral truss, and $W$ will be resisted by the vertical trusses. If $e'$ is the distance of $q$ from the center line of the bridge, the amount of $W$ that goes to the outside truss is

$$\frac{W\left(\frac{b}{2}+e'\right)}{b}, \text{ or } \frac{W}{2}+W\frac{e'}{b}$$

and the amount that goes to the inside truss is

$$\frac{W\left(\frac{b}{2}-e'\right)}{b}, \text{ or } \frac{W}{2}-W\frac{e'}{b}$$

**24. Eccentricity of Track.**—The distance $e$, Fig. 14, from the center line of the track to the center line of the bridge is called the **eccentricity** of the track. The eccentricity varies along the bridge, as shown in Fig. 15, in which $L\,M$ is the center line of the bridge; $A\,B\,C$, the center line of the curved track; and $A\,B'\,C$, the chord to the curve between the ends of the bridge, $B\,B'$ being the middle ordinate. The values of the eccentricity for the different panel points can be computed from the curve of the track; they may be called $e_1, e_2, e_3$, etc. The distance $e'$, Fig. 14, is the sum of $e$ and $e''$,

the latter being the horizontal distance from $p$, the intersection of $Q$ with the surface of the track, to $q$, its intersection with the plane of the lower lateral truss.

In Fig. 14, let $gg'$ represent the gauge of the track; $gf$, the superelevation of the outer rail, and $po$, the vertical distance from the point $p$ to the plane of the lower lateral truss, which will be assumed horizontal. Then, as the two triangles $gg'f$ and $qpo$ are similar, we have the following proportion:

$$\frac{qo}{op} = \frac{gf}{fg'};$$

whence

$$qo = op \times \frac{gf}{fg'} = e''$$

For all practical purposes, $fg'$ may be taken equal to $gg'$; letting $gf = E$, $gg' = G$, and $po = h_1$, and substituting these values in the foregoing equation, we obtain

$$e'' = h_1 \times \frac{E}{G}$$

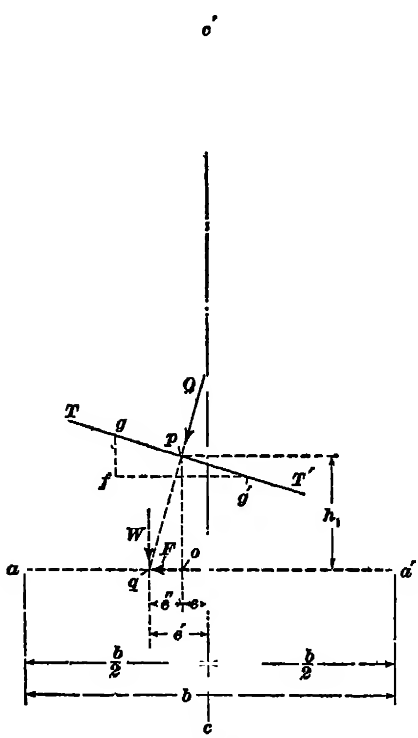


FIG 14

The value of $e''$ is the same at every point of the track.

In Fig 15, the dotted curve $q'q''$ shows the points of intersection of the resultant for the whole train with the plane of the lower lateral truss. At the floorbeam $dd'$, it lies inside the center line of the bridge; then the amount of vertical load that goes to the outside truss at this panel point is

$$\frac{W}{2} - W\frac{e'}{b}$$

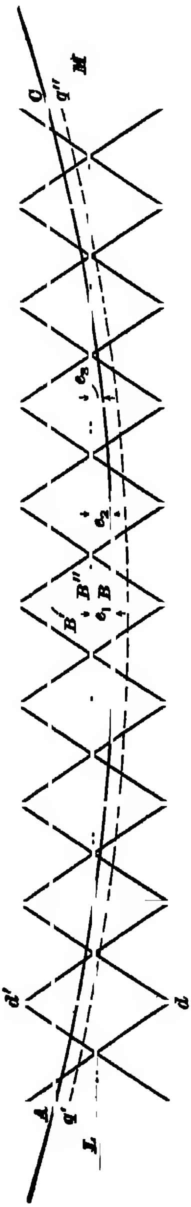

Fig 15

and the amount that goes to the inside truss is

$$\frac{W}{2} + W\frac{e'}{b}$$

For spans shorter than about 75 feet, it is customary to make the distance $B\,B''$, Fig. 15, from the center line of the track to the center line of the bridge, from one-half to one-third the middle ordinate $B\,B'$, and to assume that each girder carries one-half the vertical load, the same as for a bridge on a straight track. For longer spans, it is customary to find first the panel loads as though the track were straight, and then increase or decrease them according to the location of the center line of the track at each floorbeam.

**25. Lateral Bracing.**—In a through bridge, the centrifugal force is transmitted to the lower lateral truss by the floor, and the stresses in the members are found in the same way as those due to wind. In a deck bridge, the centrifugal force is transmitted by the floor to the upper lateral truss, and by it to the sway frames; the stresses in the members are found in the same way as those due to wind. Centrifugal force causes stresses in but one set of diagonals, as it acts in but one direction.

EXAMPLE —A train of cars weighing 5,000 pounds per linear foot, moving over a curve of 3° at the rate of 40 miles per hour, crosses an eight-panel through bridge having a span of 160 feet and a width of 20 feet If the outer rail is elevated 265 ft , and the center line of the bridge is located two-thirds of the middle ordinate from the chord to the curve ($B\,B'' = \frac{1}{3}\,B\,B'$, Fig. 15), what is. (*a*) the panel load

for the lower lateral truss caused by the centrifugal force, (*b*) the correction to be applied at each panel point of the outside truss due to the eccentricity? Assume that the distance from the surface of the track to the lower lateral truss is 4 feet, and that the gauge of the track is 4 71 feet.

SOLUTION —(*a*) $F = 00001167 \times 40 \times 40 \times 3 \times 5{,}000 = 056 \times 5{,}000 = 280$ lb per lin. ft

The panel load for the lateral truss is $280 \times 20 = 5{,}600$ lb Ans

(*b*) The perpendicular distance from the center of the curve ($R = 1{,}910\ 08$) to the chord (length = 160 ft.) is equal to 1,908 40 ft. Then, the middle ordinate is $1{,}910\ 08 - 1{,}908\ 40 = 1\ 68$ ft The ordinates $e_1$, $e_2$, and $e_3$ at the other panel points, from the chord to the curve are as follows

Point *b*, $e_1 = \sqrt{1{,}910\ 08^2 - 60^2} - 1{,}908\ 40 = \ 74$ ft.

Point *c*, $e_2 = \sqrt{1{,}910\ 08^2 - \overline{40}^2} - 1{,}908\ 40 = 1\ 26$ ft

Point *d*, $e_3 = \sqrt{1{,}910\ 08^2 - \overline{20}^2} - 1{,}908\ 40 = 1\ 58$ ft.

The center line of the bridge is two-thirds the middle ordinate, or 1 12 feet, from the chord, then, the distances $e$ from the center line of the bridge to the curve are as follows.

Point *b*, $e = \ 74 - 1.12 = -.38$ ft.
Point *c*, $e = 1\ 26 - 1\ 12 = \ .14$ ft.
Point *d*, $e = 1\ 58 - 1\ 12 = \ 46$ ft.
Point *e*, $e = 1\ 68 - 1\ 12 = \ 56$ ft

The distance $e''$ from the center line of the track to the line of intersection of the resultant with the plane of the lateral system is equal to

$$\frac{4}{4\ 71} \times .265 = .22 \text{ ft}$$

and to find the distances $e'$, it is simply necessary to add this distance to the values of $e$ just found, they are as follows

Point *b*, $e' = -\ 38 + \ 22 = -\ 16$ ft.
Point *c*, $e' = +\ 14 + \ 22 = +\ 36$ ft
Point *d*, $e' = +\ 46 + \ 22 = +\ 68$ ft.
Point *e*, $e' = +\ 56 + \ 22 = +.78$ ft

Then, the corrections at the panel points are as follows:

Point *b*, $W = (5{,}000 \times 20) \times \frac{-\ 16}{20} = -\ 800$ lb.

Point *c*, $W = (5{,}000 \times 20) \times \frac{36}{20} = +1{,}800$ lb.

Point *d*, $W = (5{,}000 \times 20) \times \frac{68}{20} = +3{,}400$ lb.

Point *e*, $W = (5{,}000 \times 20) \times \frac{.78}{20} = +3{,}900$ lb.

**EXAMPLES FOR PRACTICE**

1. A railroad train weighing 5,000 pounds per linear foot is moving at the rate of 50 miles an hour on a 2° curve, and crosses a ten-panel through bridge having a span length of 200 feet What is the panel load for the lower lateral truss due to the centrifugal force? Ans 5,835 lb

2 A railroad train weighing 4,500 pounds per linear foot is moving at the rate of 60 miles an hour on a 3° curve, and crosses a twelve-panel bridge having a span length of 180 feet. What is the panel load for the lower lateral truss due to the centrifugal force? Ans. 8,507 lb.

## SKEW BRIDGES

**26. Description.**—In all that precedes, it has been assumed that the trusses were symmetrical about the center, and that the line connecting the end joints over the supports was at right angles to the center line of the bridge. It frequently happens, however, that the center line is not at right angles to the abutments or piers, in which case the line connecting the ends of the trusses is not at right angles to the center line. Such bridges are termed **skew bridges**, and give rise to conditions that require separate consideration.

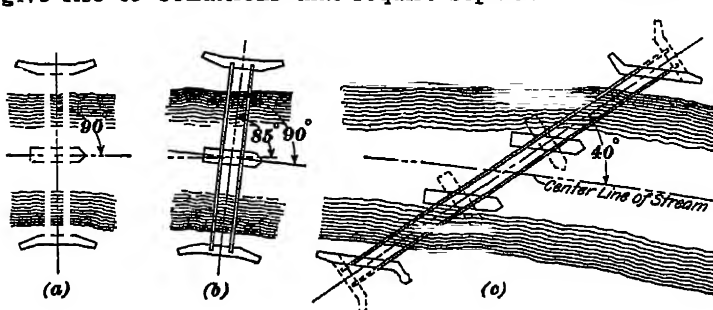


FIG 16

**27. Skew Crossings.**—In locating a new line, either for a highway or for a railroad, it is sought, as far as possible, so to arrange the alinement that all bridge crossings shall be at right angles to the obstacle, usually either a road or a stream, to be crossed, as shown in Fig. 16 (*a*);

such a crossing is economical because it is shorter than any other. Local conditions and the topography of the country occasionally render this impossible, and the line crosses at an angle other than 90°. If the angle is only slightly less, as shown in Fig. 16 (*b*), the piers and abutments may be placed at right angles to the center line of the bridge, if much less, this plan may seriously interfere with the waterway or other clearance required, and increase the length of the bridge, as shown by the dotted lines in Fig. 16 (*c*). In such a case, the piers and abutments may be placed parallel to the stream or road, thereby making the same angle as the latter with the center line of the bridge. In eliminating skew grade crossings of two existing roads, it is frequently impossible to change the alinement of either and inadvisable to increase the length of the bridge by placing the abutments at right angles to the center line, in which case the latter may be placed parallel to the lower road, and the bridge built on a skew.

28. **Arrangement of Panels, Floor, and Lateral System.**—Near the ends of a skew bridge, the panel lengths are, in some cases, not equal to those near the center. This is due to the fact that the floorbeams are placed at right angles to the center line, and are spaced the same distance apart near the center of the bridge, bringing the panel points of the two trusses directly opposite; the amount of **skew** (that is, the distance, measured along the center line, that one truss is ahead of the other) is made up in the end panels. The arrangement of the panels and of the floor and lateral system depends to a great extent on the distance center to center of the trusses and on the angle of skew, each case requiring separate treatment. The general method will be illustrated by considering a special case.

In Fig. 17, (*a*) is the top view; (*b*), the side elevation of one truss; and (*c*), the lower lateral bracing of a skew bridge; the span of the bridge is equal to $l$, the perpendicular distance from center to center of the trusses is $b$, the angle between the center line of the bridge and the

abutments is $B$, and the height of trusses is $h$. In this particular case, the angle $B$ of skew is about 45°, and the amount of skew $s$, or $b \cot B$, is very nearly equal to the panel length. Therefore, the hip vertical at one end of each truss may be located directly opposite the end joint of the other, making the end panels $f'e'$ and $fe$, Fig. 17 ($c$), equal to $s$. In order that the end posts may have the same slope, it will be well to make the end panels $a'g'$ and $ag$ at the other ends of the trusses equal

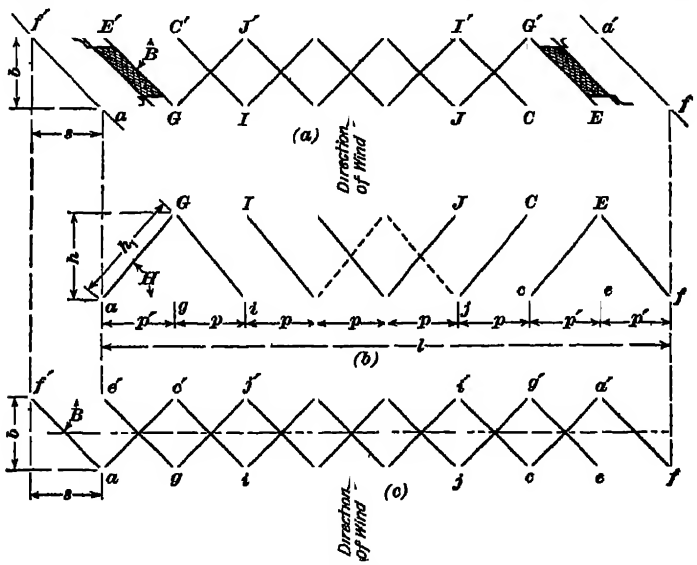


FIG 17

also to $s$; then, as the floorbeams are at right angles to the trusses, $c$ and $c'$ are, respectively, opposite $g'$ and $g$, and the panels $e'c'$ and $ec$ are also equal to $s$. For the sake of uniformity, each of these panel lengths, although equal to $s$, will be called $p'$. For each truss, then, there are three equal panels $p'$; the remaining length, $l - 3p'$, included in the area $c'gcg'$, Fig. 17 ($c$), may be divided into a number of equal panels—in this case, five.

**29. Panel Loads and Reactions.**—As the panel lengths are different, the panel loads will be different; those

from $g$ to $c$, Fig. 17 ($b$), are found by multiplying the load per linear foot for one truss by one-half the sum of the lengths of the adjacent panels. The panel load at $e$ is greatest when the truss is loaded from $c$ to $f$, and is then equal to one-quarter the load in the area $c\,g'\,a'\,e$, plus the amount that comes from the triangular area $a'\,e\,f$, Fig 17 ($c$). If the load is $w$ pounds per square foot, uniformly distributed, the total load in the area $c\,g'\,a'\,e$ is $w\,b\,p'$, and the amount of this that goes to $e$ is $\frac{w\,b\,p'}{4}$. As the area of the triangle $a'\,e\,f$ is $\frac{b\,p'}{2}$, the total load in this area is $\frac{w\,b\,p'}{2}$. To find what portion of this goes to $e$, it is well to consider it concentrated at the center of gravity of the triangle $a'\,e\,f$, at a distance from $a'\,f$ equal to one-third the distance from $e$ to $a'\,f$. Then, taking moments about the line $a'\,f$, we find that one-third of $\frac{w\,b\,p'}{2}$, or $\frac{w\,b\,p'}{6}$, goes to $e$. The panel load at $e$ is, then,

$$\frac{w\,b\,p'}{4} + \frac{w\,b\,p'}{6} = \frac{5\,w\,b\,p'}{12}$$

In a railroad bridge, the live load is usually given as a uniform amount $w''$ per linear foot of track, and it is sufficiently accurate to assume the load in the end panel equal to the load per linear foot multiplied by the distance, measured along the center line of the bridge, from the end floorbeam $a'\,e$, Fig. 17 ($c$), to the abutment. As this distance is $\frac{p'}{2}$, the load is $\frac{w''p'}{2}$; one-half of this, or $\frac{w''p'}{4}$, goes to the end floorbeam, and one-half of this portion, or $\frac{w''p'}{8}$, to $e$. As the panel lengths are unequal, they cannot be used as a unit in calculating reactions and bending moments, as heretofore.

**30. Lower Lateral Truss.**—The floorbeams $a\,e'$ and $a'\,e$, Fig. 17 ($c$), rest on the masonry, or are connected to the trusses at $a$ and $a'$, respectively, and it may be assumed that the lower lateral truss is composed of seven

panels with a span equal to $l - p'$. When the wind is blowing in the direction shown, the diagonal $a c'$ transmits the pressure directly to the support at $a$, and $g' e$ to the end of the floorbeam $e a'$, which transmits the load that comes from $g' e$, and the wind panel load at $e$, to the support. As the panel lengths are unequal, the wind and centrifugal panel loads will be unequal.

**31. Upper Lateral Truss.**—In the upper lateral system, the transverse struts $C' G$ and $C G'$, Fig. 17 ($a$), connect the hip joint of one truss with the opposite joint of the other, and may be looked on as the limit of the upper lateral truss, which will then have five panels with a span equal to $l - 3p'$. When the wind is blowing in the direction shown, the diagonal $G J'$ transmits the pressure to the portal at $G$, and $I' C$ to the strut $C G'$ at $C$, which transmits it, together with the wind panel load at $C$, to the portal at $G'$.

**32. Skew Portal.**—The portal of a skew bridge is called a **skew portal**. In Fig. 17 ($a$), the upper line $G E'$ or $G' E$ of the portal lies in the plane of the upper lateral truss, and connects the upper joints of the end posts, it is horizontal, and makes the same angle $B$ as the abutments with the center line of bridge. Each portal lies in a plane that passes through both end posts, the lower line being parallel to the upper line of the portal; the plane at the left end intersects the plane of the lower lateral bracing in the line $a f'$, which is horizontal. The portal is shown revolved about the line $a f'$ into the plane of the paper and in an upright position in Fig. 18. The general methods that have been described for finding the stresses in portals may be applied in this case; as the calculation is somewhat complicated on account of the several angles involved, the formulas for the skew portal will now be found, considering the left portal and assuming the wind blowing in the direction shown. The wind pressure that is to be resisted by the portal is the amount transmitted to joint $G$, Fig. 17 ($a$), by the diagonal $G J'$, plus the wind panel loads at $G$, $C'$, and $E'$. If the force acting at $G$ at right angles to the upper chord

(parallel to the direction of the wind), due to the tension in $GJ'$ and the wind panel loads at $G$ and $C'$, is denoted by $P$, the force $P_1$ in the direction $GE'$ is equal to $P \csc B$; if the wind panel load at $E'$ is $P'$, then the force $P_1'$ in the direction $GE'$ is equal to $P' \csc B$, as shown in Fig. 18.

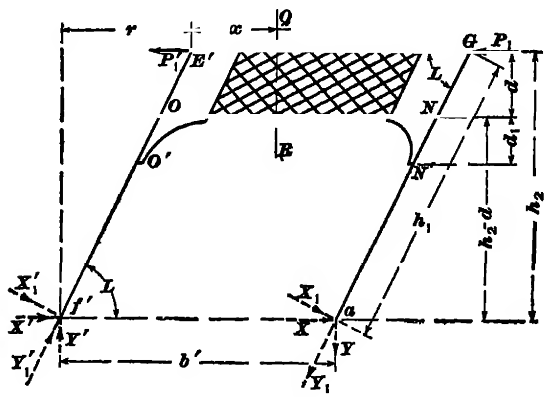


FIG 18

The reactions at $a$ and $f'$, resolved into components parallel and perpendicular, respectively, to the line of action of $P_1$ and $P_1'$, are $X$ and $X'$, $Y$ and $Y'$. Then,

$$P_1 + P_1' = X + X',$$

and, as $X$ and $X'$ may be assumed equal,

$$X = X' = \frac{P_1 + P_1'}{2}$$

Also, $$Y = Y' = (P_1 + P_1')\frac{h_2}{b'};$$

$$b' = b \csc B;\ h_2 = h_1 \sin L = h \csc H \sin L$$

The customary method of finding the stresses in the web members is to consider the portal cut by a plane, such as $QR$, Fig 18, at right angles to the flanges, and assume that the shear on the plane is evenly distributed among the web members cut by it. If there are $n$ members cut by the plane, the $Y$ component in each member is $\frac{Y}{n}$, or, in this case, $(P_1 + P_1')\frac{h_2}{b'n}$; and the stress in the member is $\frac{Y}{n}$

multiplied by the cosecant of the angle beween the web member and the flanges.

The stresses in the flanges may be found approximately in the same way as in Art. **13**; that is, by considering the portal cut by a plane at right angles to the flanges, treating the portion on one side of the plane as a free body, and applying the equation $\Sigma M = 0$ to this portion, neglecting the effect of the web. In Fig. 18, let $QR$ be at a distance $x$ from $E'$, and assume that the stress in $GE'$ is compression, and in $NO$ tension. Then, applying the equation $\Sigma M = 0$ to the portion of the portal on the left of $QR$, the stresses in $GE'$ and $NO$ are as follows:

in $GE'$,

$$\frac{Y'(r+x) - X'(h_2 - d) - P_1' d}{d}, \text{ compression;}$$

in $NO$,

$$\frac{Y'(r+x) - X' h_2}{d}, \text{ tension}$$

The value of $r$ is found by the equation

$$r = h_1 \cos L$$

In case the numerical values of these stresses come out negative, they are opposite in character to the stresses just given; that is, the stress in $GE'$ will be tension, and in $NO$ compression.

In order to find the stresses in the end posts below the portal, it is necessary to compute the components $X_1$ and $X_1'$, $Y_1$ and $Y_1'$ of $X$ and $X'$, $Y$ and $Y'$, in the direction of, and at right angles to, the end posts. Letting $L$ represent the angle $E' f' a$, the components are as follows:

at $f'$,

$$X_1' = X' \sin L - Y' \cos L; \quad Y_1' = Y' \sin L + X' \cos L;$$

and at $a$,

$$X_1 = X \sin L + Y \cos L; \quad Y_1 = Y \sin L - X \cos L$$

Then, the stresses in the end posts are as follows:

The stress in $f' O'$ is $Y_1'$, compression.

The shear in $f' O'$ is $X_1'$.

The moment at $O'$ is $X_1' [(h_2 - d - d_1) \csc L]$

The stress in $a N'$ is $Y_1$, tension.

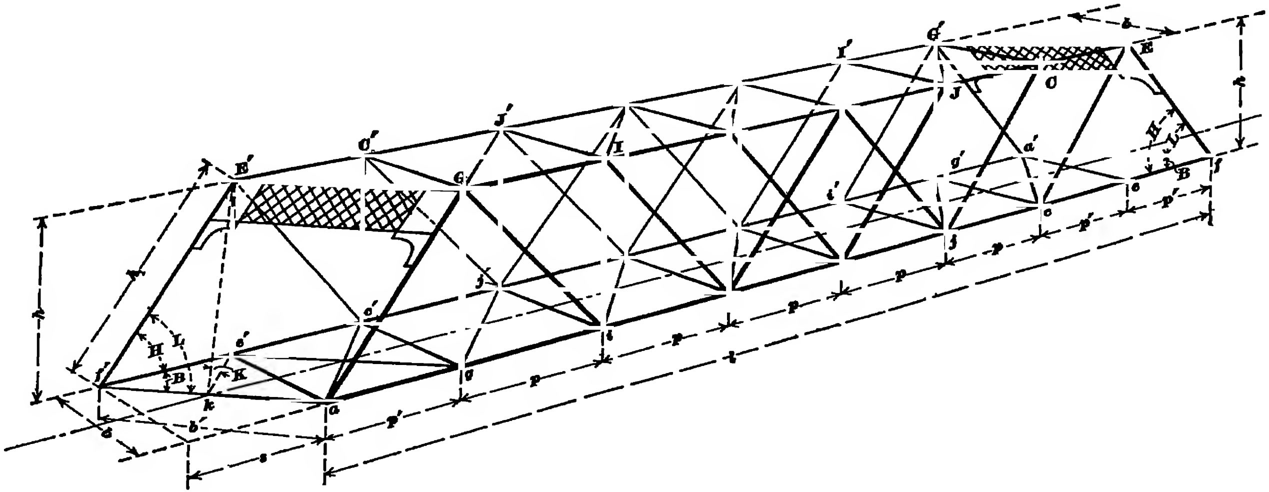

Fig 19

The shear in $a N'$ is $X_1$.

The moment at $N'$ is $X_1 [(h_2 - d - d_1) \csc L]$.

To find the value of the angle $L$ in terms of quantities that are known, it is well to consider Fig. 19, which is a perspective drawing of the bridge represented in Fig. 17. In Fig 19, $f' E' G a$ and $f E G' a'$ are the planes of the portals. At the left end, the line $e' k$ is horizontal and perpendicular to $f' a$; then, the triangle $e' k f'$ is a right triangle. As $E' e'$ is vertical, the triangle $E' e' k$ is also a right triangle. If the line $e' k$ is revolved about the line $a f'$ and kept always perpendicular to it, it will generate a plane at right angles to $a f'$ that intersects the plane of the portal in $k E'$; the triangle $f' k E'$ is, therefore, a right triangle. Then,

$$\cos L = \frac{f' k}{E' f'};\ f' k = e' k \cot B = b \cos B \cot B;$$

$$E' f' = h_1 = h \csc H;$$

whence $$\cos L = \frac{b}{h} \cos B \cot B \sin H$$

It is sometimes desired to find the angle $E' k e'$ between the plane of the portal and a horizontal plane. Let $K$ represent the angle $E' k e'$; then,

$$\tan K = \frac{E' e}{e' k};\ E' e' = h,\ e' k = b \cos B;$$

whence $$\tan K = \frac{h}{b} \sec B$$

When $K$ is known, $L$ may also be found as follows:

$$\sin L = \frac{E' k}{E' f'},\ E' k = h \csc K;\ E' f' = h_1 = h \csc H;$$

whence $$\sin L = \sin H \csc K$$

If the end posts are fixed at their lower joints, the stresses in the end post and portal may be found by assuming that the end posts are supported and hinged at points one-third to one-half the distance from their lower joints to the lowest point of the portal or brackets.

## VIADUCTS

**33. Definition.**—A **viaduct** is a bridge, commonly of stone or steel, made up of several spans that rest on intermediate supports, and is used to carry elevated railways through city streets, or to carry a highway or railroad across a valley or stream, generally at a considerable distance above it The stresses in each span may be analyzed according to the principles already discussed, and the supports are all that remain to be considered.

**34. Elevated Railways.**—In elevated railways, the main girders or trusses that carry the track are usually connected to cross-girders that rest on columns, the latter are supported on masonry piers at or about the level of the ground, and are firmly bolted to them, and so may be considered fixed in direction.

A typical elevation and section of such a structure built for two tracks is shown in Fig. 20. The cross-girders transmit the loads at $B$ and $C$ to the columns at $A$ and $D$, and receive their maximum stresses when both tracks are loaded on the two adjacent spans. The columns transmit the vertical loads from the cross-girders and the outside-track girders to the piers, and also receive their maximum loads when the adjacent spans are fully loaded.

**35. Column Bracing.**—As the clearance between the columns below the cross-girders is usually required for street traffic, it is impossible to insert any diagonal sway bracing to resist the wind pressure and centrifugal force Whenever possible, knee braces or curved brackets are inserted under the cross-girders. Then the two columns and cross-girder form a structure similar to the end posts and portal of a through bridge, and the direct, bending, and shearing stresses in the columns due to centrifugal force and to the

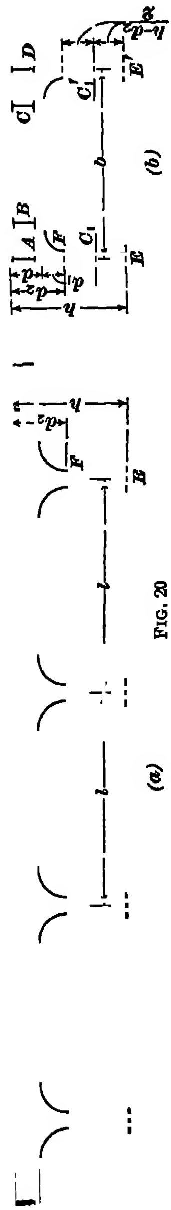

FIG. 20

pressure of the wind on the cars and girders may be found in exactly the same way as for a portal For this purpose, the formulas given in Fig. 10 may be used. In computing the wind pressure, it will be sufficient to consider the exposed area of one girder and one train of cars; the others will be sheltered. If the lower end of the column is firmly bolted to the masonry, the points of inflection $C_1$ and $C_1'$, Fig. 20 (*b*), may be assumed to be midway between the top of the pier and the bottom of curved bracket. Then, if $P$ represents the total wind pressure over one span on cars and girders, the horizontal reactions at $C_1'$ and $C_1$ are each equal to $\frac{P}{2}$, and the moment at $F$ and $F'$ is equal to

$$\frac{P}{2} \times \frac{h - d_2}{2}$$

There is also to be resisted the longitudinal thrust exerted by a train of cars coming to rest when the brakes are set. This thrust is a maximum when the brakes are set so hard that the wheels stop turning and slide on the rails; it is usually taken equal to one-fifth the weight of the cars. It is transmitted to the piers by the columns, which thereby receive bending stresses In the double-track structure shown in Fig. 20, the thrust for each track is transmitted by one row of columns; and in a single-track structure by both rows If the longitudinal thrust per linear foot for one track is $t$, and the length of span $l$, the amount $T$ that is transmitted by one column is $t\,l$, and, assuming that the columns are well bolted at the bottom, the bending moment at $F$ is

$$T \times \frac{h - d_2}{2}$$

## STEEL TRESTLES

**36. General Description.**—Fig. 21 shows the elevation (*a*) and section (*b*) of a typical steel trestle for carrying a railroad or highway across a deep valley. The track or floor is carried on girders or trusses forming alternately long and short spans, *l* and *l′*, respectively. The adjacent ends of each two consecutive spans are supported by a pair of columns, the tops of which are about under the main girders or trusses. These columns are battered, or sloped outwards, toward the ground. The two columns supporting the corresponding ends of the two trusses or girders in each span are rigidly braced by horizontal struts and diagonal members in vertical planes at right angles to the track. The combination formed by the two columns and the bracing is called a **trestle bent.** The trestle bents that come under the two ends of each short span *l′* are connected to each other by horizontal struts and diagonal bracing in inclined planes parallel to the direction of the track. The two bents, together with the bracing, form a **tower.** The short spans, which extend from one side of the tower to the other, are called the **tower spans,** and are usually about 30 feet in length; the long spans, which extend from one tower to the next, are called the **main spans,** and are usually not less than about 50 feet in

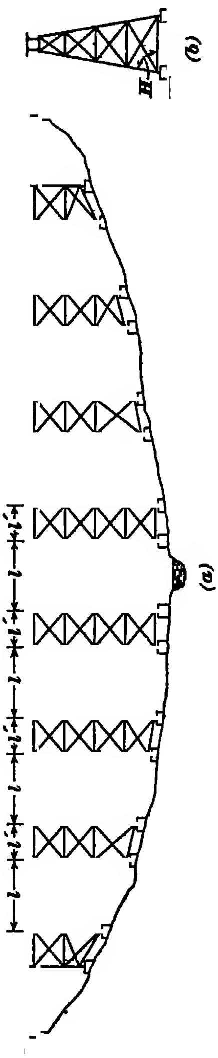

Fig. 21

length, the maximum depending to a great extent on the height of the columns.

The maximum vertical load on a bent occurs when the adjacent spans are fully loaded. If the length of the tower span is $l'$, that of the main span $l$, and the load per linear foot $w$, the vertical load $W$ on a trestle bent is $w\left(\frac{l}{2}+\frac{l'}{2}\right)$, one-half of which is carried by each column. If the angle between the center line of the column and a horizontal line at right angles to the center line of the bridge is $H$, as represented in Fig. 22 ($b$), the compression in the column due to the vertical load is $\frac{W}{2}\csc H$, and the compression in the upper transverse strut $AA'$ is $\frac{W}{2}\cot H$.

**37. Transverse, or Sway, Bracing.**—The diagonal bracing is usually arranged so that the slope is about 45°; in a high tower, the bracing divides the height into a number of **stories.** The stresses in the bracing are usually found by the graphic method. For purposes of illustration, both the graphic and the analytic method will be explained here. Fig. 22 ($a$) represents a trestle bent with dimensions as shown, the wind pressure on the train for a length equal to $\frac{l}{2}+\frac{l'}{2}$ is $P_1$, acting at a distance of $h_1'$ above the top of the bent; and the wind pressure on the rails, ties, and girders, for the same length is $P_2$, acting at a distance of $h_2'$ above the top of the bent. The wind pressures transmitted to the joints $A$, $B$, $C$, etc. by the members of the bent are $P_3$, $P_4$, $P_5$, etc As the wind is blowing from the left, the diagonals $A'B$, $B'C$, $C'D$, etc. will be in action.

To find the stress in a diagonal, such as $A'B$, the bent may be considered cut by a plane, such as $rs$, Fig. 22 ($a$), that intersects the diagonal and the two columns, and the portion above the plane may be considered as a free body. Then, applying the equation $\Sigma M = 0$ to the external forces above the section $rs$, and taking the center of

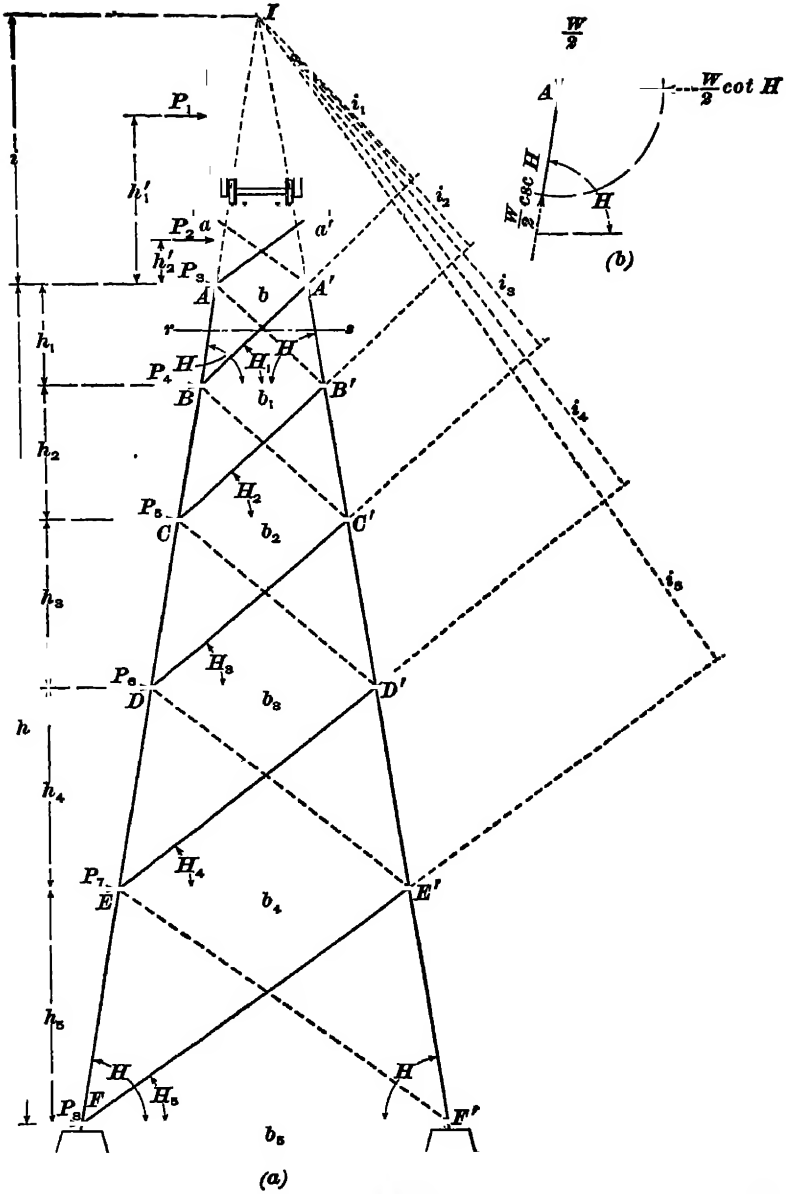
I
P1
h'1
P2 a
a'
h'2
P3
A
b
A'
r
s
h1
H
H1
H
P4
B
b1
B'
h2
H2
P5
C
b2
C'
h3
H3
P6
D
b3
D'
h
h4
H4
P7
E
b4
E'
h5
H
H5
H
P8
F
b5
F'
(a)
i1
i2
i3
i4
i5
W/2
A'
W/2 cot H
W/2 csc H
H
(b)

Fig. 22

moments at the intersection $I$ of the two columns. the stress in $A'B$ is

$$\frac{P_1(i-h_1')+P_2(i-h_2')+P_3 i}{i_1}, \text{ tension}$$

In like manner, the compression in $BB'$ is

$$\frac{P_1(i-h_1')+P_2(i-h_2')+P_3 i+P_4(i+h_1)}{i+h_1},$$

The stresses in other diagonals and transverse struts may be found in a similar way. For the stress in $FF'$, it will be well to consider that the whole horizontal load is carried to $F'$, in which case the stress in $FF'$ will be equal to the moment of all the forces above the point $F$ about $I$, including the load at $F$, divided by the lever arm $i+h$ of $FF'$.

The distances $b$, $b_1$, etc., $h$, $h_1$, $h_2$, etc., and the angle $H$ are determined by the style and general dimensions of the viaduct; from them, the distances $i$, $i_1$, $i_2$, etc. may be found as follows:

$$i = \frac{b}{2}\tan H$$

$$i_1 = IB\sin(H-H_1) = \frac{b_1}{2}\sec H\sin(H-H_1)$$

$$i_2 = \frac{b_2}{2}\sec H\sin(H-H_2), \text{ etc.}$$

**38. Wind Stresses in Columns.**—To find the wind stresses in the columns, the portion of the bent above such a section as $rs$ may be considered. The stress in either column may be found by taking moments about the intersection of the diagonal with the other column. It will be convenient to resolve the stress into its $X$ and $Y$ components at the joint opposite the center of moments, and to multiply the $Y$ component by csc $H$. Thus, with the center of moments at $A'$, the stress in $AB$ is

$$\frac{P_1 h_1'+P_2 h_2'}{b}\times \csc H, \text{ tension}$$

With the center of moments at $B$, the stress in $A'B'$ is

$$\frac{P_1(h_1'+h_1)+P_2(h_2'+h_1)+P_3 h_1}{b_1}\times\csc H, \text{ compression}$$

With the center of moments at $F$, the stress in $E'F'$ is compression equal to

$$\{[P_1(h_1'+h)+P_2(h_2'+h)+P_3 h+P_4(h-h_1)$$
$$+P_5(h_3+h_4+h_5)+P_6(h_4+h_5)+P_7 h_5]-b_5\}\times \csc H$$

The stress in any other portion of the columns may be found in like manner. The maximum compression wil' occur at the lowest section on the leeward side, in this case $E'F'$, and will be equal to the compression due to the vertical load plus that due to wind pressure; this is also the maximum vertical load on a pier.

For the minimum compression in the columns, there are two cases to be considered

1. *When the adjacent spans are fully loaded* In this case, the compression due to the live and dead loads is decreased by the tension in the windward column due to a wind pressure of 30 pounds per square foot against the exposed surface of the cars and structure.

2. *When the adjacent spans are unloaded.* In this case, the compression due to dead load alone is decreased by the tension in the windward column, due to a wind pressure of 50 pounds per square foot against the exposed surface of the structure alone. The minimum compression occurs in the lowest section of the windward column, in this case $EF$, but this is not the minimum vertical load on the pier; the latter is further decreased by the vertical component of the tension in $E'F$, so that the minimum load on the windward pier may be negative; that is, there may be a tendency to uplifting at $F$, even though the minimum stress in $EF$ is compression.

**39. Anchorage.**—To find the reaction on the windward pier, without considering the stresses in the members, the point $F'$ may be taken as the center of moments for all the external forces acting on the bent. Then, denoting by $W$ the total load on a bent, and assuming that the reaction at $F$ acts vertically upwards, its value is

$$\frac{W\times\frac{b_5}{2} - \text{moment of all wind pressure about } F'}{b_5}$$

If this comes out positive for both cases outlined in the preceding example, no anchorage is required; if it comes out negative for either case, there must be a weight of masonry in the pier equal at least to the negative reaction, although for safety it is usually made not less than twice that amount.

**40. Graphic Method.**—The stresses may be found by means of the stress diagram much more quickly than by the analytic method Those due to the vertical load may be found by drawing the triangle of forces, Fig. 23 ($d$), for a load of $\frac{W}{2}$, and the stresses in $A A'$ and $A B$ at point $A$.

In finding the wind stresses, it will be well to consider first the pressures $P_1$ and $P_2$ that act above the top of the bent. The lateral thrust due to $P_1$ is transmitted to the upper flanges of the girders at $a$ and $a'$, Fig. 22, by means of the rails, ties, and upper lateral bracing; that due to $P_2$ is transmitted to $a$ and $A$ by means of the web of the girder; the lateral thrust at $a$ and $a'$ is transmitted to the supports at $A$ and $A'$ by means of the sway frame $a a' A' A$, the diagonal $a A'$ being out of action, and there being tension in $a' A$. In the analysis of the sway frame, in Art. 20, it was explained that it is customary to consider the structure on the point of overturning about the leeward support, in which case the whole lateral thrust is exerted at $A'$, causing a compression in the lower member of the sway frame equal to $P_1 + P_2$.

Now, if the same assumption is made in the present case, there will be very little stress in $A A'$, the upper transverse strut of the tower, and to furnish a basis for the design of this member it is assumed that all the lateral thrust due to $P_1$ and $P_2$ is transmitted to the bent at $A$;* then, the reaction at $A'$ will be vertical. The problem then arises to resolve $P_1$ and $P_2$ into two components, one of which

* In reality, it matters little whether all the lateral thrust is assumed to be transmitted to $A$ or to $A'$, as all the transverse struts of the bent, including the top, are usually made the same size

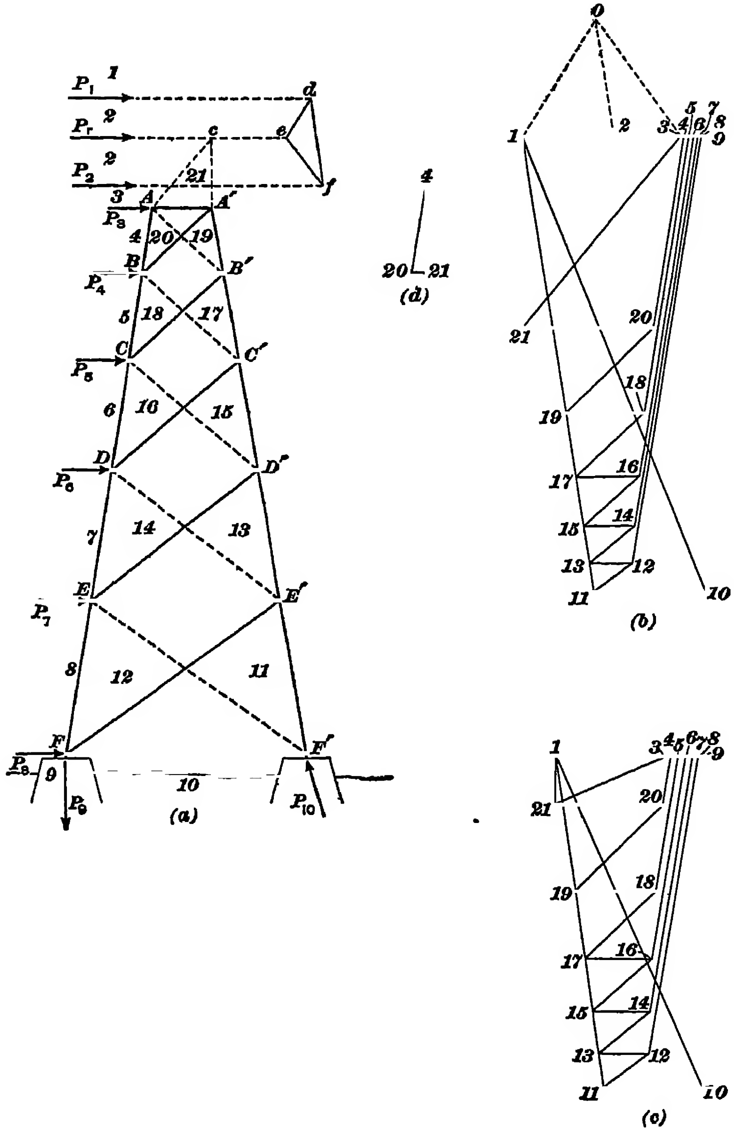


FIG 28

is vertical, passing through $A'$, and the other inclined, passing through $A$ For this purpose, $P_1$ and $P_2$ are platted to scale, as shown by *1–2* and *2–3*, respectively, in Fig. 23 (*b*), and the position of their resultant $P_r = P_1 + P_2$ is found by means of the force diagram *0–1–2–3*, Fig. 23 (*b*), and the funicular *d e f d*, Fig. 23 (*a*). The line of action of $P_r$ intersects at *c* the line of action of the vertical reaction at $A'$; therefore, $c\,A$ is the line of action of the reaction at $A$. The values of the reaction may be found by drawing the triangle of forces for the point *c*; they are *1–21* at $A'$, and *21–3* at $A$, Fig. 23 (*b*). Then, the external forces acting on the tower are: *1–21*, acting vertically downwards at $A'$; *21–3*, acting upwards in the direction $A\,c$ at $A$; $P_3, P_4, P_5$, etc., acting horizontally at $A$, $B$, $C$, etc.; the reaction at $F$, which may be assumed vertical, as explained in Art. **39**; and the reaction at $F'$, which will be inclined. The diagonals $A'\,B$, $B'\,C$, $C'\,D$, etc. will be in tension, the other diagonals will be out of action. The construction of the stress diagram presents no difficulty; it is best to start with the joint $A$, then take $A'$, and then $B$, $B'$, $C$, $C'$, $D$, $D'$, etc. in the order given. In Fig. 23, (*a*) is the elevation or space diagram of the trestle bent; (*b*), the stress diagram for wind stresses when the structure is fully loaded (30 pounds per square foot against structure and cars); (*c*), the stress diagram for wind stresses when the structure is unloaded (50 pounds per square foot against the structure); and (*d*), the force polygon for the vertical load at the joint $A$. The stresses scaled from (*b*) must be combined with those due to full dead and live loads, and those scaled from *c* must be combined with those due to dead load alone; the maximum must be used in each case.

**41. Centrifugal Force.**—The stresses in the members of the bents due to centrifugal force when the viaduct is on a curve may be found in practically the same way as the wind stresses.

**42. Longitudinal Thrust.**—The bracing referred to in Art. **36**, which connects the bents, is the **longitudinal**

**bracing,** and is designed to resist the longitudinal thrust that is exerted on the structure by a train of cars. The total thrust $T$ that acts on one tower is equal to $t\left(\frac{l}{2}+l'+\frac{l}{2}\right)$, or $t\ (l+l')$. It is transmitted to the viaduct at the level of the rails, and may be assumed to be concentrated at the top of one bent, or divided between the two; for all practical purposes, it may be assumed to be applied at one bent, as at $G$, Fig. 24 The amount to be resisted by the bracing on each side of the track is $\frac{T}{2}$; the stress in each diagonal is equal to $\frac{T}{2}\times\sec H$, tension, and in each horizontal strut it is $\frac{T}{2}$, compression. The stresses in the columns may be found by the method of moments. For example, the stress in $LI$ is $\frac{\frac{T}{2}\times l_1}{l'}$, tension, and the stress in $L'I'$ is $\frac{\frac{T}{2}(l_1+l_2)}{l'}$, compression.

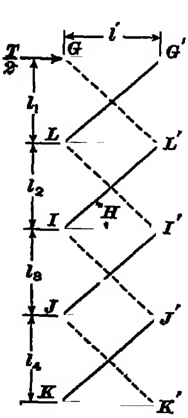

FIG 24

The distances $l_1$, $l_2$, $l_3$, etc. are not vertical heights, but actual distances measured along the columns.

EXAMPLE —In Fig 22, let $h_1' = 15$ feet, $h_2' = 4$ feet; $h_1, h_2, h_3, h_4, h_5$ = 9, 12, 15, 18, 21 feet, respectively (then $h$ = 75 feet); $b$ = 8 feet; slope of columns, 2 inches horizontal to 1 foot vertical, $P_1$ = 15,000 pounds, $P_2$ = 12,000 pounds, and let the wind pressure on the bent be 100 pounds per linear foot, vertical Determine (*a*) the vertical reaction at $F$, and the vertical and horizontal components of the reaction at $F'$, (*b*) the wind stress in the member $E'F'$, (*c*) the wind stress in the member $EF$, (*d*) the wind stress in the member $C'D$, and (*e*) the wind stress in the member $CC'$

SOLUTION —(*a*) The vertical reaction at $F$ is found by taking moments of all the wind forces about $F'$. The moments, in foot-pounds, are.

$$\begin{aligned}
&P_1(h+h_1') &&= 15{,}000\times(75+15) &&= 1\,3\,5\,0\,0\,0\,0\\
&P_2(h+h_2') &&= 12{,}000\times(75+4) &&= 9\,4\,8\,0\,0\,0\\
&P_3 h &&= \frac{9\times 100}{2}\times 75 &&= 3\,3\,7\,5\,0\\
&P_4(h-h_1) &&= \frac{21\times 100}{2}\times(75-9) &&= 6\,9\,3\,0\,0\\
&P_5(h-h_1-h_2) &&= \frac{27\times 100}{2}\times(75-9-12) &&= 7\,2\,9\,0\,0\\
&P_6(h_4+h_5) &&= \frac{33\times 100}{2}\times(21+18) &&= 6\,4\,3\,5\,0\\
&P_7 h_5 &&= \frac{39\times 100}{2}\times 21 &&= 4\,0\,9\,5\,0\\
&P_8 0 &&= \frac{21\times 100}{2}\times 0 &&= 0\\
& && && \overline{2\,5\,7\,9\,2\,5\,0}
\end{aligned}$$

The total moment about $F'$ is 2,579,250 ft.-lb. Each column slopes outwards $\frac{2}{12}\times 75 = 12\ 5$ ft , then,

$$b_8 = 12\ 5 + 8 + 12\ 5 = 33 \text{ ft}$$

The vertical reaction at $F$ is, therefore,

2,579,250 − 33 = 78,200 lb , acting downwards Ans.

Since the only vertical forces acting on the trestle bent for the wind loading are the vertical components of the reactions at $F$ and $F'$, they must be equal The vertical component of the reaction at $F'$ is, therefore, 78,200 lb. Ans

In Art **37**, it is stated that the entire horizontal load may be assumed to be carried to $F'$. Then, the horizontal component of the reaction at $F'$ is

$$15{,}000 + 12{,}000 + (75\times 100) = 34{,}500 \text{ lb} \quad \text{Ans}$$

(*b*) The stress in the member $E'F'$ can be found by taking moments about the intersection of $EF$ and $E'F$, that is, about point $F$. This is the same moment found in (*a*), 2,579,250 ft -lb.

$$\csc H = \frac{\sqrt{12^2+2^2}}{12} = 1\ 0138$$

Then, the stress in $E'F'$ is

$$\frac{2{,}579{,}250}{33}\times 1\ 0138 = +\,79{,}300 \text{ lb} \quad \text{Ans}$$

(*c*) The wind stress in the member $EF$ can be found by taking moments about the intersection of $E'F'$ and $E'F$, that is, about point $E'$. The moments, in foot-pounds, are

$$\begin{aligned}
&P_1(h_1+h_2+h_3+h_4+h_1') &&= 15{,}000\times 69 &&= 1\,0\,3\,5\,0\,0\,0\\
&P_2(h_1+h_2+h_3+h_4+h_2') &&= 12{,}000\times 58 &&= 6\,9\,6\,0\,0\,0\\
&P_3(h_1+h_2+h_3+h_4) &&= 450\times 54 &&= 2\,4\,3\,0\,0\\
&P_4(h_2+h_3+h_4) &&= 1{,}050\times 45 &&= 4\,7\,2\,5\,0\\
&P_5(h_3+h_4) &&= 1{,}350\times 33 &&= 4\,4\,5\,5\,0\\
&P_6\ h_4 &&= 1{,}650\times 18 &&= 2\,9\,7\,0\,0\\
& && && \overline{1\,8\,7\,6\,8\,0\,0}
\end{aligned}$$

The distance $b_4$ is

$$EE' = \left(2 \times \frac{54 \times 2}{12}\right) + 8 = 26 \text{ ft.}$$

Then, the stress in $EF$ is

$$\frac{1{,}876{,}800}{26} \times 1\ 0138 = -73{,}200 \text{ lb}$$

(*d*) The stress in $C'D$ is equal to the moment of $P_1$, $P_2$, $P_3$, $P_4$, and $P_5$ about $I$ divided by the perpendicular distance $i_3$ from $I$ to $DC'$ produced.

$$i = \frac{b}{2} \times \tan H = \frac{8}{2} \times 6 = 24 \text{ ft.}$$

| | | Foot-Pounds |
|---|---|---|
| $P_1(i - h_1')$ | $= 15{,}000 \times 9 =$ | 1 3 5 0 0 0 |
| $P_2(i - h_2')$ | $= 12{,}000 \times 20 =$ | 2 4 0 0 0 0 |
| $P_3\ i$ | $= 450 \times 24 =$ | 1 0 8 0 0 |
| $P_4(i + h_1)$ | $= 1{,}050 \times 33 =$ | 3 4 6 5 0 |
| $P_5(i + h_1 + h_2)$ | $= 1{,}350 \times 45 =$ | 6 0 7 5 0 |
| | | 4 8 1 2 0 0 |

According to Art **37**,

$$i_3 = \frac{b_3}{2} \times \sec H \sin\ (H - H_3)$$

In this equation,

$$b_3 = \left(2 \times \frac{36 \times 2}{12}\right) + 8 = 20 \text{ ft.}$$

$$\sec H = \frac{\sqrt{12^2 + 2^2}}{2} = 6\ 0828,$$

$$H = 80°\ 32',$$

$$\tan H_3 = \frac{h_3}{\frac{b_3}{2} + \frac{b_2}{2}} = \frac{2\,h_3}{b_3 + b_2} = \frac{2 \times 15}{20 + 15} = .85714$$

so that $H_3 = 40°\ 36'$. Then,

$$i_3 = \frac{20}{2} \times 6\ 0828 \times \sin\,(80°\ 32' - 40°\ 36') = 60\ 828 \times .64190 = 39\ 05 \text{ ft.}$$

The stress in $C'D$ is, therefore,

$$481{,}200 - 39\ 05 = -12{,}300 \text{ lb} \quad \text{Ans}$$

(*e*) The stress in $CC'$ is equal to the moment of $P_1$, $P_2$, $P_3$, $P_4$, and $P_5$ about $I$, divided by the perpendicular distance from $I$ to $CC'$, which is 45 ft The moment was found, in (*d*), to be 481,200 ft.-lb Then, the stress in $CC'$ is

$$481{,}200 \div 45 = +10{,}700 \text{ lb.} \quad \text{Ans.}$$

## EXAMPLES FOR PRACTICE

1. Suppose that the wind pressure $P_2$ on the trestle bent shown in Fig 22, when there is no live load on the trestle, is 20,000 pounds, the wind pressure on the bent is 150 pounds per foot vertical; and the

dimensions are as given in the example in Art 42. Find the vertical reaction at $F$ Ans 60,700 lb.

2 For the conditions stated in example 1, what is the horizontal component of the reaction at $F'$? Ans 31,250 lb.

3 For the same conditions as in example 1, what is the stress in the member $D'E'$? Ans $+$53,800 lb.

4 For the same conditions as in example 1, what is the stress in the member $C'D$? Ans. $-$ 14,300 lb

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